2017 Jan 1

1) $\int_0^1 \frac{dx}{x^3+x^2+7x+7} = \left[\begin{array}{l} x^3+x^2+7x+7 = x^2(x+1)+7(x+1) = (x^2+7)(x+1) \\ 1 = (Ax+B)(x+1) + C(x^2+7) \end{array} \right.$

$A+C=0$
$A+B+7C=0$
$B=1$

$A+C=0$
$A+7C=-1$
$6C=-1$
$C=-1/6$
$A=1/6$

$= \frac{1}{6}\int_0^1 \left(\frac{x+6}{x^2+7} - \frac{dx}{x+1} \right) = 1/6 \int_0^1 \left(\frac{x}{x^2+7} + \frac{6}{x^2+7} - \frac{1}{x+1} \right) dx$

$= \left[\begin{array}{l} t = x^2+7 \\ dt = 2x \end{array} \right] = 1/6 \int_0^1 \left(\frac{dt}{2t} \right) + \frac{1}{\sqrt{7}} \operatorname{arctg} \frac{x}{\sqrt{7}} \Big|_0^1 - 1/6 \ln(x+1) \Big|_0^1$

$= \frac{1}{12} \ln(x^2+7) \Big|_0^1 + \frac{1}{\sqrt{7}} \operatorname{arctg} \frac{1}{\sqrt{7}} - 1/6 \ln 2$

b) $\int x \arccos x \, dx = \left[\begin{array}{ll} u = \arccos x & dv = x \\ du = -\frac{1}{\sqrt{1-x^2}} & v = \frac{x^2}{2} \end{array} \right] = \frac{x^2}{2} \arccos x + \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} = \left[\begin{array}{l} x = \sin t \\ dx = \cos t \, dt \\ t = \arcsin x \\ dt = \end{array} \right.$

$\frac{x^2}{2} \arccos x + \frac{1}{2} \int \frac{\sin^2 t}{\cos t} \cdot \cos t \, dt = \frac{x^2}{2} \arccos x + 1/2 \int \sin^2 t$

$= \frac{x^2}{2} \arccos x + 1/2 \cdot \left(1/2 \int dt - \frac{\cos t \sin t}{2} \right)$

$= \frac{x^2}{2} \arccos x + \frac{1}{4} \arcsin x - 1/4 \, x\sqrt{1-x^2}$

c) $\int_0^{\pi/2} \operatorname{tg}(2x) dx = \int_0^{\pi/2} \frac{\sin 2x}{\cos 2x} dx = \left[\begin{array}{l} u = 2x \\ du = 2dx \end{array} \right] = \frac{1}{2} \int_0^{\pi/2} \frac{\sin u}{\cos u} = \left[\begin{array}{l} t = \cos u \\ dt = -\sin u \end{array} \right.$

$= -1/2 \int_0^{\pi/2} \frac{dt}{t} = -1/2 \ln(\cos 2x) \Big|_0^{\pi/2} = -\frac{1}{2} \left(\ln(\cos 2x) \Big|_0^{\pi/4} + \ln|\cos 2x| \Big|_{\pi/4}^{\pi/2} \right)$

divergira u $\pi/4$ → isto

2. a) $\sum_{n=1}^{\infty} \left(\cos(n\pi) \cdot \ln\left(\frac{n^2+n+\pi}{n^2+\pi} \right) \right) = \sum_{n=1}^{\infty} (-1)^n \cdot \ln\left(1 + \frac{n}{n^2+\pi}\right)$

$|a_n| = \ln\left(1+\frac{n}{n^2+\pi}\right) \sim \frac{n}{n^2+\pi} \sim \frac{1}{n}$ divergira apsolutno pa ne konv.

Uslovno $\frac{n^2+\pi}{n^2+n+\pi} \cdot \frac{(2n+1)(n^2+\pi) - 2n(n^2+n+\pi)}{(n^2+\pi)^2} = \frac{\pi + 2n\pi + 2n\pi - n^2}{\dots} = -n^2 + 4n\pi + \pi$

divergira

b) $\sum_{n=19}^{\infty} \sqrt{n} \sin \frac{1}{n^2}$ | $\sqrt{n} \sin \frac{1}{n^2} > 0$ $\quad \frac{1}{2\sqrt{n}} \sin \frac{1}{n^2} + 2\frac{1}{n^3} \cdot \sqrt{n} \cos \frac{1}{n^2} > 0$ ↗

$\sqrt{n} \sin \frac{1}{n^2} \sim \sqrt{n} \cdot \frac{1}{n^2} = \frac{1}{n^{3/2}}$ $\quad \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} < +\infty \xrightarrow[\text{kriterijum}]{\text{Poredbeni}}$ Konvergira

c) $\sum_{n=1}^{\infty} \frac{1}{2^n+n^2(-1)^n}$

$\frac{1}{|2^n+n^2(-1)^n|} \sim \frac{1}{2^n}$ $\qquad \lim_{n\to\infty} \sqrt[n]{\frac{1}{2^n}} = \frac{1}{2} < 1$ Konvergira

3. $\int_1^4 \operatorname{arctg}\sqrt{\sqrt{x}-1}\,dx = \left[\begin{matrix} \sqrt{x}-1 = t^2 & x=(t^2+1)^2 \\ \frac{dt}{2\sqrt{x}} = 2t\,dt & dx = 2(t^2+1)\cdot 2t \end{matrix} \right] = \int_1^4 \operatorname{arctg} t \cdot 4t(t^2+1)$

$= \left[\begin{matrix} u = \operatorname{arctg} t & dv = 4t(t^2+1) \\ du = -\frac{1}{t^2+1} & v = (t^2+1)^2 \end{matrix} \right] = (t^2+1)^2 \operatorname{arctg} t \Big|_1^4 + \int_1^4 (t^2+1)\,dt =$

$= (\sqrt{x}-1+1)^2 \operatorname{arctg}\sqrt{\sqrt{x}-1}\Big|_1^4 + \frac{1}{3}\left(\sqrt{\sqrt{x}-1}\right)^3\Big|_1^4 + \sqrt{\sqrt{x}-1}\Big|_1^4 =$

$= 9\pi/4 + 1/3 + 1 = 9\pi/4 + 4/3$

4. $f(x) = \operatorname{ch} x$ na $(-\pi, \pi)$

$\rightarrow 2\operatorname{sh}\pi$ $\qquad$ parna a ch ne utiče $=0$

$S_f(x) = \frac{1}{\pi}\int_{-\pi}^{\pi} \operatorname{ch} x\,dx + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \operatorname{ch} x \cdot \cos(nx) \Rightarrow I = \int_{-\pi}^{\pi} \underbrace{\operatorname{ch} x}_{=dv} \underbrace{\cos(nx)}_{=u} dx = -\operatorname{sh} x \cdot \cos(nx) + \int_{-\pi}^{\pi} n\sin(nx)\cdot \operatorname{sh} x\,dx$

$= -\operatorname{sh} x \cdot \cos(nx) + \int_{-\pi}^{\pi} \underbrace{n\sin(nx)}_{=u} \underbrace{\operatorname{sh} x}_{=dv} = -\operatorname{sh} x\cos(nx) + \underbrace{n\sin(nx)\cdot \operatorname{ch} x}_{0} - n\underbrace{\int_{-\pi}^{\pi} \operatorname{ch} x\, n\cos(nx)}_{I}$

$(1+n^2) I = -\operatorname{sh} x\cos(nx) \Big|_{-\pi}^{\pi}$

$\boxed{I = 2\frac{(-1)^n}{(n^2+1)} \operatorname{sh} x}$

$S_f(x) = \frac{2}{\pi}\operatorname{sh}\pi + \sum_{n=1}^{\infty} 2\frac{(-1)^n}{n^2+1} \operatorname{sh} x \cos(nx)$

$S_f(\pi) = \frac{2}{\pi}\operatorname{sh}\pi + \sum_{n=1}^{\infty} \frac{2\operatorname{sh}\pi}{n^2+1} \Rightarrow \frac{\operatorname{ch}\pi}{2\operatorname{sh}\pi} = \frac{1}{\pi} + \sum_{n=1}^{\infty} \frac{1}{n^2+1}$

$\frac{e^{\pi}+e^{-\pi}}{2(e^{\pi}-e^{-\pi})} - \frac{1}{\pi} = \sum_{n=1}^{\infty} \frac{1}{n^2+1}$

2017 Feb

1. a) $\int_0^1 x^3 \operatorname{arctg}\frac{1}{x}\,dx = \left[\begin{array}{l|l} \operatorname{arctg}\frac{1}{x} = u & v = \frac{1}{4}\cdot x^4 \\ \frac{1}{1+\frac{1}{x^2}}\cdot(-1)\cdot\frac{1}{x^2} = du & dv = x^3 \end{array}\right] =$

$= \operatorname{arctg}\frac{1}{x}\cdot\frac{1}{4}\cdot x^4\Big|_0^1 + \frac{1}{4}\int_0^1 \frac{x^4\cdot x^2}{(1+x^2)x^2} = \operatorname{arctg}\frac{1}{x}\cdot\frac{1}{4}x^4\Big|_0^1 + \frac{1}{4}\int_0^1 \frac{x^4+1-1}{1+x^2}$ $= (x^2-1)(x^2+1)+1$

$= \operatorname{arctg}\frac{1}{x}\cdot\frac{1}{4}\cdot x^4\Big|_0^1 + \frac{1}{4}\int_0^1\left(x^2-1+\frac{1}{1+x^2}\right) = \frac{\pi}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{3}x^3\Big|_0^1 - \frac{1}{4}x\Big|_0^1 + \frac{1}{4}\operatorname{arctg}x\Big|_0^1 =$

$= \pi/16 + \frac{1}{12} - 1/4 + \pi/16 = \pi/8 - 1/6$

b) $\int_0^e \frac{e^x+1}{e^x+x}\,dx = \left[\begin{array}{l} e^x+x = t \\ e^x+1 = dt \end{array}\right] = \int_0^e \frac{dt}{t} = \ln(e^x+x)\Big|_0^e = \ln(e^e+e)$

2. a) $\sum_{n=1}^{\infty} e^{-n^2} n!$ $\quad a_n > 0$ $\quad \lim_{n\to\infty}\frac{\cancel{e^{-n^2}}\cdot e^{-2n}\cdot e^{-1}\cdot\cancel{n!}\cdot n+1}{\cancel{e^{-n^2}}\cdot\cancel{n!}} = \lim_{n\to\infty}\frac{n+1}{e^{2n}\cdot e} \overset{n \ll a^n}{=} 0$

$a_n = |a_n|$ Konv.

b) $\sum_{n=1}^{\infty}\frac{1}{2^{\ln n}}$ $\quad a_n = |a_n|$ $\quad \frac{1}{2^{\ln n}} = \frac{1}{e^{\ln n \ln 2}} = \frac{1}{n^{\ln 2}}$ $\quad \ln 2 < \ln e$

$\ln 2 < 1 \Rightarrow$ divergira

c) $\sum_{n=1}^{\infty}(-1)^n\left(e^{\frac{1}{n-2}} - \cos\frac{n+1}{n}\right)$ $\quad \frac{e^x-1}{x}$

$\left|e^{\frac{1}{n-2}} - \cos\frac{n+1}{n}\right|$ $\quad \lim \frac{1}{n-2}\cdot\frac{e^{\frac{1}{n-2}}-1}{\frac{1}{n-2}}+1 - \cos\left(1+\frac{1}{n}\right)^{\frac{n}{n}} =$

$= \lim_{n\to\infty} \underset{\to 0}{\frac{1}{n-2}} + 1 - \cos \underset{\to\cos 1}{e^{1/n}} = |1-\cos 1| > 0$ divergira

3. $f_n(x) = \left(1-\cos\left(\frac{\pi x}{2}\right)\right)^n$

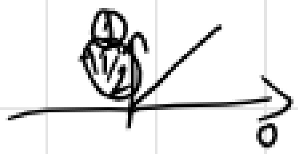
a) $[0,1]$

$\lim_{n\to\infty}\left(1-\cos\left(\frac{\pi x}{2}\right)\right)^n = \begin{cases} 1, & x=1 \\ 0, & x\in[0,1) \end{cases}$ ne konvergira ravnomjerno

b) $[0,\frac{1}{2}]$ $\quad x\in[0,\frac{1}{2}] \Rightarrow \frac{\pi x}{2}\in[0,\frac{\pi}{4}]$ $\quad \cos\left(\frac{\pi x}{2}\right)\in\left[\frac{\sqrt{2}}{2},1\right]$ $\quad 1-\cos\frac{\pi x}{2}\in\left[0,\frac{2-\sqrt{2}}{2}\right]$

$\lim_{n\to\infty}\left(1-\cos\frac{\pi x}{2}\right)^n = 0 = f(x)$ $\quad \lim_{n\to\infty}\sup_{0\le x\le 1/2}\left|\left(1-\cos\frac{\pi x}{2}\right)^n - 0\right| = \lim_{n\to\infty}\Big(\underbrace{1-\frac{\sqrt{2}}{2}}_{<1}\Big)^n = 0$

RK.



4. $f(x) = x\,\mathrm{sgn}(\sin 2x)$, $x \in (0, \pi)$

$$\frac{a_0}{\pi 2} + \frac{1}{\pi}\sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx$$

$$a_0 = \int_{-\pi}^{\pi} x\,\mathrm{sgn}(\sin 2x)\,dx = 2\int_0^{\pi} x\,\mathrm{sgn}(\sin 2x) = 2\left(\int_0^{\pi/2} x - \int_{\pi/2}^{\pi} x\right) = x^2\Big|_0^{\pi/2} - x^2\Big|_{\pi/2}^{\pi} = -\pi^2/2$$

$$a_n' = \int_{-\pi}^{\pi} x\,\mathrm{sg}(\sin 2x)\cos(nx)\,dx = 2\int_0^{\pi} x\,\mathrm{sgn}\ldots\cos(nx) = 2\left(\int_0^{\pi/2} x\cos(nx) - \int_{\pi/2}^{\pi} x\cos(nx)\right)$$

$u = x \quad v = \frac{1}{n}\sin nx$

$dv = \cos(nx)$

$$= 2\left(\frac{x}{n}\sin(nx)\Big|_0^{\pi/2} - \frac{x}{n}\sin nx\Big|^{\pi} - \frac{1}{n}\int \sin nx + \frac{1}{n}\int \sin nx\right)$$

$$= 2\left(\frac{\pi}{n}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2}\cos nx\Big|_0^{\pi/2} - \frac{1}{n^2}\cos nx\Big|_{\pi/2}^{\pi}\right) = 2\left(\frac{\pi}{n}\sin\left(\frac{n\pi}{2}\right) + \frac{1}{n^2}\cos\left(\frac{n\pi}{2}\right) - \frac{1}{n^2} - \frac{1}{n^2}\cos \pi n + \frac{1}{n^2}\cos\left(\frac{\pi n}{2}\right)\right)$$

$$= 2\left(\frac{\pi}{n}\sin\left(\frac{n\pi}{2}\right) + \cos\left(\frac{\pi n}{2}\right)\frac{2}{n^2} - \frac{1}{n^2}\left(1 + (-1)^n\right)\right)$$

$$b_n = \int_{-\pi}^{\pi} x\,\mathrm{sgn}(\sin 2x)\cdot\sin(nx) \qquad f(-x) = -x\,\mathrm{sgn}(\sin 2x)\cdot\sin(nx) = 0$$

$$S_n(x) = -\frac{\pi}{4} + \frac{2}{\pi}\sum_{n=1}\left(\frac{(-1)^{n+1} - 1}{n^2} + \frac{\pi}{n}\sin\left(\frac{n\pi}{2}\right) + \frac{2}{n^2}\cos\left(\frac{\pi n}{2}\right)\right)\cos(nx)$$

b) $x = \pi/2$

$$2\sum_{n=0}^{\infty}\left(\frac{1}{(4n+1)^2} + \frac{1}{(4n+3)^2}\right) = \pi\sum_{n=0}^{\infty}\left(\frac{1}{4n+1} - \frac{1}{4n+3}\right)$$

$$S_n\left(\frac{\pi}{2}\right) = -\frac{\pi}{4} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1} - 1}{n^2} + \frac{\pi}{\ldots}\ldots\right)\cdot\cos\left(\frac{n\pi}{2}\right) =$$

$$\frac{\pi}{2}\cdot\mathrm{sgn}(\sin\pi) > 0 = -\frac{\pi}{4} + \frac{2}{\pi}\sum$$

$$\cos\left(\frac{\pi n}{2}\right) = \begin{cases} 0 & n = 4k+1,\ 4k+3 \\ 1 & n = 4k \\ -1 & n = 4k+2 \end{cases}$$

$$0 = -\frac{\pi}{4} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1} - 1}{n^2} + \frac{\pi}{n}\sin\left(\frac{n\pi}{2}\right) + \frac{2}{n^2}\cos\left(\frac{\pi n}{2}\right)\right)\cos\left(\frac{\pi n}{2}\right)$$

$$\sin\left(\frac{\pi n}{2}\right) = \begin{cases} 0 & n = 4k+2 \text{ ili } 0 \\ 1 & n = 4k+1 \\ -1 & n = 4+3 \end{cases}$$

$$0 = -\frac{\pi}{4} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1} - 1}{16n^2} + \frac{2}{16n^2}\right) - \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1} - 1}{(4n+1)^2} - \frac{2}{4(2+1)^2}\right)$$

. . .

