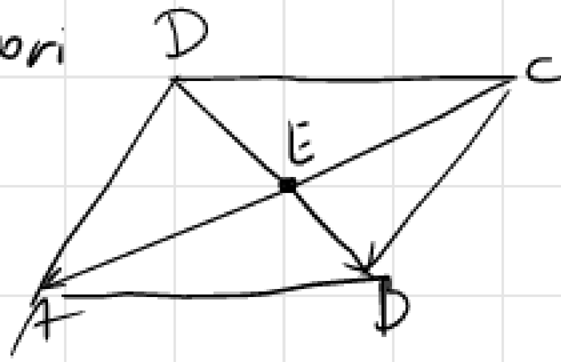
Ventori

2.1.



$\Rightarrow \overrightarrow{AE} = \overrightarrow{EC} \quad \overrightarrow{BE} = \overrightarrow{ED}$

$\overrightarrow{AB} = \overrightarrow{AE} + \overrightarrow{EB} = \overrightarrow{EC} + \overrightarrow{DE} = \overrightarrow{DC}$

$\Leftarrow \quad \overrightarrow{AB} = \overrightarrow{DC}$

$\overrightarrow{AB} = \overrightarrow{AE} + \overrightarrow{EB} = \overrightarrow{AE} + \lambda \overrightarrow{DB} = \mu \overrightarrow{AC} + \lambda \overrightarrow{DB}$

$\overrightarrow{AB} = \overrightarrow{DC} = \overrightarrow{DE} + \overrightarrow{EC} = \overrightarrow{EC} + (1-\lambda)\overrightarrow{DB} = (1-\mu)\overrightarrow{AC} + (1-\lambda)\overrightarrow{DB}$

$(2\mu-1)\overrightarrow{AC} = (1-2\lambda)\overrightarrow{DB} \qquad \overrightarrow{AE} = \frac{\overrightarrow{AC}}{2} = \overrightarrow{EC}$

$\mu = 1/2 \qquad \lambda = 1/2 \Rightarrow \qquad \overrightarrow{EB} = \overrightarrow{ED}$

2.2 $\quad \overrightarrow{OA} \; ; \; \overrightarrow{OB}$

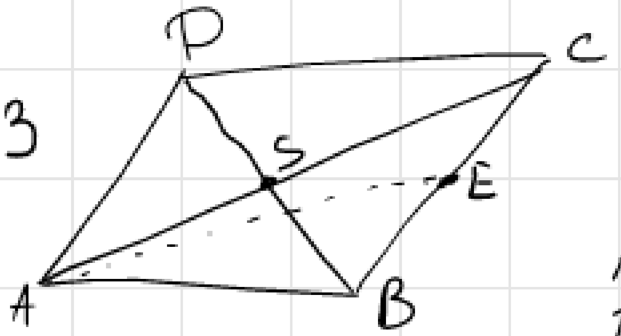
a) $\overrightarrow{AC} = \lambda \overrightarrow{CB} \Rightarrow \overrightarrow{AO} + \overrightarrow{OC} = \lambda(\overrightarrow{CO} + \overrightarrow{OB}) \quad \overrightarrow{OC} + \lambda \overrightarrow{OC} = \lambda \overrightarrow{OB} + \overrightarrow{OA}$

$\overrightarrow{OC} = \frac{\lambda \overrightarrow{OB} + \overrightarrow{OA}}{1+\lambda}$

b) $\frac{\overrightarrow{AC}}{\overrightarrow{CB}} = \frac{p}{q} \qquad q \cdot (\overrightarrow{OC} - \overrightarrow{OA}) = p(\overrightarrow{OB} - \overrightarrow{OC}) \quad (q+p)\overrightarrow{OC} = p\overrightarrow{OB} + q\overrightarrow{OA}$

$\overrightarrow{OC} = \frac{p\overrightarrow{OB} + q\overrightarrow{OA}}{q+p}$

2.3



$\overrightarrow{AE} \; ; \; \overrightarrow{AS} \quad ; AB, AC, AD$

$\overrightarrow{AB} = \overrightarrow{AE} + \overrightarrow{EB} = \overrightarrow{AE} + \overrightarrow{CE} = \overrightarrow{AE} + \overrightarrow{CA} + \overrightarrow{AE} = 2\overrightarrow{AE} - 2\overrightarrow{AS}$

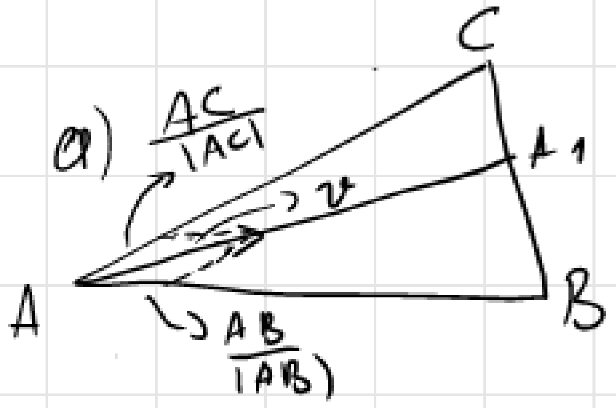
$\overrightarrow{AC} = 2\overrightarrow{AS}$

$\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD} = 2\overrightarrow{AS} - 2\overrightarrow{AE} + 2\overrightarrow{AS} = 4\overrightarrow{AS} - 2\overrightarrow{AE}$

