I KOLOKVIJUM, 10.4.2019.

GRUPA 1

1 a) 48 KUGLICA BIRAMO 35

$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{6}\ \underbrace{\qquad\qquad}_{29}$$

6 UNAPRED ODABRANIH (ZNA SE TAČNO KOJE SU)

PREOSTALIH 29 BIRAMO OD PREOSTALIH 48-6 =42

Rešenje: $\binom{42}{29}$ (JER NIJE BITAN REDOSLED)

b) 8 ISTIH GUMICA
5 RAZL. BOJICA

NIKOJE 2 BOJICE NISU SUSEDNE

POREDAMO GUMICE (1 NAČIN, JER SU ISTE): $X\ \underline{G1}\ X\ \underline{G2}\ X\ \underline{G3}\ X\ \underline{G4}\ X\ \underline{G5}\ X\ \underline{G6}\ X\ \underline{G7}\ X\ \underline{G8}\ X$

Bojice ne smeju biti jedna do druge pa njih smeštamo na nekim od mesta obeleženim sa X

Rešenje: $\binom{9}{5} \cdot 5!$

$9 \to$ BR. MESTA

$5 \to$ BR. BOJICA

$5! \to$ JER SU RAZLIČITE

② $$\sum_{k=0}^{n} \frac{1}{k^2+3k+2}\binom{n}{k} = \frac{2^{n+2}-n-3}{(n+2)(n+1)}$$

$$\sum_{k=0}^{n} \frac{1}{(k+2)(k+1)}\cdot\binom{n}{k} = \sum_{k=0}^{n} \frac{n!}{(k+2)!\,(n-k)!} =$$

$$= \frac{1}{(n+1)(n+2)}\cdot \sum_{k=0}^{n} \frac{(n+2)(n+1)\cdot n!}{(k+2)!\,(n-k)!} =$$

$$= \frac{1}{(n+1)(n+2)}\cdot \sum_{k=0}^{n} \frac{(n+2)!}{(k+2)!\,(n-k)!} = \frac{1}{(n+1)(n+2)}\cdot \sum_{k=0}^{n} \binom{n+2}{k+2}$$

SMANJUJEMO INDEKS U SUMI ZA 2

POVEĆAVAMO INDEKS ZA 2 U GRANICAMA

$$\frac{1}{(n+1)(n+2)}\cdot \sum_{\boxed{k=2}}^{n+2} \binom{n+2}{k}$$

$$= \frac{1}{(n+1)(n+2)}\cdot\left(\sum_{\boxed{k=0}}^{n+2}\binom{n+2}{k} - \binom{n+2}{1} - \binom{n+2}{0}\right)$$

$$= \frac{1}{(n+1)(n+2)}\left(2^{n+2} - (n+2) - 1\right)$$

$$= \frac{1}{(n+1)(n+2)}\left(2^{n+2} - n - 3\right)$$

3 a) $\left(0, 1, \frac{1}{2}, 0, \frac{1}{8}, 0, \frac{1}{32}, \dots\right)$

$$A(x) = x + \frac{1}{2}x^2 + \frac{1}{8}x^4 + \frac{1}{32}x^6 + \dots$$

$$= x\left(1 + \frac{1}{2}x + \frac{1}{8}x^3 + \frac{1}{32}x^5 + \dots\right)$$

$$= x\left(1 + \frac{1}{2^1}x + \frac{1}{2^3}x^3 + \frac{1}{2^5}x^5 + \dots\right)$$

$$= x \cdot \sum_{n=0,1,3,5,\dots}^{\infty} \frac{1}{2^n} x^n = x \cdot \left(\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n - \sum_{n=2,4,6,\dots}^{\infty}\left(\frac{x}{2}\right)^n\right)$$