2017 Jun 1

1) a) $\int x \ln(1+x^2) = \left[\begin{matrix} 1+x^2=t \\ 2x\,dx=dt \end{matrix} \right] = \frac{1}{2}\int \ln(t)\cdot dt = \frac{1}{2}\left(\ln(t)\cdot t - \int \frac{t}{t}\right)$

$= \frac{1}{2}(t\ln t - t)$

b) $\int_0^{\ln 5} \frac{e^x\sqrt{e^x-1}}{e^x+3} = \left[\begin{matrix} t=e^x \\ x=\ln t \\ dx=\frac{1}{t}dt \end{matrix} \right] = \int_1^5 \frac{\sqrt{t-1}}{t+3} = \left[\begin{matrix} t-1=u^2 \\ dt=2u\,du \end{matrix} \right] = \int_0^2 \frac{2u^2+8-8}{u^2+4}$

$= \int_0^2 \left(2 - 8\frac{1}{u^2+2^2}\right) = 2u - \frac{8}{2}\cdot \operatorname{arctg}\frac{u}{2}\Big|_0^2 = 4 - 4\cdot \frac{\pi}{4} = 4-\pi$

2 a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^{1/n^2}}$ $\quad |a_n| = \frac{1}{e^{1/n^2}} \sim \frac{1}{1+\frac{1}{n^2}} \sim \frac{1}{n^{-2}}$ divergira

$\lim \frac{1}{e^{1/n^2}} = \infty$ Div

b) $\frac{(-1)^n}{\sqrt[3]{n} + (-1)^{\frac{3^n-1}{2}}}$ $\quad 2 \mid \frac{5^n-1}{2}$ $\quad \frac{(5\cdot 5^n - 1 - 4) + 4}{2} = 5\cdot$ deljivo sa 2 $+ 2$ $\to$ deljivo sa 2

$\sim \frac{(-1)^n}{n^{1/3}} \sim \frac{1}{n^{1/3}}$ div aps.

$\frac{1}{n^{1/3}} > 0 \quad -\frac{1}{3}\cdot n^{-4/3} < 0$ opada $\Rightarrow$ Konv. uslovno

c) $\sum_{n=1}^{\infty}\left(\sin\left(\frac{1}{n}\right) - \ln\left(1+\frac{1}{n}\right)\right) \sim \frac{1}{n} - \frac{1}{6n^3} - \frac{1}{n} - \frac{1}{2n^2} = \frac{1}{2n^2}$ konv suroz

3. $\sum_{n=1}^{\infty} \frac{(n+1)^2((2n+1)!!)^p}{n!}\cdot x^n$ $\quad x_0 = 0$

$\frac{\cancel{(n+1)^2}((\cancel{2n+1})!!)^p(n+1)}{(n+2)^2(2n+3)^p(\cancel{(2n+1)!!})^p} = \frac{n+1}{(2n+3)^p} = \frac{n+1}{(2n)^p(1+\frac{3}{2n})^p} = \frac{1}{2^p\cdot n^{p-1}}$

Za $p>1$ $\{0\}$ ili $\emptyset$

Za $p=1$ $\left(-\frac{1}{2}, \frac{1}{2}\right)$ $\quad$ Za $p<1$ $(-\infty, +\infty)$

$p>1 \quad x=0 \Rightarrow \boxed{0}$ $\quad x=-\frac{1}{2}$ $\quad \frac{(-1)^n(n+1)^2(2n+1)!!}{n!}$ $\quad \frac{(2n+3)(n+2)^2}{(n+1)(n+1)^2}\ \boxed{>}\ 1$

$f_n \nearrow$ $\quad \frac{(n+1)^2(-1)^n(2n+1)!!}{\sqrt{2\pi n}\ e^{-n}\cdot n^n}$ $\quad 2n^3 + \cancel{8n^2} + \cancel{8n} + 3n^2 + \cancel{12n} + 12 - \cancel{n^3} - \cancel{3n^2} - 3n - 1 \ \square\ 0$

$n^3 + 8n^2 + 17n + 11 > 0$

Nemam blage $-\frac{1}{2}$?? ali def konv za $\frac{1}{2}$

$p=0 \quad \sum_{n=1}^{\infty}\frac{(n+1)^2}{n!}\cdot x^n$ Nešto sa e^x

4 a) $y=0,\ x=\frac{\pi}{4},\ y=tg\,x$ površina

$$\left|\int_0^{\pi/4} tg\,x\,dx\right| = \left|-\int_0^{\pi/4} \frac{-\sin x}{\cos x}dx\right| = \begin{bmatrix}\cos x = t\\ -\sin x = dt\end{bmatrix} = \left|\int_{\frac{\sqrt2}{2}}^{1} \frac{1}{t}\right| = \left|\ln 1 - \ln\frac{\sqrt2}{2}\right| = \frac{\ln\sqrt2}{2}$$

b) $$\pi\int_0^{\pi/4} tg^2x\,dx = \pi\int_0^{\pi/4}\frac{1-\cos^2x}{\cos^2x} = \pi\int_0^{\pi/4}\left(\frac{1}{\cos^2x}-1\right) = \pi\left(tg\,x - x\right)\Big|_0^{\pi/4} = \boxed{\pi - \frac{\pi^2}{4}}$$

2018 jan 1

1. a) $$\int_0^{\frac{9\pi^2}{4}}|\sin\sqrt{x}|dx = \int_0^{\pi^2}\sin\sqrt{x} - \int_{\pi^2}^{\frac{9\pi^2}{4}}\sin\sqrt{x} \begin{bmatrix}x=t^2\\ dx=2t\end{bmatrix} = \int_0^{\pi}2t\cdot\sin t - \int_{\pi}^{\frac{3\pi}{2}}2t\sin t = 2t\cos t\Big|_{\pi}^{\frac{3\pi}{2}} - 2t\cos t\Big|_0^{\pi}$$

$$+2\int_0^{\pi}\cos t - 2\int_{\pi}^{\frac{3\pi}{2}}\cos t = +2\pi + 2\pi - 2\cdot(-1) = \boxed{4\pi+2}$$

b) $$\int\frac{dx}{4\sin^2x+\cos^2x} = \begin{bmatrix}t = tg\,x & dx=\frac{1}{1+t^2}\\ \sin^2x = \frac{t^2}{1+t^2}\\ \cos^2x = \frac{1}{1+t^2}\end{bmatrix} = \int\frac{dt}{(1+t^2)\left(\frac{4t^2+1}{1+t^2}\right)} = \int\frac{dt}{(2t)^2+1} = \frac{1}{2}\operatorname{arctg}2t$$
$$= \frac{1}{2}\operatorname{arctg}(2\cdot tg\,x)$$

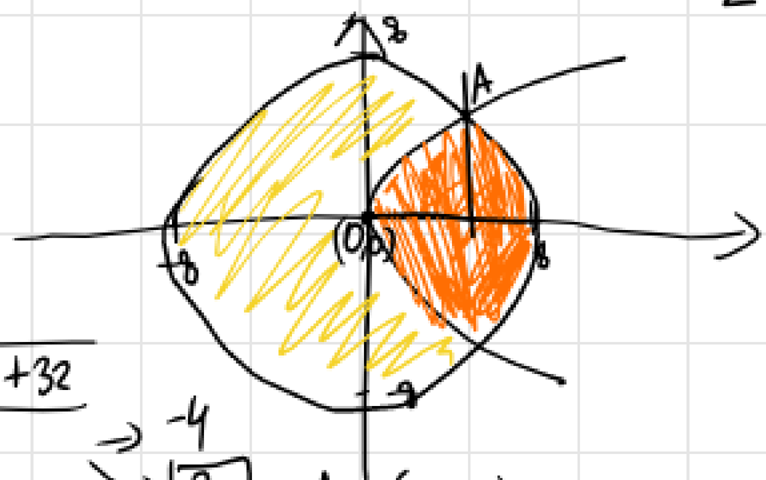
2. $x^2+y^2\le 8$ i $y^2=2x$

Narandžasto: $A = \cap$

$x^2+2x-8=0$

$x_{1/2} = \frac{-2\pm\sqrt{4+32}}{2} \to -4,\ \boxed{2}$

$A=(2,2)$



$x^{1/2+1} = 3/2$

$$\sqrt2\int_0^2\sqrt{x}\,dx + \int_2^{8}\sqrt{8-x^2}\,dx = \frac{2\sqrt2}{3}x^{3/2}\Big|_0^2 + \sqrt8\int_2^8\sqrt{1-\left(\frac{x}{2\sqrt2}\right)^2} = \frac{8}{3} + \begin{bmatrix}\frac{x}{2\sqrt2}=\sin t\\ dx = 2\sqrt2\cos t\,dt\end{bmatrix} =$$

$$= \frac{8}{3} + 8\int_{\pi/4}^{\pi/2}\cos^2t = \frac{8}{3} + 8\left(\frac{\cos t\sin t}{2}+\frac{1}{2}\cdot t\right)\Big|_{\pi/4}^{\pi/2} = \frac{8}{3} + 8\cdot\left(-\frac{1}{4}+\frac{\pi}{8}\right) = \boxed{\frac{8}{3}-\frac{8}{4}+\pi}$$

$$8\cdot\pi \longrightarrow \frac{8/3-2+\pi}{7\pi+2-8/3} = \frac{\text{Narandžasto}}{\text{žuto}}$$

$$\pi\int_0^2 x\,dx + \int_2^8|8-x^2| = 2\pi + \int_2^{\sqrt8}(8-x^2) + \int_{\sqrt8}^{8}(x^2-8) = 2\pi + 8\sqrt8 - 16 - \frac{1}{3}8\sqrt8 + \frac{1}{3}8 + \frac{64}{3} - \frac{8\sqrt8}{3} - 64 + 8\sqrt8$$
$$= \pi\left(-14+16\sqrt8 - \frac{16\sqrt8}{3} + 24 - 64\right) = \pi\left(\frac{16\sqrt8\cdot2}{3} - 54\right)$$

3.

a) $\sum_{n=1}^{\infty}\left(\sqrt[4]{n+1}-\sqrt[4]{n}\right) \longrightarrow \dfrac{\sqrt{n+1}-\sqrt{n}}{\left(\sqrt[4]{n+1}+\sqrt[4]{n}\right)} = \dfrac{n+1-n}{\left(\sqrt[4]{n+1}+\sqrt[4]{n}\right)\left(\sqrt[2]{n+1}+\sqrt{n}\right)} \approx \dfrac{1}{2n^{1/4}\cdot 2n^{1/2}}$

$\sim \dfrac{1}{n^{3/4}}$ $\quad 3/4 < 1$ divergira

b) $\sum_{n=1}^{\infty}(-1)^{n-1} n^2 \sin\frac{\pi}{3^n}$ $\qquad \lim n^2\cdot\sin\frac{\pi}{3^n} = \frac{\pi n^2}{3^n} = 0$

$\lim \dfrac{n^2(1+\frac{1}{n})^2 \sin\frac{\pi}{3^n\cdot 3}}{\cancel{n^2}\cdot\sin\frac{\pi}{3^n}} = \dfrac{\frac{\pi}{3^{\cancel{n}}}\cdot\frac{1}{3}}{\pi/\cancel{3^n}} = 1/3 < 1$ Konv. aps.

pa i obično, pa ne uslovnom

c) $\sum_{n>2}\left(\dfrac{(-1)^n}{n^2}+\dfrac{1}{n+(-1)^n}\right)$

$\underbrace{\dfrac{(-1)^n(1+n)+n^2}{n^2(n+(-1)^n)}}$

$n^2-n-1=0$

$n_{1/2}=\dfrac{1\pm\sqrt{1+4}}{2} \nearrow \dfrac{1+\sqrt{5}}{2}\sim 2$

$n^2+n+1 \quad D<0$ uvek (+)

Uvek će biti +

$\underbrace{-\frac{1}{n^2}+\frac{1}{n+1}}_{\text{divergira}} < a_n \searrow$ divergira

4. a) $f_n(x)=nx^n(1-x)$ $\quad x\in[0,1]$

$f_n(x)=\begin{cases}0, & x\in[0,1)\\ 0, & x=1\end{cases}$ $\qquad \lim \quad = 0$

$\lim \ldots = 0$

b) $[0,1/2]$

$\lim_{n\to\infty}\frac{n}{2^n}\cdot\frac{1}{2}=0$ $\qquad \lim_{n\to\infty} f(x_0)=0$

$\lim_{n\to\infty}\sup_{[0,1/2]}\left|nx^n(1-x)\right| \Rightarrow$

$\underline{x^n+n^2x^{n-1}-x^{n+1}-n^2x^n-nx^n}$

$x^n(1-n-n^2)+n^2x^{n-1}-x^{n+1}$

$x^{n-1}\left(x+n^2-x^2-n^2x-nx\right)=0$

$x+n^2-x^2-n^2x-nx=0$

$x^2+x(n^2+n-1)-n^2=0$

$x_{1/2}=\dfrac{n^2+n-1\pm\sqrt{n^4+n^2+1+2n^3-2n^2-2n+4n^2}}{2} = \dfrac{n^2+n-1}{2}\pm\dfrac{\sqrt{n^4+2n^3+3n^2-2n+1}}{2}$

$\lim_{n\to\infty}\left|\ldots\right|>0$

$$\lim_{n\to\infty} \sup_{x\in[0,+\infty)} \left| \frac{n^{2024}\cdot x \cdot \sin(6nx)}{(8e^x)^n(nx+5)} \right| [x=0] = 0$$

b) $$\lim_{x\to\infty} \sum_{n=1}^{\infty} \frac{(-1)^n\, n^{2024}\, x\sin(6nx)}{(8e^x)^n\,(nx+5)} = \sum_{n=1}^{\infty} \frac{(-1)^n\, n^{2024}\, \sin(6nx)\cdot \cancel{x}}{(8e^x)^n\,(\underbrace{n+\frac{5}{x}}_{n})\cdot\cancel{x}} = \sum_{n=1}^{\infty} \frac{(-1)^n\cdot n^{2023}\sin(6nx)}{(8e^x)^n}$$

$$-1 \le \sin(6nx) \le 1 \quad \cdot(-1)^n$$

$$(-1)^{n+1} \le \sin(6nx) \le (-1)^n$$

$$0$$

$$= 0$$

5. $f(x) = 4-\cos\frac{x}{3}$ $\qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{9n^2-1}$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(4-\cos\frac{x}{3}\right)dx = \frac{1}{2\pi}\left(4x - \sin\frac{x}{3}\cdot 3\right)\Big|_{-\pi}^{\pi} = \frac{1}{2\pi}(8\pi) = 4$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(4-\cos\frac{x}{3}\right)\cos(nx)\,dx = \frac{1}{\pi}\left(\frac{4}{n}\underbrace{\sin(nx)\Big|_{-\pi}^{\pi}}_{0} - I\right)$$

$$I = \int_{-\pi}^{\pi} \underbrace{\cos\frac{x}{3}}_{dv}\cdot \underbrace{\cos(nx)}_{u} = 3\cos(nx)\cdot\sin\frac{x}{3} + 3\underbrace{\int \underbrace{\sin\frac{x}{3}}_{dv}\cdot n\underbrace{\sin(nx)}_{u}}_{I_2} =$$

$$I_2 = \int \underbrace{\sin\frac{x}{3}}_{dv}\cdot\underbrace{\sin(nx)}_{u} = -\sin(nx)\cos\frac{x}{3}\cdot 3n + \int\cos\frac{x}{3}\cdot 3\cos(nx)\cdot n =$$

$$I = 3n\cos(nx)\cdot\sin\frac{x}{3} + 3n\left(3n\,I - 3n\sin(nx)\cos\frac{x}{3}\right) = (1-9n^2)I = 3n\left(\cos(nx)\sin\frac{x}{3} - 3n\sin nx\cos\frac{x}{3}\right)$$

$$I = \frac{3n}{1-9n^2}\left(-\sin\frac{\pi}{3} - \sin\frac{\pi}{3}\right) = -\frac{6n}{1-9n^2}\cdot\frac{\sqrt3}{2} = \frac{3\sqrt3}{9n^2-1}\ \#\ \frac{1}{\pi}\ \#$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\underbrace{\left(4-\cos\frac{x}{3}\right)}_{\text{parna}}\underbrace{\sin(nx)}_{\text{neparna}} = 0$$

parna $\cdot$ neparna = neparna

$$Sf(x) = 4 + \sum_{n=1}^{\infty}\frac{3\sqrt3}{9n^2-1}\cdot\pi\cdot\cos(nx) = 4-\cos\left(\frac{x}{3}\right)$$

Hoćemo $\sum_{n=1}^{\infty}\frac{(-1)^n}{9n^2-1}$ $\qquad$ Nađimo $Sf(\pi) = 4-\frac12 = 4+\sum\frac{3\sqrt3\pi\cdot(-1)^n}{9n^2-1}$

$$\underbrace{-\frac12\cdot\frac{1}{3\sqrt3\pi}}_{} = \sum\frac{(-1)^n}{9n^2-1}$$

$$\boxed{-\frac{1}{6\sqrt3\pi}}$$

II tok

1. 2) $\int \sqrt[3]{3x - x^3} = \int x \sqrt[3]{\frac{3}{x^2} - 1} = \int x \sqrt[3]{\frac{3-x^2}{x^2}}$ $\begin{bmatrix} x^2 = t \\ 2x = dt \\ xdx = \frac{dt}{2} \end{bmatrix} = \frac{1}{2}\int \sqrt[3]{\frac{3-t}{t}}$ $\left| \begin{array}{l} \frac{3-t}{t} = u^3 \\ \frac{3}{t} - 1 = u^3 \\ \frac{3}{u^3+1} = t \end{array} \right.$

$\frac{-3u^2}{(u^3+1)^2} = dt$ $= -\frac{3}{2}\int \frac{u^3+1-1}{(u^3+1)^2} = -\frac{3}{2}\left(\int \frac{1}{u^3+1} - \frac{1}{(u^3+1)^2}\right) = -\frac{3}{2}\int\left(\frac{1/3}{u+1} + \frac{\frac{2u}{3} - 1/3}{u^2-u+1} + \dots\right)$

$= -\frac{1}{2}\left(\ln(u+1) + \ln(u^2-u+1)\right) + -\frac{3}{2}\int\left(\frac{2/9}{u+1} + \frac{1/9}{(u+1)^2} + \frac{-2u+3}{9(u^2-u+1)} + \frac{-u+1}{3(u^2-u+1)^2}\right)$ $u^2 - u + \frac{1}{4} + \frac{3}{4}$, $(u+\frac{1}{2})^2 + \frac{3}{4}$

$= -\frac{1}{2}\left(\ln(u^3+1)\right) - \frac{3}{2}\left(\frac{2}{9}\ln(u+1) - \frac{1}{9}\frac{1}{(u+1)}\right) + \frac{3}{2}\int\left(\frac{2u-1-2}{9(u^2-u+1)} + \frac{u-1-1}{6(u^2-u+1)^2}\right)$

$= -\frac{1}{2}\ln(u^3+1) - \frac{1}{3}\ln(u+1) + \frac{1}{6}\frac{1}{(u+1)} + \frac{3}{2\cdot 9}\ln(u^2-u+1) + \frac{3}{2}\left(\int -\frac{2}{9(u^2-u+1)} + \frac{2u-1}{6(u^2-u+1)^2} - \frac{1}{6(u^2-u+1)^2}\right)$

$= -\ln\left(\sqrt{u^3+1}\cdot\sqrt[3]{u+1}\right) + \frac{1}{6}\ln(u^2-u+1) + \frac{1}{6(u+1)} + \frac{1}{6(u^2-u+1)}$

$-\frac{1}{3}\cdot\frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2u+1}{\sqrt{3}} - 4\int\left((2u+1)^2+3\right)^{-2}$ $\begin{array}{l} t = (2u+1)^2 \\ u = \frac{\sqrt{t}-1}{2} \\ du = \frac{1}{4\sqrt{t}} \end{array}$ $-\int (t+3)^{-2} t^{-1/2}$ $t = y^2$ $dt = 2y$

$-2\int (y+3)^{-2}\cdot y^{-1}\cdot y' = -2\int \frac{1}{(y+3)^2}2 = +2\frac{1}{y+3}$

SAD SE TO SREDI (lol):

$y = 2u+1$ $u = \sqrt[3]{\frac{3-t}{t}} = \sqrt[3]{\frac{3-x^2}{x^2}}$

$I = -\ln\left(\sqrt{\frac{3}{x^2}}\cdot\sqrt[3]{\sqrt[3]{\frac{3-x^2}{x^2}} + 1}\right) + \frac{1}{6}\ln\left(\sqrt[3]{\left(\frac{3-x^2}{x^2}\right)^2} - \sqrt[3]{\frac{3-x^2}{x^2}} + 1\right) + \frac{1/6}{\sqrt[3]{\frac{3-x^2}{x^2}} + 1} + \frac{1/6}{\sqrt[3]{\left(\frac{3-x^2}{x^2}\right)^2} - \sqrt[3]{\frac{3-x^2}{x^2}} + 1}$

$-\frac{2}{3\sqrt{3}}\operatorname{arctg}\left(\frac{1}{\sqrt{3}}\left(2\sqrt[3]{\frac{3-x^2}{x^2}} + 1\right)\right) + 2\cdot\frac{1}{\left(2\sqrt[3]{\frac{3-x^2}{x^2}} + 3\right)}$

Verujem da dosta može da se skrati, ali ja to ne mogu

2. $f: [-1,1] \to R$ $f(x) = 1 - x^2$

$\ell(y) = \int_{-1}^{1} \sqrt{1 + (-2x)^2} = 2\int_0^1 \sqrt{1+4x^2} = 2\int_0^1 x\sqrt{4 + \frac{1}{x^2}}$ $\begin{array}{l} 4 + \frac{1}{x^2} = t^2 \\ \frac{1}{t^2-4} = x^2 \\ 2x\,dx = \frac{-2t}{(t^2-4)^2} \end{array}$ $=$

$= -\int_0^1 \frac{t^2}{(t^2-4)^2} = -\int_0^1 \left(\frac{-1/8}{t+2} + \frac{1/4}{(t+2)^2} + \frac{1/8}{(t-2)} + \frac{1/4}{(t-2)^2}\right)$

$= -\left(-\frac{1}{8}\ln(t+2) + \frac{1}{4}\frac{1}{t+2} + \frac{1}{8}\ln(t-2) + \frac{1}{4(t-2)}\right)\Big|_0^1 =$

$= -\left(-\frac{1}{8}\ln 3 + \frac{1}{12} + \frac{-1}{4} - \cancel{\frac{1}{8}\ln 2} + \cancel{\frac{1}{8}} - \cancel{\frac{1}{8}} + \cancel{\frac{1}{8}\ln 2}\right) = \boxed{\frac{1}{8}\ln 3 + \frac{1}{6}}$??? Bog te pitaj

b) $V = \pi\int_{-1}^{1}(1-x^2)^2dx = 2\pi\int_0^1(1-2x^2+x^4)dx = 2\pi\left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{2\pi(15-10+3)}{15} = \frac{16\pi}{15}$

$I = \frac{\frac{1}{8}\ln 6 + \frac{1}{6} + 2\int x \text{ (ovo gore)}}{2}$

$P = 2\pi\int_0^1 (1-x^2)\sqrt{1+4x^2} = 2\pi\left(\frac{1}{8}\ln 3 + \frac{1}{6}\right) - 4\pi\int_0^1 \underbrace{x^2}_{u}\underbrace{\sqrt{1+4x^2}}_{dv}$

#

NMG STVARNO

3. a1) $\sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{n-1}$ $\qquad \frac{\sqrt{n}}{n-1} \sim \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$ $\quad \frac{1}{2} < 1$ divergira apsolutno

$$f(x)' = \frac{\frac{1}{2\sqrt{n}}(n-1) - \sqrt{n}\cdot 1}{(n-1)^2} = \frac{n-1-2n}{2\sqrt{n}(n-1)^2} = \frac{-n-1}{2\sqrt{n}(n-1)^2} \quad n>2$$

niz opada i svuda je poz

Pa po Lajbnicu uslovno konvergira.

a2) $\sum_{n=1}^{\infty} \frac{n^4 \cdot 4^n}{3^n + 5^n}$ $\qquad \lim_{n\to\infty} \sqrt[n]{\frac{4^n \cdot \cancel{n^4}}{5^n(1+(\frac{3}{5})^n)}} = 4/5 < 1$ KONVERGIRA

b) $x_1 = 1$, $x_2 = \frac{4}{3}$, $x_3 = 2$ $\qquad \sum_{n=1}^{\infty} \frac{n^n \cdot n!}{(2n)!} \cdot x^n$

I $x = 1$ $\quad \sum_{n=1}^{\infty} \frac{n^n \cdot n!}{(2n)!}$

$$\lim \frac{\frac{(n+1)^n \cdot (n+1) \cdot \cancel{n!} \cdot \cancel{(n+1)}}{2\cancel{(n+1)} \cdot (2n+1) \cdot \cancel{(2n)!}}}{\frac{n^n \cdot \cancel{n!}}{\cancel{(2n)!}}} = \lim_{n\to\infty} \frac{\overbrace{\cancel{n^n}(1+\frac{1}{n})^n}^{e} \cdot (n+1)}{\cancel{n^n} \cdot 2 \cdot (2n+1)} =$$

$$= \lim_{n\to\infty} \frac{e \cdot n(1+\frac{1}{n})}{4n(1+\frac{1}{2n})} = \frac{e}{4} < 1 \text{ konvergira}$$

II $x = 4/3$ $\quad \lim \ldots \frac{e}{4} \cdot \frac{4}{3} \cdot \frac{4^n}{3^n} \cdot \frac{3^n}{4^n} = \frac{e}{3} < 1$ KONVERGIRA

III $x = 2$ $\quad \lim \ldots \frac{e}{4} \cdot 2^n \cdot 2 \cdot \frac{1}{2^n} = \frac{e}{2} > 1$ DIVERGIRA

4. a) $\lim_{x\to 0} \sum_{n=1}^{\infty} \frac{n^3}{3^n} e^{3x} \sin(nx)$

$$\frac{n^3}{3^n} \cdot e^{3x} \sin(nx) \sim \frac{n^4 x}{3^n} \cdot 1 = x \frac{n^4}{3^n}$$

$\lim_{x\to 0} x \cdot \sum_{n=1}^{\infty} n^4 \cdot (\frac{1}{3})^n$ $\qquad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad /' \cdot x$

$$\sum_{n=0}^{\infty} n x^n = \frac{x}{(1-x)^2} \quad /' \cdot x$$

$$n^2 x^n = \frac{x((1-x)^2 + x(1-x)\cdot 2)}{(1-x)^4} = \frac{x(\overbrace{1-x+2x}^{x+x^2})}{(1-x)^3} \quad /'$$

$$n^3 x^n = \frac{x((1+2x)(1-x)^3 + (x+x^2)(1-x)^2 \cdot 3)}{(1-x)^4} \quad /' \cdot x$$

$$n^4 \cdot x^n = \left(\frac{x(1+2x-x-2x^2+3x+3x^2)}{(1-x)^4}\right)' \cdot x = \frac{\left((1-x)(1+8x+3x^2) + 4(1-x)^{3}(x+4x^2+x^3)\right) \cdot x}{(1-x)^5}$$

$(x+4x^2+x^3)$ $\qquad x = 1/3$ $\qquad \frac{\left(\frac{8}{3} + 4(\frac{1}{3} + \frac{4}{9} + \frac{1}{27})\right) \cdot \frac{1}{3}}{(\frac{2}{3})^5}$ $\qquad \frac{1+12+9}{27}$

$$\Rightarrow \frac{(\frac{8}{3} + \frac{22}{27}) \cdot \frac{1}{3} \cdot 27 \cdot 8 \cdot 3}{32} = \frac{(72+22)\cdot 3}{32} = \boxed{\frac{57 \cdot 3}{16}} - 1$$

$$\lim_{x\to 0} \left(\frac{57 \cdot 3}{16} - 1\right) x = 0 \quad ???$$

4 b) $\sum_{n=1}^{\infty}\left(\frac{4^2}{n^4}+\frac{2^2}{n^2}\right)$ $\quad x^2+x=\frac{\pi^2}{3}+\sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{n^2}\cos(nx)+\frac{2}{n}(-1)^{n+1}\sin(nx)\right)$

$\left(\frac{\pi^4-2\pi^3+2\pi^2+3\pi}{3}\right)$

MOŽDA??

$x=\pi \quad S_4(x)=\pi^2+\pi-\frac{\pi^2}{3}=\sum_{n=1}^{\infty}\frac{4}{n^2}=\frac{2\pi^2+3\pi}{3}$ #

$\left(x^2+x-\frac{\pi^2}{3}\right)=\sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{n^2}\cdot\cos(nx)+\ldots\right) \quad \int$

$\frac{1}{3}x^3+\frac{1}{2}x^2-x\frac{\pi^2}{3}=\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^3}\sin nx+\ldots \quad \int$

$\frac{1}{12}x^4+\frac{1}{6}x^3-\frac{1}{6}x^2\pi^2=\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^4}\cdot\cos(nx)$

$x=\pi \quad -\pi^4\frac{1}{12}+\frac{\pi^3}{6}=-\sum\frac{4}{n^4} \quad \cdot|{-4}$

$-\frac{2\pi^3}{3}+\frac{\pi^4}{3}=\sum\frac{16}{n^4}$

2023 jun 1

1) $\int\frac{x\ln x^2}{\sqrt{1-x^2}}=\int\frac{2x\ln x}{\sqrt{1-x^2}}=\left[\begin{matrix}1-x^2=t^2\\-2x=tdt\cdot 2\end{matrix}\right]=\int\frac{-2tdt\ln(1-x^2)\cdot\frac{1}{2}}{t}=$

$=-\int\ln(1-x^2)=-\int\left(\ln(1-x)+\ln(1+x)\right)dx=-\left(x\ln(1-x)+x\ln(1+x)-x+x\right)$

2) $\int\frac{dx}{\sqrt[6]{x+2}+\sqrt[3]{x+2}+1}=\left[\begin{matrix}x+2=t^3\\x=t^3-2\\dx=3t^2\end{matrix}\right]=\int\frac{3t^2}{t+\sqrt{t}+1}=\left[\begin{matrix}t=u^2\\dt=2u\end{matrix}\right]=$

$=\int\frac{3u^4\cdot 2u}{u^2+u+1}=6\int\left(\frac{u^3(u^2+u+1)}{u^2+u+1}-\frac{\overbrace{u^4}^{=u^2(u^2+u-1)+u^3+u^2}}{u^2+u+1}-\frac{u^3}{u^2+u+1}\right)=$

$=6\int\left(u^3-u^2+\frac{u^2+u+1}{u^2+u+1}-\frac{u+1}{u^2+u+1}\right)=6\left(\frac{1}{4}u^4-\frac{1}{3}u^3+u-\ln(u)\right)=$

$=\frac{3}{2}\sqrt[3]{(x+2)^2}-2\sqrt{x+2}+\frac{1}{6}\sqrt[6]{x+2}-\ln(x+2)$

3) $\int_0^{2023\pi/2}\frac{dx}{|\sin x|+|\cos x|}$ Znamo da je π periodična, ispitajmo za $\frac{\pi}{2}$:

$|\sin(\frac{\pi}{2}+x)|+|\cos(\frac{\pi}{2}+x)|$

$=|\sin\frac{\pi}{2}\cos x|+|\sin\frac{\pi}{2}\sin x|=|\cos x|+|\sin x|=|\sin x|+|\sin x|$

Jeste! :)

$2023\frac{\pi}{2}=0+T\cdot k$

$\int_0 =0 \longrightarrow \int_0^{\pi/2}\left[\begin{matrix}tg\frac{x}{2}=t\\\sin x=\frac{2t}{1+t^2}\\\cos x=\frac{1-t^2}{1+t^2}\\dx=\frac{2}{1+t^2}\end{matrix}\right]=\int_0^{\pi/2}\frac{2dt}{\cancel{(1+t^2)}\left(\frac{1+2t-t^2}{\cancel{1+t^2}}\right)}=-\int_0^{\pi/2}\frac{2dt}{(t-1)^2-2}=\frac{1}{(\ldots)(\ldots)}=A(t-1-\sqrt{2})+B(t-1+\sqrt{2})$

$-\sqrt{2}-\sqrt{2}=-2\sqrt{2}$

na 0 do $\frac{\pi}{2}$ su oba +

$=\frac{1}{\sqrt{2}}\left|\left(\ln(t-1+\sqrt{2})-\ln(t-1-\sqrt{2})\right)\right|_0^{\pi/2}$