DOPUNA

$$\overset{\frac{x}{2}=t}{=} 2t \cdot \left(\sum_{n=0}^{\infty} t^n + \sum_{n=2,4,6,\dots}^{\infty} t^n\right) \rightarrow \boxed{(*)}$$

$$\overset{(*)}{=} 2t\left(\frac{1}{1-t} + \frac{t^2}{1-t^2}\right) = 2t\,\frac{1+t+t^2}{1-t^2} =$$

$$= x \cdot \frac{1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2}{1 - \left(\frac{x}{2}\right)^2}.$$

$\boxed{(*)}$: $t^2 + t^4 + t^6 + \dots = t^2 \cdot (1 + t^2 + t^4 + \dots) \underline{\underline{t^2 = u}}$

$$= u \cdot (1 + u + u^2 + u^3 + \dots) = u \cdot \frac{1}{1-u} = t^2 \cdot \frac{1}{1-t^2}$$

b) $\left(0, 2, \frac{3}{2}, 0, \frac{5}{8}, 0, \frac{7}{32}, \dots\right)$

$$B(x) = 2x + \frac{3}{2}x^2 + \frac{5}{8}x^4 + \frac{7}{32}x^6 + \dots$$

$$= (x^2)' + \left(\frac{1}{2}x^3\right)' + \left(\frac{1}{8}x^5\right)' + \left(\frac{1}{32}x^7\right)' + \dots$$

$$= \left(x^2 + \frac{1}{2}x^3 + \frac{1}{8}x^5 + \frac{1}{32}x^7 + \dots\right)'$$

$$= \Big(x \cdot \underbrace{\left(x + \frac{1}{2}x^2 + \frac{1}{8}x^4 + \frac{1}{32}x^6 + \dots\right)}_{A(x)\ (\text{POD } a))}\Big)'$$

$$= (x \cdot A(x))' = \left(x^2 \cdot \frac{1 + \frac{x}{2} + \left(\frac{x}{2}\right)^2}{1 - \left(\frac{x}{2}\right)^2}\right)'$$

$$= \left(\frac{4x^2 + 2x^3 + x^4}{4 - x^2}\right)' = \frac{-2x^5 - 6x^4 + 16x^3 + 28x^2 + 32x}{(4 - x^2)^2}$$

$$\overbrace{a_{n+3}-6a_{n+2}+9a_{n+1}-4a_n}^{L}=36\cdot 4^n,$$

4) $a_0=2,\ a_1=10,\ a_2=51$

HOMOGENA: $L=0$

KARAKTERISTIČNA J-NA: $t^3-6t^2+9t-4=0$

$t_1=1$ rešenje

$(t^3-6t^2+9t-4):(t-1)=t^2-5t+4$ → $t_2=1$, $t_3=4$

OPŠTE REŠENJE HOMOGENE: $\boxed{a_H=C_1+C_2\cdot n+C_3\cdot 4^n}$

$f(n)=36\cdot 4^n$; $t=4$ rešenje karakt. j-ne? $\boxed{\text{DA}}\longrightarrow S=1$ (JEDNOSTRUKO)

36 → POLINOM ST. 0

$a_p=n^1\cdot 4^n\cdot A$

$$A(n+3)\cdot 4^{n+3}-6\cdot A(n+2)\cdot 4^{n+2}+9A(n+1)\cdot 4^{n+1}-4An\cdot 4^n=36\cdot 4^n \quad /:4^{n+1}$$

$$A(n+3)\cdot 16-24A(n+2)+9A(n+1)-An=9$$

OVA J-NA MORA DA VAŽI ZA SVAKO $n\in\mathbb{N}$, PA SPECIJALNO I ZA $n=0$, NPR.

$n=0$: $48A-48A+9A=9 \Rightarrow \boxed{A=1}$

OPŠTE REŠENJE NEHOMOGENE:

$$a_n=a_H+a_p=C_1+C_2\cdot n+C_3\cdot 4^n+n\cdot 4^n$$

$a_0=2\overset{n=0}{=}C_1+C_3$

$a_1=10=C_1+C_2+4C_3+4$

$a_2=51=C_1+2C_2+16C_3+32$

$\Rightarrow$ $C_1=1$, $C_2=1$, $C_3=1$