$=\frac{1}{\sqrt{2}}\ln\left|\frac{t-1+\sqrt{2}}{t-1-\sqrt{2}}\right|_0^{\pi/2}=\frac{1}{\sqrt{2}}\left(\ln\left|\frac{\frac{\pi}{2}-1+\sqrt{2}}{\frac{\pi}{2}-1-\sqrt{2}}\right|-\ln\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right|\right)$

2. 1) $\int_0^4 \frac{1-\cos\sqrt{x}}{\ln(1+4\sqrt{x})\sqrt[4]{16-x^2}}\,dx$

$\int_0^1 \sim \frac{x}{4\sqrt{x}\cdot(16+(-x^2))^{1/4}} \sim \frac{\sqrt{x}}{8\left(\frac{1}{4}\cdot\left(-\left(\frac{x}{4}\right)^2+1\right)\right)} \sim \frac{\sqrt{x}}{2\left(1/4-\frac{x^2}{16}\right)} \sim \frac{\sqrt{x}}{x^2} = \frac{1}{x^{3/2}}$

$\frac{3}{2} > 1$ divergira

$\int_1^4 \quad t = x-4 \quad = \int_{-3}^{0} \frac{1-\cos\sqrt{t+4}}{\ln(1+4\sqrt{t+4})\sqrt[4]{16-t^2-8t-\cancel{16}}} \sim \frac{1-\cos\sqrt{t+4}}{\ln(1+4\sqrt{t+4})\cdot\sqrt[4]{-t8}\left(1-\frac{t}{8}\right)^{1/4}}$

$\sim \frac{1-\cos 2}{\ln(1+8)\cdot\sqrt[4]{-t8}\left(\frac{1}{4}\cdot\left(-\frac{t}{8}\right)+1\right)} \sim \frac{1}{\sqrt[4]{-t}\cdot t}$ $\quad 3/4 > 1$ divergira

b) $\int_0^{+\infty} \frac{\ln(e+x)\operatorname{arctg}\sqrt{x}}{\sqrt[3]{x^5+x^2}}$ $\quad \int_0^1 \frac{\ln e \cdot \operatorname{arctg}\sqrt{x}}{\sqrt[3]{x^2}\left(1+x^3\right)^{1/3}} \sim \frac{\sqrt{x}}{\sqrt[3]{x^2}\cdot\left(\frac{1}{3}x^3-1\right)} \sim \frac{}{x^{\frac{2}{3}-\frac{1}{2}+\frac{18}{8}}} =$

$\frac{18-3+4}{6} > 1$ divergira

$\int_1^\infty \dots \sim \frac{\ln\left(1+\frac{x}{e}\right)\pi/2}{x^5\left(1+\frac{1}{x^3}\right)^{\frac{x^3}{x^3}\cdot\frac{1}{3}}} \sim \frac{\ln(x)}{x^3\cdot e^{\frac{1}{x^3\cdot 3}}} \sim \frac{\ln x}{x^5}$ konvergira

3. $a_1)$ $\sum_{n=1}^{\infty}\left(\sin\frac{1}{\sqrt{n}}+\frac{1}{n^2}\right)$ $\qquad a_2)$ $\sum_{n=1}^{\infty}\frac{(2n)!!}{(2n-1)^{n+e}}$

a) $\lim_{n\to\infty}\left(\sin\frac{1}{\sqrt{n}}+\frac{1}{n^2}\right) = \lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}+\frac{1}{n^2}\right) = 0$ Opada i uvek je pozitivan pa konvergira

$a_2)$ $\lim_{n\to\infty} \frac{(2n+2)\,\cancel{(2n)!!}\cdot(2n-1)^n\cdot(2n-1)^e}{\cancel{(2n)!!}\cdot(2n+1)^n\,(2n+1)\,(2n+1)^e} = \frac{\cancel{2n}\left(1+1/n\right)^{n/n}\cdot\cancel{(2n)^n}\left(1-\frac{1}{2n}\right)^{\frac{n\cdot 2\cdot(-1)}{2\cdot(-1)}}\cdot(2n-1)^e}{\cancel{2n}\left(1+\frac{1}{2n}\right)^{\frac{2n}{2n}}\,\cancel{(2n)^n}\left(1+\frac{1}{2n}\right)^{\frac{n\cdot 2}{2}}(2n+1)^e} =$

$= \frac{e^{1/n}\cdot e^{-1/2}\cdot(2n)^e\left(1-\frac{1}{2n}\right)^{e\cdot\frac{-2n}{-2n}}}{e^{1/2n}\cdot e^{1/2}\,(2n)^e\left(1+\frac{1}{2n}\right)^{e\cdot\frac{2n}{2n}}} = \frac{e^{\frac{1}{2n}}\cdot e^{e/2n}}{e\cdot e^{e/2n}} = 1/e < 1$ KONVERGIRA

b) $\sum_{n=1}^{\infty}\frac{\operatorname{arctg} n}{n}(x+1)^n$ $\quad \boxed{x_0=-1}$ $\quad \lim_{n\to\infty}\sqrt[n]{\frac{n}{\operatorname{arctg} n}} = 1$ $\quad D = (-2, 0)$

Za $x=0$ $\quad \frac{\frac{n}{n^2+1}-\operatorname{arctg} n}{n^2} = \frac{n-n^2(n^2+1)\operatorname{arctg} n}{n^2(n^2+1)}$?

arctg n je najmanji u $n=1$ i tada je izraz također negativan kao i na ostatku $n\in[1,+\infty)$

Opada, uvek pozitivan konvergira

Za $\boxed{x=-2}$ $\quad \lim_{n\to\infty}\frac{2^n\cdot 2\cdot\operatorname{arctg}(n+1)\cdot n}{2^n\operatorname{arctg}(n)\cdot(n+1)} = 2 > 1$ divergira pa je

$D = [-2, 0]$

4. $f_n(x) = x^n(1+\ln x)$

a) $(0, 1]$

Za $x=1$ $f_n(1) = 1$

$$\lim_{x\to 0} x(1+\ln x) = \lim_{x\to 0} \frac{1+\ln x}{\frac{1}{x}} \overset{\frac{-\infty}{\infty}}{=} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -x = \boxed{0} \quad \text{Ne R.K.}$$

b) $(0, \frac{9}{10}]$

$$\underbrace{\left(\frac{9}{10}\right)^n \left(1+\ln\left(\frac{9}{10}\right)\right) \le \left(\frac{9}{10}\right)^n \longrightarrow x^n = \frac{1}{1-x} = \frac{10}{1} = 10}$$

Ovo konvergira pa po Vajerštrasu i ovo levo konvergira

5. $f(x) = -4x^2-7$ $\quad A = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cos\frac{2n}{5}}{n^2}$ $\quad B = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}(-4x^2-7) = \frac{1}{\pi}\int_0^{\pi}\left(-\frac{4}{3}x^3-7x\right) = \frac{1}{\pi}\left(-\frac{4}{3}\pi^3 - 7\pi\right) = \boxed{-\frac{4}{3}\pi^2 - 7}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(-4x^2-7)\cos nx = \frac{2}{\pi}\int_0^{\pi}\underbrace{(-4x^2-7)}_{u}\cos nx = \frac{2}{\pi}\left(\left(-4x^2-7\right)\frac{1}{n}\sin nx - \frac{1}{n}\int_0^{\pi}\sin nx \cdot \underbrace{(-8x)}_{u}\right) =$$

$$= -\frac{2}{\pi n}\cdot\left(8x\cdot\frac{1}{n}\cdot\cos nx - 8/n\int_0^{\pi}\cos nx\right) \qquad = -\frac{2}{\pi}\left(\frac{8\pi(-1)^n}{n^2} - \frac{8}{n^2}\sin nx\right) =$$

$$= \left(-\frac{16(-1)^n}{\pi n^2}\right)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\underbrace{(-4x^2-7)\sin x}_{\text{neparna}} = 0$$

$$S_f(x) = -\frac{4}{3}\pi^2 - 7 + \sum_{n=0}^{\infty}\left(-\frac{16(-1)^n}{\pi n^2}\right)\cos nx$$

$$4x^2+7 = \frac{4}{3}\pi^2 + 7 + \sum_{n=0}^{\infty}\left(\frac{16}{\pi n^2}(-1)^n\right)\cos nx$$

A: $x = \frac{2}{5}$ $\qquad \frac{16}{25} = \frac{4}{3}\pi - \frac{16}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot\cos\frac{2n}{5}}{n^2} \Rightarrow A = \left(\frac{4}{3}\pi - \frac{16}{25}\right)\cdot\frac{\pi}{16}$

$$4x^2+7=\frac{4}{3}\pi^2+7+\sum_{n=0}^{\infty}\left(\frac{16}{\pi n^2}(-1)^n\right)\cos nx \qquad B=\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$x=0$$

$$0=\frac{4}{3}\pi^2+\frac{16}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$-\frac{\pi^3}{12}=\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

II tou razlike

1 2) $\int \frac{x^3}{x^3-3x-2}\,dx=\int 1+\frac{3x+2}{(x-2)(x+1)^2}$

$$\frac{3x+2}{\cdots}=\frac{A}{x-2}+\frac{B}{x+1}+\frac{C}{(x+1)^2} \quad /\cdot\ldots$$

$$3x+2=A(x^2+2x+1)+B(x^2-x-2)+C(x-2)$$

$A+B=0 \Rightarrow A=-B$

$2A-B+C=3 \quad 3A+C=3$

$A-2B-2C=2 \quad 3A-2C=2 \quad \Rightarrow 3C=5$

$C=5/3$

$A=4/9$

$B=-4/9$

$$=x+\frac{1}{9}\left(4\ln(x-2)-4\ln(x+1)+\frac{15}{(x+1)}\right)$$

2. $l: y=\frac{x^2}{4}-\ln\sqrt{x} \quad 1\le x\le 2$

a) $l(y)=\int_1^2\sqrt{1+\left(\frac{x}{2}-\frac{1}{\underbrace{2\sqrt{x}\cdot\sqrt{x}}_{2x}}\right)^2}=\int_1^2\sqrt{1+\frac{1}{4}\left(x^2-2+\frac{1}{x^2}\right)}=\int_1^2\sqrt{1+\frac{1}{4}\,\frac{(x^4-2x+1)}{x^2}}=\int_1^2\sqrt{\frac{4x^2+x^4-2x+1}{4x^2}}$

$$=\int_1^2\frac{x^2+1}{2x}=\frac{1}{2}\left(\frac{1}{2}x^2+\ln x\right)\Big|_1^2=\frac{1}{2}\left(2+\ln 2+\frac{1}{2}\right)=\frac{5}{4}+\ln\sqrt{2}$$

b) $P=\pi\int_1^2\left(\frac{x^2}{4}-\ln\sqrt{x}\right)(x^2+1)=\pi\int_1^2\left(\frac{x^4}{4}+\frac{x^2}{4}-\ln\sqrt{x}-x^2\ln\sqrt{x}\right)=\pi\left(\frac{x^5}{20}+\frac{x^3}{12}-I_1-I_2\right)\Big|_1^2$

$I_1=\int\ln\sqrt{x}\,dx=x\ln\sqrt{x}-\int\frac{x}{2x}=x\ln\sqrt{x}-\frac{1}{2}x$

$I_2=\int x^2\ln\sqrt{x}=\frac{1}{3}x^3\ln\sqrt{x}-\frac{1}{6}\int\frac{x^3}{x}=\frac{1}{3}x^3\ln\sqrt{x}-\frac{1}{18}x^3$

$$P=\pi\left(\frac{8}{5}+\frac{8}{12}-\ln 2+1-\frac{4}{3}\ln 2-\frac{4}{9}-\frac{1}{20}-\frac{1}{12}-\frac{1}{2}-\frac{1}{18}\right)$$

2023 sept 1

$1+x^4=t$

$4x^3=dt$

$x^3=dt/4$

1) $\int \underbrace{x^5}_{dv}\underbrace{\operatorname{arctg}(x^2)}_{u}=\frac{1}{6}x^6\operatorname{arctg}(x^2)-\frac{1}{6}\int\frac{x^6\cdot 2x}{1+x^4}=\frac{1}{6}x^6\operatorname{arctg}x^2-\frac{1}{3}\int\frac{x^3(x^4+1)}{1+x^4}-\frac{x^3}{1+x^4}$

$$=\frac{1}{6}x^6\operatorname{arctg}x^2-\frac{1}{12}x^4-\frac{1}{12}\ln(1+x^4)$$

2) $\int\frac{x^4}{x^3+x^2-5x+3}=\int\frac{x^4}{(x-1)(x+3)(x-1)}=\int x+\frac{-x^2+5x-3}{\cdots}=\frac{1}{2}x^2+\int\frac{11}{16(x-1)}+\frac{1}{4(x-1)^2}-\frac{27}{16(x+3)}$

$$=\frac{1}{2}x^2+\frac{11}{16}\ln(x-1)+\frac{1}{4(x-1)}-\frac{27}{16}\ln(x+3)$$

3) $$\int_{-10\pi}^{10\pi} \frac{\sin^3 x(|x|+\cos^2 x)+\cos x}{\cos^2 x+|\sin x|} \qquad \frac{\sin^3(-x)(|-x|+\cos^2 -x)+\cos -x}{\cos^2 -x \ +|\sin -x|}$$

neparno $=0$ $\leftarrow \frac{\sin^3(|x|+\cos^2 x)}{\cos^2 x+|\sin x|} + \frac{\cos x}{\ldots}$ → parno

$$= 2\underbrace{\int_{0=0}^{10\pi = 0+5\cdot(2\pi)} \frac{\cos x}{\cos^2 x+|\sin x|}}_{I} \overset{2\pi \text{ period}}{=\!=} 10\int_0^{2\pi} \frac{\cos x}{\cos^2 x+|\sin x|} = 10\int_0^{2\pi} \frac{\cos x}{}$$

$$I = \int \frac{\cos x}{\cos^2 x+|\sin x|} = \begin{matrix} tg\frac{x}{2}=t \rightsquigarrow \frac{x}{2} = \frac{2}{1+t^2} \\ \sin x = \frac{2t}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \end{matrix} = \int \frac{2(1-t^2)}{(1+t^2)^2\left(\frac{(1-t)^2}{(1+t^2)^2}+\frac{|1-t^2|}{1+t^2}\right)} =$$

$$= \int \frac{2-2t^2}{(1-2t^2+t^4+|1-t^4|)}$$

$1-t^4 \geq 0 \Rightarrow t = tg\frac{x}{2}$

$1-t^4<0 \Rightarrow \frac{2-2t^2}{-2t^2+2t^4} = -\frac{1-t^2}{t^2(1-t^2)} \Rightarrow \frac{1}{t} = ctg\frac{x}{2}$

$1-(tg\frac{x}{2})^4 \geq 0$

$\pi/2, -\pi/2 \qquad 0=1$

$$10\int_0^{2\pi} \frac{\cos x}{\cos^2 x+|\sin x|} =$$

$$10\left(\int_0^{\pi/2} + \int_{\pi/2}^{3\pi/2} + \int_{3\pi/2}^{2\pi}\right) = 10\left(tg\frac{x}{2}\Big|_0^{\pi/2} + \frac{1}{tg\frac{x}{2}}\Big|_{\pi/2}^{3\pi/2} + tg\frac{x}{2}\Big|_{\frac{3\pi}{2}}^{2\pi}\right) =$$

$$10(\cancel{1+(-1)} - 1 - \cancel{1+1}) = -10 \ ?,?$$

2. 1 $$\int_0^{+\infty} \frac{x^3}{(x^3+1)\ln^2(x+1)} \qquad \int_0^1 + \int_1^{\infty} \quad \int_1^{\infty} \sim \frac{x^3}{x^3\ln^2(x+1)} \sim \frac{1}{x^2} \quad 2>1 \text{ KONV.}$$

$$\int_0^1 \frac{1}{\frac{(x^3+1)}{x^3}\cdot x^2} \sim \frac{1}{(x^3+1)\cdot x} \sim \frac{1}{x} \quad \text{divergira} \quad 1=1$$

2) $$\int_1^{+\infty} \frac{\ln^2 x}{x(\ln^3 x+1)\ln^2(\ln x+1)}dx \sim \frac{\cancel{x^2}}{x\cdot x^3\cdot \cancel{x^2}} \sim \frac{1}{x^4} \quad 4>1 \text{ KONVERGIRA}$$

3 1) $$\sum_{n=1}^{\infty} \frac{n^{2023}(2+(-1)^n)^n}{3^{2n}}$$

dva → $n=2k \quad \frac{n^{2023}}{3^n} \quad \lim \frac{(n+1)^{2023}\cdot \cancel{3^n}}{3\cdot \cancel{3^n}\, n^{2023}} = \frac{(1+\frac{1}{n})^{2023}}{3} = \frac{1}{3}$ KONV.

→ $n=2k+1 \quad \frac{n^{2023}}{3^{2n}} \quad \lim \frac{\cancel{3^{2n}}}{9\cdot \cancel{3^{2n}}} = \frac{1}{9}$ KONV

Red podelimo na dva i s obzirom da oba konvergiraju i njihov zbir konvergira.

2) $\sum_{n=1}^{\infty} \left(\cos\frac{1}{n} - 1\right)(2n+1) \quad \sim \left(-\frac{1}{n^2 2}\right)(2n+1) \sim \frac{2n}{2n^2} \sim \frac{1}{n}$ divergira

3) $\sum_{n=1}^{\infty} \frac{(-e)^n}{n(e^n+1)}$ Aps: $\frac{e^n}{n(e^n+1)} \underset{>1}{\sim} \frac{1}{n}$ divergira

Uslovno: $\frac{e^n}{n(e^n+1)} \to 0$ i $e^n(ne^n+n-e^n-1-ne^n) = n-1-e^n$ opada
pa po Lajbnicu konvergira (Nadam se)

4 a) $\lim_{x\to a} \sum_{n=1}^{\infty} \frac{x \ln n}{x^2+n^3}$ 1) $a=0$ 2) $a=+\infty$

1) $\lim_{x\to 0} x \sum_{n=1}^{\infty} \frac{\ln n}{x^2+n^3} = \lim_{x\to 0} x \cdot \sum_{n=1}^{\infty} \frac{1}{n^2} = 0$

2) $\lim_{x\to\infty} \sum_{n=1}^{\infty} \frac{x \ln n}{x^2+n^3} = \lim_{x\to\infty} \sum_{n=1}^{\infty} \frac{\ln n}{x} = 0$

Ako znam...

b) $f(x)=\sin^2 x$ $\quad a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin^2 x\, dx \overset{P}{=} \frac{1}{\pi}\int_0^{\pi} \sin^2 x\, dx = \frac{1}{\pi}\left(\underbrace{\int_0^{\pi} dx}_{x} - \frac{\cos x \sin x}{2}\right)$

$a_0 = \frac{1}{\pi 2}(\pi) = 1/2$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin^2 x \cos(nx) = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin x\left(\frac{1}{2}\sin(nx+x) + \sin(x-nx)\right) =$

$= \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{1}{2}\left(\frac{1}{2}(\cos(nx+2x) - \cos(nx)) + \frac{1}{2}(\cos(2x-nx) - \cos(nx)\right) =$

$= \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{1}{4}\left(\cos(nx+2x) + \cos(2x-nx) - 2\cos(nx)\right) = \frac{1}{\pi 2}\left(\frac{1}{2+n}\sin(nx+2x) + \frac{1}{2-n}\sin(2x-nx) - \frac{2}{n}\sin nx\right)\Big|_0^{\pi}$

$= \frac{1}{\pi 2}\left(\frac{1}{2+n}\sin(\pi n) - \frac{1}{2-n}\sin(n\pi) - \frac{2}{n}\sin(n\pi) - 0\right) = 0$ (Ovo ne važi za $n \leq 2$)

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin^3 x \overset{neparna}{=} 0$

$\boxed{n=1}$ $\frac{1}{\pi}\int_{-\pi}^{\pi} \sin^2 x \cos x = \frac{2}{\pi}\int_0^{\pi}\left(\frac{1-\cos(2x)}{2}\right)\cos x =$

$= \sin\pi = 0 \quad -\frac{1}{2}\int \cdot \frac{1}{2}\left(\cos(3x) + \cos x\right) =$

$= -\frac{1}{12}\sin 3x - \frac{1}{4}\sin x = 0$

$\boxed{n=2}$ $\frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{1-\cos(2x)}{2}\right)\cos(2x) =$

$\frac{1}{\pi}\left(\frac{1}{2}\cdot\frac{1}{2}\underbrace{\sin(2x)}_{=0} - \frac{1}{2}\int\left(\frac{1}{2}(\cos(0) + \cos 4x)\right)\right) = \frac{1}{\pi}\left(-\frac{1}{4}(2\pi) + \frac{1}{4}\underbrace{\sin x}_{=0}\right) = -\frac{1}{\pi}$

$\Rightarrow \sin^2 x = 1/2 - \frac{1}{2}\cos(2x)$ $\blacksquare$

2024 jan 1

1. a) $\int_0^{\sqrt3} \frac{4x + \operatorname{arctg}^7 x + e^{\operatorname{arctg} x}}{2x^2+2}dx = \int_0^{\sqrt3}\frac{4x}{2x^2+2} + \frac12\int_0^{\sqrt3}\frac{\operatorname{arctg}^7 x}{1+x^2} + \frac12\int_0^{\sqrt3}\frac{e^{\operatorname{arctg}x}}{1+x^2}$

$= \ln|2x^2+2|\Big|_0^{\sqrt3} + \frac{1}{16}\cdot \operatorname{arctg}^8 x\Big|_0^{\sqrt3} + \frac12 \cdot e^{\operatorname{arctg}x}\Big|_0^{\sqrt3} = \ln 4 + \frac{1}{16}\frac{\pi^8}{3^8} + \frac12 e^{\pi/3} - \frac12$

b) $\int \frac{x^2}{\sqrt{1+x^2}}dx = \begin{matrix} x=\operatorname{tg}t \\ dx = \frac{dt}{\cos^2 t}\end{matrix} = \int \frac{\operatorname{tg}^2 t}{\sqrt{1+\operatorname{tg}^2 t}}\cdot\frac{dt}{\cos^2 t} = \int \frac{\sin^2 t}{\sqrt{\frac{1}{\cos^2 t}}}\frac{dt}{\cos^4 t} = \int\frac{\sin^2 t}{\cos^3 t}$

$= \begin{bmatrix} \sin t = u \\ dt = (\arcsin u)' \\ dt = \frac{1}{\sqrt{1-u^2}} \end{bmatrix} = \int \frac{u^2}{(1-u^2)^{3/2}}\cdot\frac{dt}{(1-u^2)^{1/2}} = \int\frac{u^2}{(1-u^2)^2} = \frac14\int\left(\frac{-1}{u+1} + \frac{1}{(u+1)^2} + \frac{1}{u-1} + \frac{1}{(u-1)^2}\right)$

$= \frac14\left(-\ln|\sin(\operatorname{arctg}x)+1| + \frac{1}{\sin(\operatorname{arctg}x)+1} + \ln|\sin(\operatorname{arctg}x)-1| + \frac{1}{(\sin(\operatorname{arctg}x)-1)}\right)$

c) $\int_{-\pi/2}^{\pi/2}\frac{x\ln(x^2+1)+\cos^3 x}{4|\sin x| + \cos^2 x - 5}dx \qquad \frac{x\ln(x^2+1)}{4|\sin x|+\cos^2 x-5}$ neparna $=0 \qquad \frac{\cos^3 x}{\cdots}$ parna

$= 2\int_0^{\pi/2}\frac{\cos^3 x}{4\sin x + \cos^2 x - 5}dx = 2\int_0^{\pi/2}\frac{\cos^3 x}{4\sin x - \sin^2 x - 4} = 2\int_0^{\pi/2}\frac{-\cos^3 x}{(\sin x-2)^2} = -2\int_0^{\pi/2}\frac{(1-\sin^2 x)\cos x}{(\sin x - 2)^2}dx$

$\left[t = \sin x\right] = -2\int_0^{\pi/2}\frac{(1-t^2)}{(2-t)^2} = 2\int_0^{\pi/2}\frac{t^2-1-4t+4-4+4t}{4-4t+t^2} = 2t\Big|_0^1 + 2\int_0^1\frac{4t+5-2\cdot2+2(-1)+1}{4-4t+t^2}$

$= 2 + 2\int_0^1 \frac{2t-4}{t^2-4t+4} + \frac{2t+1-5+5}{t^2-4t+4} = 2 + 2\ln(t-2)^2\Big|_0^1 + 10\int_0^1\frac{1}{(t-2)^2} =$

$= 2 - 4\ln 4 + \frac{10}{-2} + \frac{10}{-1} = 2 - 4\ln 4 - 5 - 10 = -4\ln 4 - 13$

2. a) $\int_1^{+\infty}\frac{\ln(1+x)}{x^3}dx \sim \frac{1}{\ln^{-1}x\, x^3}$ KONV.

$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{1+x}$

$Ax + Ax^2 + B + Bx + Cx^2 = 1$

$B=1,\ A=-1,\ C=+1$

$= \begin{matrix} u=\ln(1+x) & dv = \frac{1}{x^3} \\ du = \frac{1}{1+x} & v = -\frac{1}{2x^2}\end{matrix} = -\frac{\ln(1+x)}{2x^2}\Big|_1^{+\infty} + \frac12\int\frac{1}{x^2(1+x)} = +\frac12\ln2 + \frac12\left(-\ln x - \frac1x + \ln(1+x)\right) =$

$= \frac12\ln 2 + \frac12 - \frac12\ln 2 + \frac12\lim_{a\to+\infty}\left(\frac1x + \frac{\ln\left(1+\frac1x\right)\frac{1}{x}}{\frac1x}\right) = +\frac12 + \frac12\lim_{a\to\infty}\frac{2}{x} = +\frac12$

b) $\int_0^{\pi}\frac{(\pi-x)\sqrt[3]{1-\cos x}}{\sin x\sqrt{2x+x^3}}dx = \underbrace{\int_0^1}_{I_1} + \int_1^{\pi}$

$I_1 :\sim \frac{\sqrt[3]{1-1+\frac{x^2}{2}}}{x\cdot x^{1/2}} = \frac{x^{2/3}}{x^{3/2}} = \frac{1}{x^{\frac56}} \qquad \frac56 < 1$ KONV.

I_2: $t=\pi-x$, $dt=-dx$ $= -\int_{\pi-1}^{0} \frac{t\sqrt[3]{1+\cos t}}{\sqrt{2(\pi-t)+(\pi-t)^3}\sin t} = \int_0^{\pi-1} \frac{t\sqrt[3]{1+\cos t}}{\sqrt{2(\pi-t)+(\pi-t)^3}\sin t} \sim \frac{t\sqrt[3]{2}}{\sin t\sqrt{2\pi+\pi^3}} \sim \frac{t}{\sin t}$

$\sim \frac{1}{t^0}$ $0<1$ konv.

3.) $\sum_{n=1}^{\infty} \left(\frac{\ln n}{\ln(n+1)}\right)^{\frac{1}{n^2}}$ $|a_n| = a_n$ $\ln(n+1) = \ln n + \ln(1+\frac{1}{n}) \sim \ln n$

$\left(\frac{\ln n}{\ln(n+1)}\right)^{\frac{1}{n^2}} \sim 1^{\frac{1}{n^2}} = 1 \neq 0$ divergira

b) $\sum_{n=1}^{\infty} \frac{n \operatorname{arctg}\frac{1}{n^2}}{\ln(n+8)} \sim \frac{n\cdot\frac{1}{n^2}}{\ln n} = \frac{1}{n\cdot\ln n}$ divergira

c) $\sum_{n=1}^{\infty} (-1)^n \frac{2^{n+3}\cdot n!}{(n+5)^n}$ $|a_n| \to$ $\frac{\cancel{8}\cdot\cancel{2^n}\cdot 2\cdot\cancel{n!}\cdot(n+1)\cdot(n+5)^n}{\cancel{8}\cdot\cancel{2^n}\cdot\cancel{n!}\cdot(n+6)^n\cdot(n+6)} = \frac{2\cdot n\cancel{(1+\frac{1}{n})}\cdot\cancel{n^n}\cdot(1+\frac{5}{n})^{\frac{n}{5}\cdot 5}}{n\cancel{(1+\frac{6}{n})}\cdot\cancel{n^n}(1+\frac{6}{n})^{\frac{n}{6}\cdot 6}}$

$= \frac{2e^5}{e^6} = \frac{2}{e} < 1$ konv. aps.

$\sum a_n$ (AU) $\Rightarrow \sum a_n$ (U) $\Rightarrow$ Ne konv uslovno

4 a) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+9}(x+4)^n$ $x_0=-4$

$R = \limsup_{n\to\infty} \frac{1}{\sqrt[n]{|a_n|}} = \sqrt[n]{\frac{n^3+9}{n^2}} = \sqrt[n]{\frac{n^3(1+\frac{9}{n^3})}{n^2}} = \sqrt[n]{n} = 1$ } Potencijal $[-5,-3]$

U $\boxed{-5}$ $\frac{(-1)^n n^2}{n^3+9}$ $\frac{2n(n^3+9)-3n^4}{(n^3+9)^2} = \frac{18n-n^4}{(n^3+9)^2} \to n(18-n^3)$ $n=\sqrt[3]{18}$ $2\cdot 8^2$ a_n opada $n\geq 3$ konv.

U $\boxed{-3}$ $\frac{n^2}{n^3+9} \sim \frac{1}{n}$ divergira $\Rightarrow D=[-5,-3)$

b) $\lim_{x\to 0^+} \sum_{n=1}^{\infty} \frac{e^{-2nx^2}}{n(n+x)}$ $\lim_{x\to+\infty} \sum_{n=1}^{\infty} \frac{e^{-2nx^2}}{n(n+x)}$

$|f_n(x)| = \frac{e^{-2nx^2}}{n(n+x)} \leq \frac{1}{n^2} =: \underbrace{C_n}_{\text{konvergira}}$

pa $\sum f_n$ ravnomerno konvergira

$\lim_{x\to 0^+} f_n(x) = \lim_{x\to 0} = \frac{1}{n^2} \Rightarrow \sum f_n \frac{1}{n^2} = \frac{\pi^2}{6}$

$\lim_{x\to\infty} = \frac{e^{-nx^2}}{n(n+x)} = 0 \Rightarrow \sum f_n\ 0 = 0$

2024 Jan 2

1 a) $\int \frac{\ln(x^2+6x+10)}{(x+1)^2}dx = \frac{-\ln(x^2+6x+10)}{x+1} + \int \frac{2x+6}{(x+1)(x^2+6x+10)} = \frac{-\ln(x^2+6x+10)}{x+1}$

$+\int \frac{4/5}{x+1} - \frac{1}{5}\cdot\frac{4x+10}{x^2+6x+10}$

$\frac{A}{x+1} + \frac{Bx+C}{x^2+6x+10} = Ax^2 + Bx^2 = 0 \qquad A+B=0$

$6Ax + Bx + Cx = 2 \qquad 5A+C=2 \qquad A=4/5$

$10A + C = 6 \qquad 10A+C=6 \qquad B=-4/5 \qquad C=-2$

$= \frac{-\ln(x^2+6x+10)}{x+1} + \frac{4}{5}\ln|x+1| - \frac{1}{5}\int \frac{2(2x+6)}{x^2+6x+10} - \frac{2}{x^2+6x+10} \qquad = (x+3)^2+1$

—/— $-\frac{2}{5}\ln|x^2+6x+10| + \frac{2}{5}\,\mathrm{arctg}(x+3) + C$

b) $f(x) = \begin{cases} \cos x \ln\frac{1+x}{1-x} & x\in[-\frac{1}{2},\frac{1}{2}) \rightarrow I_1 \\ 5 & x=\frac{1}{2} \rightarrow \int 5 = 5x \\ \sqrt{1-\cos^2 x} & x\in(\frac{1}{2}, 2\pi] \rightarrow \int\sqrt{1-\cos^2 x} = \int|\sin x| = |\cos x| \end{cases}$

$I = \int_{-1/2}^{2\pi} f(x)dx \qquad I_1 = \int_{-1/2}^{1/2} \underbrace{\cos x}_{\text{parna}} \cdot \underbrace{\ln\frac{1+x}{1-x}}_{\text{neparna}} \overset{\text{neparna}}{=} 0 \qquad \int_{1/2}^{1/2} 5 = 0$

$\int_{1/2}^{\pi}\sin x - \int_{\pi}^{2\pi}\sin x = -\cos x\Big|_{1/2}^{\pi} + \cos x\Big|_{\pi}^{2\pi} = 1 + \cos\frac{1}{2} + 1 + 1 = 3 + \cos\frac{1}{2}$

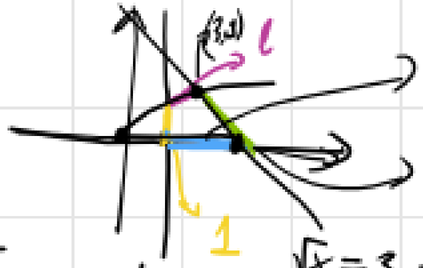
c) $\int_0^{\pi/2} \frac{x\sin 2x}{\sin^4 x + \cos^4 x}dx = \left[\begin{matrix} t = \frac{\pi}{2} - x \\ dt = -dx \end{matrix}\right] = -\int_{\pi/2}^{0} \frac{(\frac{\pi}{2}-t)\sin 2t}{\cos^4 t + \sin^4 t}dt = -\underbrace{\int_0^{\pi/2} \frac{t\cdot\sin 2t}{\cos^4 t + \sin^4 t}}_{I} + \int_0^{\pi/2}\frac{\frac{\pi}{2}\sin 2t}{\cos^4 t+\sin^4 t} =$

$\Rightarrow 2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{2\sin t\cos t}{1-2\cos^2 t + 2\cos^4 t} = \frac{\pi}{2}\int_0^{\pi/2}\frac{2\sin t\cos t}{2\left((\cos^2 t - \frac{1}{2})^2 + \frac{1}{4}\right)} = \left[\begin{matrix}\cos^2 t - \frac{1}{2} = u \\ -2\cos t\sin t = du\end{matrix}\right]$

$= -\frac{\pi}{4}\int_{1/2}^{-1/2} \frac{du}{(u^2+\frac{1}{4})} = -\frac{\pi}{4}\left(2\,\mathrm{arctg}(2\cdot(\cos^2 t - \frac{1}{2})\right) = -\frac{\pi}{2}\left(\mathrm{arctg}(2\cos^2 t - 1)\right)\Big|_0^{\pi/2}$

$\Rightarrow I = -\frac{\pi}{8}\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = -\frac{\pi^2}{16}$

2 a) $y=\sqrt{x} \quad x=1,\ y=0 \quad x+y-12=0 \to y=12-x$



$12-1=11$

$\sqrt{x}=12-x$

$x+\sqrt{x}-12=0$

$x_{1/2}=\frac{-1\pm\sqrt{1+48}}{2} \to -4,\ 3$

$\sqrt{x}=3 \Rightarrow x=9$

$(\sqrt{x})'=\frac{1}{2\sqrt{x}}$

$$l(y)=\int_1^9\sqrt{1+\frac{1}{4x}}\,dx=\int_1^9\sqrt{\frac{4x+1}{4x}}\,dx=\left[\begin{array}{l} t^2=\frac{4x+1}{4x}\\ t^2 4x-4x=1\\ 4x(t^2-1)=1\\ x=\frac{1}{4(t^2-1)}\\ dx=\frac{-2t}{4(t^2-1)^2}\end{array}\right] \quad \int_{\frac{\sqrt5}{2}}^{\frac{\sqrt{37}}{12}} \frac{-t^2}{2(t^2-1)^2}$$

$$=-\frac{1}{2}\int_{\frac{\sqrt5}{2}}^{\frac{\sqrt{37}}{12}}\left(-\frac{1/4}{t+1}+\frac{1/4}{(t+1)^2}+\frac{1/4}{(1-t)}+\frac{1/4}{(1-t)^2}\right)=$$

$$=-\frac{1}{8}\left(-\ln(t+1)-\frac{1}{t+1}+\ln(1-t)+\frac{1}{(1-t)}\right)\Bigg|_{\frac{\sqrt5}{2}}^{\frac{\sqrt{37}}{12}}=$$

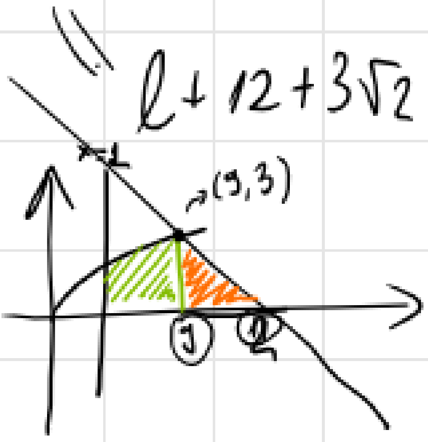
$\frac{2t}{1-t^2}$

$$=-\frac{1}{8}\left(\ln\frac{12-\sqrt{37}}{12+\sqrt{37}}+\ln\frac{2-\sqrt5}{2+\sqrt5}+\frac{\frac{\sqrt{37}}{6}}{\frac{144-37}{12}}-\frac{\sqrt5}{-1}\right) \Rightarrow \text{Označimo sa } \boxed{l}$$

$\text{OBIM}=l+11+1+\left[A(12,0)\quad B(9,3)\right]\ d=\sqrt{(12-9)^2+9}=3\sqrt2$

$\parallel$

$l+12+3\sqrt2$ je obim



$$P=\int_1^9|\sqrt{x}|\,dx=\frac{1}{1+\frac12}\cdot x^{1+\frac12}\Big|_1^9=\frac{2\sqrt[3]{x^2}}{3}\Big|_1^9=\boxed{\frac{6\cdot\sqrt[3]{3}-2}{3}}$$ zelena površina

Narandžasta: $\frac{3\cdot3}{2}=\frac{9}{2}$ Ukupna površina $=\frac{131}{6}$

b) $y=\sin x$

$$P=2\pi\int_0^\pi\sin x\sqrt{1+\cos^2x}=\left[\cos x=t\right]=-2\pi\int_1^{-1}\sqrt{1+t^2}\,dt=4\pi\int_0^1\sqrt{1+t^2}\,dt=$$

$$\left[\begin{array}{l} t=\operatorname{sh}u\\ dt=\operatorname{ch}u\end{array}\right]=4\pi\int_0^{\ln(1+\sqrt2)}\sqrt{1+\operatorname{sh}^2u}\cdot\operatorname{ch}u=4\pi\int_0^{\ln(1+\sqrt2)}\operatorname{ch}^2u\,du=\pi\int_0^{\ln(1+\sqrt2)}\left(e^{u}+e^{-u}\right)^2du=$$

$$\left[\operatorname{sh}=\frac{e^x-e^{-x}}{2}\right]=\pi\int_0^{\ln(1+\sqrt2)}\left(e^{2u}+2e^{u-u}+e^{-2u}\right)du=\pi\left(\frac12e^{2u}+2u-\frac12e^{-2u}\right)\Bigg|_0^{\ln(1+\sqrt2)}$$

$$=\pi\left(\frac12(1+\sqrt2)^2-\frac12+2\ln(1+\sqrt2)-\frac{1}{2(1+\sqrt2)^2}+\frac12\right) \quad \cdot\left(\frac{\sqrt2-1}{\sqrt2-1}\right)^2$$

$$=\pi\left(\cancel{\frac12}+\sqrt2+\cancel{1}-\frac12(\cancel{2}-2\sqrt2+\cancel{1})+2\ln(1+\sqrt2)\right)=\pi\left(2\sqrt2+2\ln(1+\sqrt2)\right)=2\pi\left(\sqrt2+\ln(1+\sqrt2)\right)$$

3. a) $\sum_{n=1}^{\infty}\left(e^{\frac{1}{n^2}}-\cos\frac{2}{n}\right)(n-\ln(1+n))$ $\quad \sum_{n=1}^{\infty}\frac{n\cos(n\pi)}{\sqrt{n^4+2024}}$

$\left(1+\frac{1}{n^2}-1+\frac{2}{n^2}\right)\cdot\left(\frac{n^2}{n}\right)$

$\sim \frac{1}{n}$ divergira

$\left(1+\frac{1}{n^2}-1+\frac{2}{n^2}\right)(n-\ln n-\frac{1}{n}) = \frac{3}{n^2}(n-\ln n-\frac{1}{n}) = \frac{3}{n}\left(1-\frac{\ln n}{n}-\frac{1}{n^2}\right) = \lim \frac{3}{n} = 0$

b) $(-1)^n\cdot\frac{n}{\sqrt{n^4+2024}} \sim \frac{n}{n^2}\sim\frac{1}{n}$ divergira aps.

$\left(\frac{x}{\sqrt{x^4+2024}}\right)' = \frac{\sqrt{x^4+2024}-x\cdot\frac{4x^3}{2\sqrt{x^4+2024}}}{(x^4+2024)} = \frac{x^4+2024-2x^4}{(x^4+2023)^{3/2}} = \frac{2024-x^4}{>0}$ $\quad <0 \quad x>\sqrt[4]{2024} \sim 7$

Opada i ide u 0 pa po Lajbnicu konv uslovno

$\sum_{n=1}^{\infty}\sqrt{2}^n\left(\frac{3n-2}{3n+1}\right)^{n^2+n}$ $\quad \limsup\sqrt[n]{\sqrt{2}^n\left(\frac{3n-2}{3n+1}\right)^{n^2+n}} = \sqrt{2}\cdot\left(1+\frac{-3}{3n+1}\right)^{n+1} = \sqrt{2}\cdot e^{\frac{-3(n+1)}{3n+1}}$

$=\sqrt{2}\cdot e^{-1} = \frac{\sqrt{2}}{e}<1$ KONV

uvek pozitivno $|a_n|=a_n$ $\quad$ AK $\Rightarrow$ K pa ne konv. uslovno

4. $f_n(x)=x^n e^{-(n+4)x}$ $\quad x\in[0,1]$ $\quad x^n=\begin{cases}0, & x\in[0,1)\\ 1, & x=1\end{cases}$

$\lim_{n\to\infty} x\cdot e^{-x\cdot(n+4)} = $ $\to 0\cdot e^{-\infty}=0$; $\to 1\cdot e^{-\infty}=0$

$L=\lim_{n\to\infty}\sup|x^n\cdot e^{-x(n+4)}-0| = x^n e^{-x(n+4)}$ $\quad f_n'(x)=nx^{n-1}+x^n\cdot e^{-x(n+4)}\cdot(-n-4)$
$=x^{n-1}\cdot e^{-x(n+4)}(n-x(n+4))$

$f_n\nearrow$ $\quad n>x(n+4)$ tj $x<\frac{n}{n+4}$ tj. $\sup=\frac{n}{n+4}$

$\sup_{x\in[0,1]}|f_n(x)-f(x)| = f_n\left(\frac{n}{n+4}\right) = \frac{n}{n+4}\cdot e^{-n} = \left(1+\frac{-4}{n+4}\right)e^{-n} = e^{\frac{-4-n^2-n4}{n+4}} = e^{-n} = \boxed{0}$

b) $|x^n e^{-(n+4)x}| \le |e^{-x(n+4)}| \le e^{-(n+4)}$

$\sum_{n=1}^{\infty} e^{-(n+4)} = \frac{1}{e^4}\sum_{n=1}^{\infty}\frac{1}{e^n}$ konvergira $\lim\sqrt[n]{\frac{1}{e^n}} = 1/e < 1$ konv.

Po Vajerštrasu ravnomerno konvergira

a) $\int_{2}^{+\infty} \frac{2x^2+1}{x^2(x^2-1)}$

$\frac{2x^2+1-2+2}{x^2(x^2-1)} = \frac{2}{x^2} - \frac{2}{x^2(x^2-1)} \longrightarrow \frac{Ax+B}{x^2} + \frac{Cx+D}{x^2-1} = \frac{2}{\ldots}$

$= \int \left(\frac{4}{x^2} + \frac{2}{x^2-1}\right) = -4\frac{1}{x} + \frac{2}{2}\ln\left|\frac{1-x}{1+x}\right|$

$Ax^3 - Ax + Bx^2 - B + Cx^3 + Dx^2 = 2$

$B = -2$ $\quad A+C=0$

$D+B=0 \longrightarrow D=-2$

$A=0, \quad C=0$

$\lim_{b\to\infty} \left(\ln\left|\frac{1-b}{1+b}\right| - \frac{4}{b} + 2 + \ln 3\right) = \lim_{b\to\infty}\left(2+\ln 3 - 0 + \ln\left|\frac{1}{1}\right|\right) = 2+\ln 3$ $\quad \frac{1}{x} + \frac{-x}{1+x^2}$

b) $\lim_{a\to 0} \int_a^1 \frac{x-\operatorname{arctg}x}{x^2}dx = \lim_{a\to 0}\int_a^1 \frac{1}{x} - \int_0^1 \frac{\operatorname{arctg}x}{x^2} = \lim_{a\to 0}(-\ln a) + \frac{\operatorname{arctg}x}{x}\Big|_a^1 - \int_a^1 \frac{1}{x(1+x^2)}$ $\quad \frac{A}{x} + \frac{Bx+C}{x^2+1} = 1$

$Ax^2 + A = 1$, $A = 1$, $Bx^2 + Cx = 0$, $B = -1$

$= \lim_{a\to 0}\left(\frac{\pi}{4} - \cancel{\ln a} - \frac{\operatorname{arctg}a}{a} - \cancel{\ln x\Big|_a^1} + \int_a^1 \frac{2x}{2(1+x^2)}\right)$

$2y\cdot y' = (y^2)'$

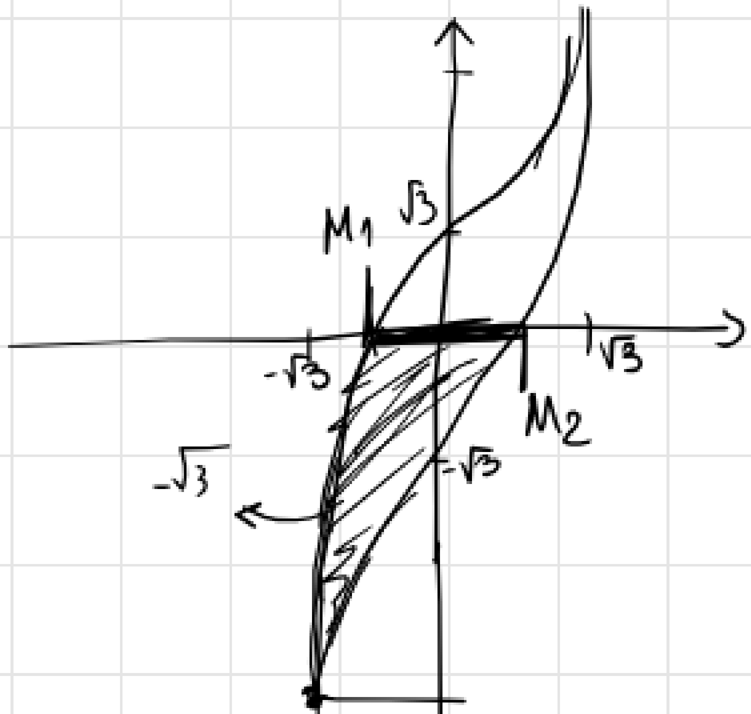
$= \lim_{a\to 0}\left(\frac{\pi}{4} - 1 + \frac{1}{2}\ln(1+x^2)\Big|_a^1\right) = \frac{\pi}{4} + \frac{1}{2}\ln 2 - 1$

c) $\int_0^1 \frac{x-\operatorname{arctg}x}{x^2(1+x^2)} = \lim_{a\to 0}\int_a^1 \frac{x-\operatorname{arctg}x}{x^2}dx - \int_a^1 \frac{x-\operatorname{arctg}x}{1+x^2}dx = \frac{\pi}{4} + \frac{1}{2}\ln 2 - 1 - \int_a^1 \frac{2x}{2(1+x^2)}dx + \frac{\operatorname{arctg}x^2}{2}\Big|_a^1$

$= \frac{\pi}{4} + \frac{1}{2}\ln 2 - 1 - \frac{1}{2}\ln(1+x^2)\Big|_a^1 + \frac{\pi^2}{32} = \frac{\pi^2}{32} + \frac{\pi}{4} - 1$

$y - 3x^3 = \pm\sqrt{3-x^2}$

$y = 3x^3 \pm \sqrt{3-x^2} \longrightarrow \pm\sqrt{3} \longrightarrow 9\sqrt{3}$, $-9\sqrt{3}$



$9x^6 - 3 + x^2 = 0 \qquad x^2 = r$

$9r^3 - 3 + r = 0$

$r(9r^2+1) = 3$

$2\int_{-9\sqrt{3}}^{a}\left(3x^3 - \sqrt{3-x^2} - 3x^3 - \sqrt{3-x^2}\right)dx = -2\int_{-9\sqrt{3}}^{a}\sqrt{3-x^2} = -2\sqrt{3}\int^{a}\sqrt{1-\left(\frac{x}{\sqrt{3}}\right)^2} \longrightarrow \frac{x}{\sqrt{3}} = \sin t$, $\frac{dx}{\sqrt{3}} = \cos t$

$\cos 2t = 2\cos^2 t - 1 \longrightarrow \frac{\cos 2t+1}{2}$

$= -2\sqrt{3}\cdot\frac{1}{\sqrt{3}}\cdot\int \cos t\cdot\cos t = -2\int\frac{\cos 2t+1}{2} = \boxed{\left|-\arcsin\frac{x}{\sqrt{3}} + \frac{1}{2}\sin\left(2\arcsin\frac{x}{\sqrt{3}}\right)\Big|_{9\sqrt{3}}^{a}\right| = P}$

$\to a_n \qquad \to x_0 = 2020$

$$R = \lim_{n\to\infty} \frac{n^2 \sqrt{(n+1)^6+1}}{(n+1)^2\sqrt{n^6+1}} = 1 \longrightarrow (2020-1,\ 2020+1)$$

Konv u $x = 2019 \to \sum_{n=0}^{n} \frac{1^n n^2}{\sqrt{n^6+1}}$ $\quad \frac{n^2}{\sqrt{n^6}} \sim \frac{n^2}{n^3} \sim \frac{1}{n}$ divergira

$x = 2021 \to \frac{(-1)^n n^2}{\sqrt{n^6+1}}$ ne konv apsolutno $\quad (2019, 2021]$

$\sim \frac{n^2}{n^3} \sim \frac{1}{n}$ opada i $\lim \frac{1}{n} = 0$ Uslovno konv

a) $$\int_1^2 e^{\sqrt{x-1}}dx \qquad \begin{aligned}\sqrt{x-1} &= y\\ x &= y^2+1\\ dx &= 2y\,dy\end{aligned} \quad = \int_0^1 e^y \cdot 2y\,dy = 2(e^y\cdot y)\Big|_0^1 - 2\int_0^1 e^y dy = 2e - 2e + 2 = \boxed{2}$$

b) $$\int_{-1}^1 \frac{2x+1-1}{2(x^2+x+1)} = \left[\begin{aligned}x^2+x+1 &= y\\ 2x+1 &= dy\end{aligned}\right] = \int_1^3 \frac{dy}{2y} - \int_{-1}^1 \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}} =$$

$$= \frac{1}{2}\ln 3 - \int_{-1}^1 \frac{\frac{2}{\sqrt3}\cdot\frac{2}{\sqrt3}dx}{(\frac{2}{\sqrt3}x+\frac{1}{\sqrt3})^2+1} = \frac{1}{2}\ln 3 - \frac{2}{\sqrt3}\operatorname{arctg}\left(\frac{2}{\sqrt3}x + \frac{1}{\sqrt3}\right)\Big|_{-1}^1$$

$$= \frac{1}{2}\ln 3 - \frac{2}{\sqrt3}\cdot\frac{\pi}{3} + \frac{2}{\sqrt3}\frac{\pi}{6}$$

c) $$\int x^3(1+x^2)^{1/3}dx = \frac{1}{2}\int x^2\cdot 2\cdot x(1+x^2)^{1/3} = \frac{1}{2}\left(x^2\cdot\frac{3}{4}(1+x^2)^{4/3} - \int\frac{3\times 2}{4}(1+x^2)^{4/3}\right)$$

$$= \frac{3}{8}x^2(1+x^2)^{4/3} - \frac{3}{8}\cdot\frac{3}{7}(1+x^2)^{7/3}$$

$$2\pi\int_0^{\pi/4} \operatorname{tg}x\sqrt{1+\frac{1}{\cos^4 x}}\,dx = 2\pi\int_0^{\pi/4}\frac{\sin x}{\cos x}\sqrt{\frac{\cos^4 x+1}{\cos^4 x}} = 2\pi\int_0^{\pi/4}\frac{\sin x\cdot\cos^4 x}{\cos^5 x}\sqrt{1+\frac{1}{\cos^4 x}}$$

$$u = 1+\frac{1}{\cos^4 x} \qquad du = \frac{\sin x}{\cos^5 x}$$

$$\to 2\pi\int_2^5 du\cdot\frac{\sqrt u}{u-1} \qquad \begin{aligned}y &= \sqrt u\\ 2y &= du\end{aligned} \quad = 2\pi\int_{\sqrt2}^{\sqrt3}\frac{2y^2-2+2}{y^2-1} = 2\pi\int_{\sqrt2}^{\sqrt5} 1 + 2\frac{1}{y^2-1})$$

$$= 2\pi\left(\sqrt5-\sqrt2 + \ln\left|\frac{1-\sqrt5}{1+\sqrt5}\right| - \ln\left|\frac{1-\sqrt2}{1+\sqrt2}\right|\right)$$

$$\int_1^\infty \frac{dx}{x\ln^a x(\ln^b x+1)\ln^2(\ln x+1)} = \begin{bmatrix} \ln x = u \\ dx\frac{1}{x} = du \end{bmatrix} = \int_0^\infty \frac{du}{u^a(u^b+1)\ln^2(u+1)}$$

U blizini 0 $\int_0^1 \frac{du}{u^a \ln^2 u} = \frac{du}{u^{a+2}}$ $\quad a+2<1$ tj $a<-1$

U blizini ∞ $\int_1^\infty \frac{du}{u^{a+b}\ln^2 u}$ $\to a+b=1$, $2>1$ kon; $\to a+b>1$

x^2 $\quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + b_n\sin nx$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{\pi}\cdot\frac{1}{3}(\pi^3+\pi^3) = \frac{2\pi^2}{3}$$

x^2 je parna fja parna·neparna = neparna pa $b_n = 0$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos(nx)dx = \begin{bmatrix} du = \cos(nx)dx & v = x^2 \\ u = \frac{1}{n}\sin(nx) & dv = 2x\,dx \end{bmatrix} = \frac{2}{\pi}\left(\frac{x^2}{n}\sin(nx)\Big|_0^\pi - 2\int_0^\pi \frac{x}{n}\sin(nx)dx\right) =$$

$$= \frac{2}{\pi}\left(\frac{\pi^2}{n}\sin(n\pi) + 2\frac{x}{n}\cdot\frac{1}{n}\cdot\cos(nx)\Big|_0^\pi - \frac{2}{n^2}\int_0^\pi \cos(nx)\,dx\right)$$

$$= \frac{2}{\pi}\left(\underbrace{\frac{\pi^2}{n}\sin(n\pi)}_{\in\mathbb{N}\to=0} + 2\frac{\pi}{n^2}\underbrace{\cos(nx)}_{(-1)^n} + \frac{2}{n^3}\underbrace{\sin nx\Big|_0^\pi}_{0}\right) = \frac{4}{n^2}(-1)^n$$

$$\to f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n\cos nx \quad \blacksquare$$

Jun 1 2024

1

(a) $$\int_0^{\frac{\pi}{4}} \frac{(e^{\operatorname{tg} x}+e^{\operatorname{tg}^2 x})\sin x}{\cos^3 x}dx = \int_0^{\pi/4} \frac{e^{tgx}+e^{tg^2x}}{\cos^2 x}\cdot tgx\,dx =$$

$$\left[\begin{aligned} t &= e^{tg^2x} \\ \ln t &= tg^2x \\ \frac{1}{t}dt &= \frac{2tgx}{\cos^2 x} \end{aligned}\right] \qquad \left[\begin{aligned} u &= e^{tgx} \\ \ln u &= tgx \\ \frac{1}{u}du &= \frac{1}{\cos^2 x} \end{aligned}\right]$$

$$= \int_0^{\pi/4} \frac{e^{tgx}\cdot tgx}{\cos^2 x}dx + \int_0^{\pi/4}\frac{e^{tg^2x}\,tgx}{\cos^2 x}\cdot dx =$$

$$= \int_1^e \frac{u\cdot\ln u}{u}du + \int_1^e \frac{t\,du}{2t} = u\cdot\ln u - u\Big|_1^e + \frac{t}{2}\Big|_1^e =$$

$$= e^{tgx}(tgx - e^{tgx})\Big|_0^{\pi/4} + \frac{e^{tg^2x}}{2}\Big|_0^{\pi/4} = e(1-e)-(0-1)+\frac{e}{2}-\frac{1}{2}$$

$$= \boxed{\frac{3e}{2}-e^2+\frac{1}{2}}$$

(б) $$\int \frac{dx}{\sqrt[6]{x^5}\,(2+\sqrt{x}-\sqrt[3]{x})} = \left[\begin{aligned} x &= t^6 \\ dx &= 6t^5dt\end{aligned}\right] = \int \frac{6\cancel{t^5}dt}{\cancel{t^5}(2+t^3-t^2)} = \int\frac{6dt}{t^3-t^2+2}$$

$$t^3-t^2+2 = (t+1)(t^2-2t+2) \qquad \frac{A}{t+1}+\frac{Bt+C}{t^2-2t+2} = \frac{1}{\dots}$$

$$\left.\begin{aligned} A + B &= 0 \to A=-B \\ -2A+B+C &= 0 \\ 2A + C &= 1 \to C = 1-2A \end{aligned}\right\} \quad \begin{aligned} -2A+1-2A-A&=0 \\ A &= 1/5 \\ B &= -1/5 \\ C &= 3/5 \end{aligned}$$

$$= \frac{6}{5}\int \frac{1}{t+1} - \frac{t-3}{t^2-2t+2}dt = \frac{6}{5}\ln|t+1| - \frac{6}{5}\int \frac{2t-2}{2(t^2-2t+2)} - \frac{2}{(t-1)^2+1}dt =$$

$$\frac{6}{5}\left(\ln|t+1| - \frac{1}{2}\ln|t^2-2t+2| - 2arctg(t-1)\right) + C$$

(в) $$\int_0^{2\pi}\frac{\sin 2x+|\cos x|}{\sin^2 x+\sin x+1}dx = \int_0^{\pi/2} \overset{\cos}{I_1} + \int_{\pi/2}^{3\pi/2} \overset{-\cos}{I_2} + \int_{3\pi/2}^{2\pi}\overset{+\cos}{I_1} = \ln 3 - 2\sqrt2 arctg\left(-\frac{3}{2\sqrt2}\right) - \ln 3 + 2\sqrt2 arctg\left(\frac{1}{2\sqrt2}\right)$$

$$+\ln|1| - \ln|2-1| = 2\sqrt2\left(arctg\left(\frac{3}{2\sqrt2}\right) + arctg\left(\frac{1}{2\sqrt2}\right)\right)$$

→ Nakon ovoga ispod

$$I_1 = \int\frac{2\sin x\cos x+\cos x}{\sin^2x+\sin x+1}dx = \left[\begin{aligned} \sin^2x+\sin x+1 &= t \\ (2\sin x\cos x+\cos x)dx &= dt\end{aligned}\right] = \int\frac{dt}{t} = \ln|\sin^2x+\sin x+1|$$

$$I_2 = \int\frac{\cos x(2\sin x-1)}{\sin^2x+\sin x+1} = \left[\begin{aligned}\sin x &= t \\ \cos x &= dt\end{aligned}\right] = \int\frac{2t-1+1-1}{t^2+t+1} = \ln|\sin^2x+\sin x+1| - \int\frac{2}{(t-\frac12)^2+(\frac{1}{\sqrt2})^2}$$

$$= \ln|\sin^2x+\sin x+1| - 2\int\frac{1}{\frac12\left(\left(\frac{t-\frac12}{\sqrt2}\right)^2+1\right)}dt = \dots - 4\int\frac{dt}{\left(\frac{t-\frac12}{\sqrt2}\right)^2+1}$$

$$= \dots - 4\frac{1}{\sqrt2}arctg\left(\frac{t-\frac12}{\sqrt2}\right) = \dots -2\sqrt2 arctg\left(\frac{\sin x-\frac12}{\sqrt2}\right)$$

$$-\frac{3}{2\sqrt2}$$

2. а) Испитати конвергенцију интеграла $\int_1^{+\infty} \frac{\sqrt{x^3-3x^2+4x-2}\,\mathrm{arctg}\,x}{\sqrt[3]{\ln x}(e^{x-1}-1)}dx = \int_1^2 + \int_2^{+\infty}$

U 1: $\frac{\sqrt{x^3-3x^2+4x-2}\,\mathrm{arctg}x}{\sqrt[3]{\ln x}\,(e^{x-1}-1)} \sim \frac{\pi}{4}\frac{\sqrt{(x-1)(x^2-2x+2)}}{\sqrt[3]{\ln x}\,(x-1)} \sim \frac{\pi\,\cancel{(x-1)}\sqrt{x-1}}{4\,\cancel{(x-1)}\sqrt[3]{\ln x}}$

$$\lim_{x\to1}\frac{(x-1)^{1/2}}{\sqrt[3]{\ln x}} \overset{0/0}{=} \lim_{x\to1}\frac{\frac12 (x-1)^{-1/2}}{\frac13\cdot\frac1x\cdot\frac{1}{\sqrt[3]{\ln^2 x}}} = \lim_{x\to1}\frac{3x^{1/2}\sqrt[3]{\ln^2 x}}{2\cancel{x^{1/2}}} = 0 \to \text{konvergira}$$

U $+\infty$: $\frac{\sqrt{x^3\cdot\pi/2}}{\sqrt[3]{\ln x}\,(e^{x-1}-1)} \sim \frac{x^{3/2}}{(\ln x)^{1/3}\cdot e^x}$ $\quad e^x \gg x^{3/2} \to$ konvergira

б) Израчунати дужину лука криве $y=e^x$ на интервалу $x\in[0,\frac32\ln 2]$

$$\int_0^{\frac32\ln2}\sqrt{1+e^{2x}}\,dx = \left[\begin{array}{l} 1+e^{2x}=t^2\\ e^{2x}=t^2-1\\ 2x=\ln(t^2-1)\\ dx=\frac{t}{t^2-1}\end{array}\right] = \int_{\sqrt2}^{3}\frac{t^2-1+1}{t^2-1}dt = \int_{\sqrt2}^{3} 1+\frac{1}{t^2-1}\,dt$$

$$= t+\frac12\ln\left|\frac{1-t}{1+t}\right| = \left(\sqrt{1+e^{2x}}+\frac12\ln\left|\frac{1-\sqrt{1+e^{2x}}}{1+\sqrt{1+e^{2x}}}\right|\right)\Bigg|_0^{\frac32\ln2} = 3+\frac12\ln\frac12-1 = \boxed{2-\frac12\ln2}$$

(а) $\sum_{n=1}^{\infty}\frac{7^n\,\mathrm{arctg}\,n}{5^n+2^{3n}} \Rightarrow \lim_{n\to\infty}\sqrt[n]{\frac{7^n\,\mathrm{arctg}\,n}{5^n+8^n}} = \lim_{n\to\infty}\sqrt[n]{\frac{8^n(\frac78)^n\pi}{8^n(1+(\frac58)^n)2}} = \lim_{n\to\infty}\sqrt[n]{\frac{(\frac78)^n\cdot\pi}{(1+(\frac58)^n)\cdot2}} = 0$ konv.

(б) $\sum_{n=1}^{\infty}\frac{2^n(n!)^2 5^{-n}}{(2n+1)!} \Rightarrow \lim_{n\to\infty}\frac{2\cdot\cancel{2^n}(n+1)^2\cdot\cancel{(n!)^2}\,\cancel{5^{-n}}\,\cancel{(2n+1)!}}{5\cdot\cancel{2^n}\cdot\cancel{(n!)^2}\cdot\cancel{5^{-n}}\,(2n+3)(2n+2)\cancel{(2n+1)!}} = \lim_{n\to\infty}\frac{\cancel{n}(1+\frac1n)}{\cancel{n}(10+\frac{10}{n})} = \frac{1}{10}$ konv.

(в) $\sum_{n=2}^{\infty}(-1)^n\underbrace{\sin\frac3n\ln\left(\frac{2n^2+n}{n^2-1}\right)}_{\geq0} \Rightarrow \lim_{n\to\infty}(-1)^n\cdot\frac3n\ln\left(\frac{2\cancel{n^2}(1+\frac{1}{2n})}{\cancel{n^2}(1-\frac{1}{n^2})}\right) = \lim_{n\to\infty}\frac{3\ln2\,(-1)^n}{n} = 0$

$$\cos\frac3n\cdot\left(-\frac{3}{n^2}\right)\ln\left(2+\frac{n+2}{n^2-1}\right)+\sin\frac3n\cdot\frac{\frac{(4n+1)(n^2-1)-2n(2n^2+n)}{(n^2-1)^{\cancel{2}}}}{\frac{2n^2+n}{\cancel{n^2-1}}}$$

$$=\cos\frac3n\left(-\frac{3}{n^2}\right)\ln\left(2+\frac{n+2}{n^2-1}\right)+\sin\frac3n\cdot\frac{\cancel{4n^3}-\cancel{4n^3}+n^2-1-4n-2n^2}{(n^2-1)n(2n+1)}=$$

poz za n=3 poz — poz — za n=1 poz

$$=\cos\frac3n\left(-\frac{3}{n^2}\right)\ln\left(2+\frac{n+2}{n^2-1}\right)-\sin\frac3n\cdot\frac{n^2+4n+1}{(n^2-1)n(2n+1)} \to \text{poz}$$

$\to$ poz

Za n=2 ili 3 opada

$\Longrightarrow a_n\geq0,\ a_n\searrow,\ \lim_{n\to\infty}a_n=0$ konv.

4. Развити функцију $f(x) = x^2 + x$ у Фуријеов ред на $(-\pi, \pi)$ и израчунати суме $A = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ и $B = \sum_{n=1}^{\infty} \frac{n^2+4}{n^4}$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (x^2+x)\,dx = \frac{1}{\pi \cdot 3}(\pi^3 + \pi^3) = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x^2+x)\cos(nx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos(nx)\,dx = \frac{1}{\pi}\left(\frac{1}{n}x^2\sin(nx) - \int_{-\pi}^{\pi}\frac{1}{n}\sin(nx)2x\,dx\right)$$

$$= \frac{1}{\pi}\left(\frac{x^2}{n}\overset{n\pi \to 0}{\sin(nx)} - \frac{2}{n}\left(\frac{-1}{n}\cos(nx)\cdot x - \int_{-\pi}^{\pi}\left(-\frac{1}{n}\right)\cos(nx)\,dx\right)\right) =$$

$$= \frac{1}{\pi}\left(\frac{2}{n^2}\cdot x\cdot\cos(nx)\Big|_{-\pi}^{\pi} + \frac{1}{n^2}\sin(nx)\Big|_{-\pi}^{\pi}\right) = \frac{1}{\pi}\left((-1)^n\cdot\frac{2}{n}\cdot 2\pi + 0\right) = \frac{4}{n^2}(-1)^n$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(\overbrace{\underbrace{x^2+x)\sin(nx)}_{\text{parna}}}^{\text{odd}=0}\,dx = \frac{2}{\pi}\int_0^{\pi} x\sin(nx) = \frac{2}{\pi}\left(\frac{1}{n}x\cos(nx) - \frac{1}{n}\int_0^{\pi}\cos(nx)\,dx\right)$$

$$= \frac{-2}{\pi}\left(\frac{1}{n}x\cos(nx) - \frac{1}{n^2}\sin(nx)\right)\Big|_0^{\pi} =$$

$$= \frac{-2}{\pi}\left(\frac{1}{n}\pi(-1)^n - \frac{1}{n^2}0\right) = \frac{2}{n}(-1)^n(-1)$$

$$\Rightarrow f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left(\frac{(-1)^n\cdot 4}{n^2}\cos(nx) - \frac{2(-1)^n}{n}\sin(nx)\right)$$

$$\sum\frac{(-1)^n}{n^2} \qquad x=0 \to 0 = \frac{\pi^3}{3} + 4\sum\frac{(-1)^n}{n^2} \to \sum\frac{(-1)^n}{n^2} = -\frac{\pi^3}{12}$$

$$\frac{n^2+4}{n^4}? \qquad \frac{a_0^2}{2} + \sum a_n^2 + b_n^2 = \frac{2\pi^4}{9} + \sum\frac{16}{n^4} + \frac{4}{n^2} = \frac{1}{\pi}\int_{-\pi}^{\pi}|f(x)|^2\,dx$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} x^4+2x^3+x^2\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^4+x^2\,dx = \frac{1}{\pi}\left(\frac{1}{5}x^5 + \frac{1}{3}x^3\right)\Big|_{-\pi}^{\pi} = \left(\frac{\pi^4}{5}+\frac{\pi^2}{3}\right)2$$

$$\frac{1}{4}\left(\left(\frac{\pi^4}{5}+\frac{\pi^2}{3}\right)2 - \frac{2\pi^4}{9}\right) = \sum\frac{4+n^2}{n^4} = \frac{1}{2}\left(\frac{4\pi^4+15\pi^2}{45}\right)$$

1. Израчунати интеграл $\int \frac{\sqrt{1-x^2}}{x^2+1}dx = \begin{matrix} x=\sin t \\ dx=\cos t \end{matrix} = \int \frac{\sqrt{1-\sin^2 t}}{\sin^2 t+1}\cdot \cos t = \int \frac{\cos^2 t}{\sin^2 t+1}$

$$= \int \frac{\cos^2 t}{1+\sin^2 t} = \begin{matrix} \operatorname{tg} t = u \\ dt = \frac{1}{u^2+1} \\ \sin^2 t = \frac{u^2}{1+u^2} \\ \cos^2 t = \frac{1}{1+u^2} \end{matrix} = \int \frac{\frac{1}{1+u^2}}{\frac{1+2u^2}{1+u^2}} \cdot \frac{1}{u^2+1}\, du = \int \frac{1}{(1+2u^2)(u^2+1)}$$

$$\frac{Au+B}{1+2u^2} + \frac{Cu+D}{u^2+1} = \frac{1}{\ldots}$$

$$\begin{cases} A+2C=0 & u^3 \\ B+2D=0 & u^2 \\ A+C=0 & u \\ B+D=1 & 1 \end{cases}$$

$D=-1$
$B=2$ $A=C=0$

$$= \int \frac{2}{1+2u^2} - \frac{1}{u^2+1} = \sqrt{2}\operatorname{arctg}(\sqrt{2}\operatorname{tg}(\arcsin x)) - \operatorname{arctg}(\operatorname{tg}(\arcsin x)) + C$$

2. Испитати апсолутну конвергенцију следећих редова:

$$\text{(а)} \sum_{n=1}^{+\infty} \frac{(-1)^n}{\ln(n^2+1)}, \qquad \text{(б)} \sum_{n=1}^{+\infty} \frac{n!}{2^n n^2}.$$

a) $\lim_{n\to\infty} \frac{1}{\ln(n^2+1)} = 0$ ✓ $\quad \frac{1}{\ln(n^2+1)} \sim \frac{1}{\ln n^2} \sim \frac{1}{2\ln n}$ $\quad \frac{1}{n^p \ln^q n}$ konv za $p>1$ i $p=1, q>1$

$\Rightarrow$ Ne konv.

b) $\frac{n!}{2^n \cdot n^2}$ $\quad \lim_{n\to\infty} \frac{(n+1)\cancel{n!}\cdot \cancel{2^n}\cdot n^2}{\cancel{n!}\, 2\cdot \cancel{2^n}\cdot \cancel{(n+1)}(n+1)} = \infty$ Ne konvergira

3. Функцију $f(x) = x^3$ развити у Фуријеов ред на $(-\pi, \pi)$ и израчунати $\sum_{n=1}^{+\infty} \frac{(-1)^n(6-\pi^2(2n+1)^2)}{(2n+1)^3}$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^3\,dx = 0 \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^3 \cos(nx) = 0 \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^3 \sin(nx)\,dx = \frac{-1}{n}\cos(nx)x^3 + \frac{1}{n}\int 3x^2\cos(nx)$$

$$= \frac{1}{n\pi}\left(-\cos(nx)x^3 + \frac{3}{n}\left(+\frac{1}{n}\sin(nx)\cdot x^2 - \int\left(+\frac{1}{n}\sin(nx)2x\right)\right)\right) = \frac{1}{n\pi}\left(-x^3\cos(nx) - \frac{6}{n}\left(-x\frac{1}{n}\cos(nx) + \frac{1}{n}\int_{-\pi}^{\pi}\cos nx\,dx\right)\right)$$

$$= \frac{1}{n\pi}\left(-2\pi^3(-1)^n + \frac{12}{n^3}\pi(-1)^n + \frac{1}{n^2}\sin(nx)\,\Big|\right) = \frac{1}{n\pi}\left(-2\pi^3(-1)^n + \frac{12}{n^2}\pi(-1)^n\right)$$

$$= \left(-\frac{\pi^2}{n} + \frac{6}{n^3}\right)(-1)^n = (-1)^n\frac{1}{n^3}(6-\pi^2 n^2)$$

$$f(x) = \sum_{n=1}^{\infty}\left(\frac{6-\pi^2 n^2}{n^3}\right)(-1)^n \sin(nx)$$

$$\frac{-\pi^3}{8} = \sum_{n=0}^{\infty} \frac{6-\pi^2(2n+1)^2}{(2n+1)^3}(-1)^{2n+1}\cdot(-1)^n$$

$$\frac{-\pi^3}{8} = \sum_{n=0}^{\infty} \frac{6-\pi^2(2n+1)^2}{(2n+1)^3}(-1)^n$$

$$\sum_{1}^{\infty} = \frac{-\pi^3}{8} - \pi^2 + 6$$

1. Израчунати запремину обртног тела насталог ротацијом функције $f(x)=\sqrt{\dfrac{x\ln(1+x)}{x^4+8x^2+16}}$, $x\in[0,1]$ око x-осе.

$$V=\frac{\pi}{2}\int_0^1\frac{2x\ln(1+x)}{(x^2+4)^2}dx=\left[\begin{array}{ll}\frac{2x}{(x^2+4)^2}=dv & \ln(1+x)=u\\ v=-\frac{1}{(x^2+4)} & \frac{1}{1+x}=du\end{array}\right]=\frac{\pi}{2}\left(-\frac{\ln(1+x)}{x^2+4}+\int\frac{dx}{(1+x)(x^2+4)}\right)$$

$$\left[\ \frac{A}{1+x}+\frac{Bx+C}{x^2+4}=\frac{1}{\dots}\right.$$

$$\begin{array}{lll} A+B=0 & x^2 & A=C\\ B+C=0 & x & \\ 4A+C=1 & 1 & 5A=1\\ & & A=1/5=C\\ & & B=-1/5\end{array}$$

$$=\frac{\pi}{2}\left(-\frac{\ln(1+x)}{x^2+4}+\frac15\left(\ln(1+x)+\int\frac{1-x}{x^2+4}\right)\right)$$

$$=\frac{\pi}{2}\left(-\frac{\ln(1+x)}{x^2+4}+\frac15\left(\ln(1+x)+\int-\frac12\frac{2x}{x^2+4}+\frac{\frac12}{4\left(\left(\frac{x}{2}\right)^2+1\right)\frac12}\right)\right)$$

$$=\frac{\pi}{2}\left(-\frac{\ln(1+x)}{x^2+4}+\frac15\ln(1+x)-\frac12\ln(x^2+4)+\frac12\operatorname{arctg}\frac{x}{2}\right)\Bigg|_0^1=$$

$$=\frac{\pi}{2}\left(-\cancel{\frac{\ln 2}{5}}+\cancel{\frac15\ln(2)}-\frac12\ln(5)+\frac12\ln4+\frac12\operatorname{arctg}\frac12\right)=\frac{\pi}{4}\left(\ln\frac45+\operatorname{arctg}\frac12\right)$$

2. Испитати конвергенцију следећих интеграла:

$$\text{(а)}\ \int_1^{+\infty}\frac{1}{\sqrt[3]{x^4-x}}\mathrm{d}x,\qquad\text{(б)}\ \int_2^{+\infty}\frac{\ln(1+x)}{x^2\sin\left(\frac{1}{\sqrt{x}}\right)}\mathrm{d}x.$$

a) U 1 $\quad t=x-1,\ t\to0\quad \displaystyle\int_0^1\frac{1}{\sqrt[3]{(t+1)(t^3+3t^2+3t)}}=\int_0^1\frac{1}{\sqrt[3]{1\cdot t(3)}}\quad t^{1/3}\quad 1/3<1$ Konv.

U ∞ $\quad\sim\dfrac{1}{\sqrt[3]{x^4}}\sim\dfrac{1}{x^{4/3}}\quad 4/3>1$ Konv

b) $\dfrac{\ln(1+x)}{x^2\sin\left(\frac{1}{\sqrt{x}}\right)}\sim\dfrac{\ln x}{x^2\cdot x^{-1/2}}\sim\dfrac{1}{\ln^{-1}x\cdot x^{3/2}}$ Konvergira

3. Дат је степени ред $\displaystyle\sum_{n=1}^{+\infty}\frac{(-1)^{n+1}x^n}{n(n+2)}$.

(а) Одредити његову област конвергенције.

$x_0=0\qquad\displaystyle\lim_{n\to\infty}\frac{\cancel{(-1)^{n+1}}(n+1)(n+3)}{-\cancel{(-1)^{n+1}}\cancel{(n+2)}\cancel{n}}=\boxed{-1}\ \to\ D\subseteq[-1,1]$

U (1) $\to a_n=\dfrac{1}{n(n+2)}\qquad\displaystyle\lim_{n\to\infty}\dots=0\qquad\left(\frac{1}{n^2+2n}\right)'=-\frac{2n+2}{(n^2+2n)^2}$ Uvek neg $\searrow$

Konv po Lajbnicu

U (−1) Σ bez − $\dfrac{1}{n(n+2)}\sim\dfrac{1}{n^2}$ konv aps pa konv

1. У зависности од реалног параметра $a \in (1,2)$ израчунати

$$\int_0^{\frac{\pi}{2}} \cos^a(x) \underbrace{\sin^{4-2a}\left(\frac{x}{2}\right)}_{\left(\sin^{2-a}\left(\frac{x}{2}\right)\right)^2 = \frac{1-\cos^{2-a} x}{2}} \operatorname{tg}(x) \mathrm{d}x.$$

$1 - 2\sin = \cos 2$

$\frac{1-\cos 2}{2}$

$$= \frac{1}{2^{2-a}} \int_0^{\pi/2} \frac{\cos^a(x)(1-\cos x)^{2-a} \sin x}{\cos x} dx = \left[\begin{array}{l}\cos x = t \\ -\sin x = dt\end{array}\right] = \frac{1}{2^{2-a}} \int_0^1 t^a (1-t)^{2-a} t^{-1} dt =$$

$$= \frac{1}{2^{2-a}} \int_0^1 t^{a-1} (1-t)^{2-a = 3-a-1} dt = \frac{1}{2^{2-a}} B(a, 3-a) = \frac{1}{2^{2-a}} \frac{\Gamma(a)\Gamma(3-a)}{\Gamma(3)} = \frac{(3-a)(2-a)}{2^{3-a}} \cdot \Gamma(a)\Gamma(1-a)$$

$\Gamma(3-a) \to (3-a)\Gamma(2-a)$

$(3-a)(2-a)\Gamma(1-a)$

$$= \frac{(3-a)(2-a)}{2^{3-a}} \frac{\pi}{\sin(\pi a)}$$

2. Испитати условну и апсолутну конвергенцију следећих редова:

$$\text{(а)} \sum_{n=4}^{+\infty} \frac{(-1)^n}{\sqrt{[\frac{n}{2}]} + (-1)^n}, \qquad \text{(б)} \sum_{n=1}^{+\infty} \sin\left(\frac{1}{n}\right) - \frac{1}{n}.$$

Zaokruživanje na niže

$5/2 = 2$ NPR

$$a) \sum_{n=4}^{+\infty} \frac{(-1)^n}{\sqrt{[\frac{n}{2}]} + (-1)^n} = \underbrace{\sum_{n=2}^{\infty} \frac{1}{1+\sqrt{n}}}_{\text{parni}} + \sum_{n=2}^{\infty} \frac{-1}{\sqrt{n}-1}$$

$$= \sum_{n=2}^{\infty} \frac{\sqrt{n}-1-1-\sqrt{n}}{n-1} = -2\sum_{n=2}^{\infty} \frac{1}{n-1} \qquad \frac{1}{n-1} \sim \frac{1}{n}$$ divergira. Okej num, ipak

$$\frac{(-1)^n}{\sqrt{\cdot} + (-1)^n} \sim \frac{1}{\sqrt{n}}$$ divergira lmao

$$b) \sin\left(\frac{1}{n}\right) - \frac{1}{n} \sim \frac{1}{n} - \frac{1}{6n^3} - \frac{1}{n} \sim \frac{1}{n^3}$$ konv.

3.

(а) Развити функцију $f(x) = \cosh x$ у Фуријеов ред на интервалу $[-\pi, \pi]$.

$$\frac{e^x + e^{-x}}{2} \qquad a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} \frac{e^x + e^{-x}}{2} dx = \frac{e^x - e^{-x}}{2} \Big|_{-\pi}^{+\pi} = \frac{e^{\pi} - e^{-\pi} - e^{-\pi} + e^{+\pi}}{2} = \frac{1}{\pi}\left(e^{\pi} - e^{-\pi}\right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \cosh x \cdot \cos(nx) \, dx =$$

$I = \int \cosh x \cdot \cos(nx) = n\cos(nx) \cdot \sinh x + n^2 \int \sin(nx) \cdot \sinh x \, dx$

$= n\cos(nx) \sinh x + n^2 (\sin(nx) \cosh x - n \int \cos hx \cosh x)$

$(1+n) I = n\cos(nx) \sinh x + n \sin(nx) \cosh x$

$$a_n = \frac{1}{1+n^2}\left((-1)^n\left(\frac{e^{\pi}-e^{-\pi}}{2} - \frac{e^{-\pi}-e^{\pi}}{2}\right)\right) = \frac{(-1)^n (e^{\pi} - e^{-\pi})}{(1+n^2)}$$

$\cosh x \cdot \sin(nx) \to$ neparna $= 0$

$$\frac{e^{\pi} - e^{-\pi}}{\pi} + \sum_{n=1}^{\infty} \frac{(-1)^n (e^{\pi} - e^{-\pi})}{(1+n^2)} \cos(nx)$$