2.4 a) $\|\vec{v}\| = \sqrt{\frac{6}{36} + \frac{2}{4} + \frac{3}{9}} = \sqrt{\frac{6+18+12}{36}} = 1 \qquad v = \left(\frac{\sqrt{6}}{6}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{3}\right) \; \vec{u} = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{6}}{6}, \frac{\sqrt{3}}{3}\right)$

b) $\arccos \dfrac{\frac{2\sqrt{3}}{12} - \frac{2\sqrt{3}}{12} + \frac{3}{9}}{1 \cdot \sqrt{\frac{1}{2} + \frac{1}{6} + \frac{1}{3}}} = \boxed{\arccos \frac{1}{3}}$

2.5 a)



$AA_1 = k \cdot \vec{v} = k\left(\frac{\overrightarrow{AC}}{|AC|} + \frac{\overrightarrow{AB}}{|AB|}\right)$

$= k\left(\frac{AC}{b} + \frac{AB}{c}\right)$

$AA_1 = AB + BA_1 = AB + mBC = AB + m(AC - AB)$

$= (1-m)AB + mAC = \frac{k}{c}AB + \frac{k}{b}AC$

$AB\left(1 - m - \frac{k}{c}\right) = \left(\frac{k}{b} - m\right)AC$ Neud. $\Rightarrow k = mb$ i $c - mc - k = 0$

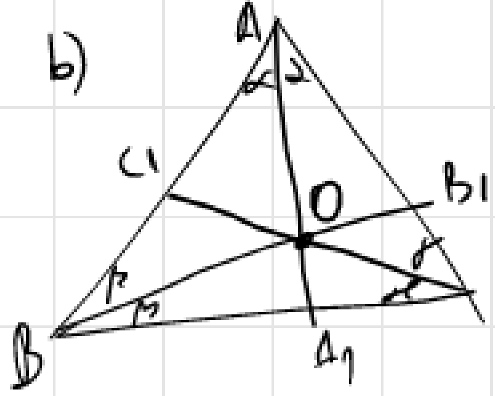
$k = c(1-m) \wedge m = \frac{k}{b} \Rightarrow k = c\left(1 - \frac{k}{b}\right) \Rightarrow k = \frac{cb - kc}{b}$

$kb + kc = cb \Rightarrow k = \frac{bc}{(b+c)}$

$AA_1 = \frac{bc}{b+c} \cdot \frac{AC}{b} + \frac{bc}{b+c} \frac{AB}{c} = \frac{c \cdot AC + b + AB}{(b+c)}$

b) ↑ 9 DoudE ano

b)



$AA_1 = \frac{b}{(b+c)} \cdot AB + \frac{c}{c+b} \cdot AC$ $\qquad O \in AA_1 \cap BB_1 \Rightarrow O \in CC_1$?

$BB_1 = \frac{a}{(a+c)} BA + \frac{c}{(a+c)} BC$

O je centar upisanog $\Rightarrow$ centar mase $A(a), B(b), C(c)$

$CC_1 = \frac{a}{(a+b)} CA + \frac{b}{(a+b)} CB$

$CO = CA + AO$ $\qquad |AO| : |OA_1| = (b+c) : a \qquad |OA_1| = \frac{a}{b+c} |AO|$

$AA_1 = AO + OA_1 = \left(1 + \frac{a}{b+c}\right) AO = \frac{a+b+c}{b+c} AO$

$CO = CA + AO = CA + \frac{b+c}{a+b+c} AA_1 = CA + \frac{b}{a+b+c} AB + \frac{c}{a+b+c} AC$

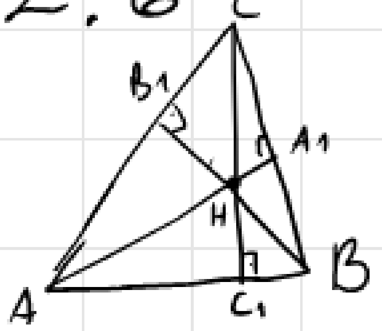
$= \frac{b}{a+b+c} AB + \frac{a+b}{a+b+c} CA = \frac{b}{a+b+c}(AC + CB) + \ldots =$

$= \frac{b}{a+b+c} CB + \frac{a}{a+b+c} CA$

$CO = \frac{a+b}{a+b+c}\left(\frac{b}{a+b} \cdot CB + \frac{a}{a+b} CA\right) = \frac{a+b}{a+b+c} \cdot CC_1$

CO i CC_1 su nolin $\Rightarrow CC_1 \in O$

2.6



Neka je $AA_1 \cap BB_1 = \{H\}$

Hoćemo da pokažemo $H \in CC_1$

$H \in AA_1 \Rightarrow AH \perp CB \Rightarrow \vec{AH} \circ \vec{CB} = 0$

$H \in BB_1 \Rightarrow BH \perp CA \Rightarrow \vec{BH} \circ \vec{CA} = 0$

$\vec{CH} = \vec{CA} + \vec{AH} / \circ \vec{CB} \qquad \vec{CH} = \vec{CB} + \vec{BH} / \circ \vec{CA}$

$\left.\begin{array}{l} \vec{CH} \circ \vec{CB} = \vec{CA} \circ \vec{CB} + \vec{AH} \circ \vec{CB} \\ \vec{CH} \circ \vec{CA} = \vec{CB} \circ \vec{CA} + \vec{BH} \circ \vec{CA} \end{array}\right\} \quad \left.\begin{array}{l} \vec{CH} \circ \vec{CB} = \vec{CA} \circ \vec{CB} \\ \vec{CH} \circ \vec{CA} = \vec{CB} \circ \vec{CA} \end{array}\right\}$

$\vec{CH} \circ \vec{CB} - \vec{CH} \circ \vec{CA} = \vec{CA} \circ \vec{CB} - \vec{CB} \circ \vec{CA} = 0$

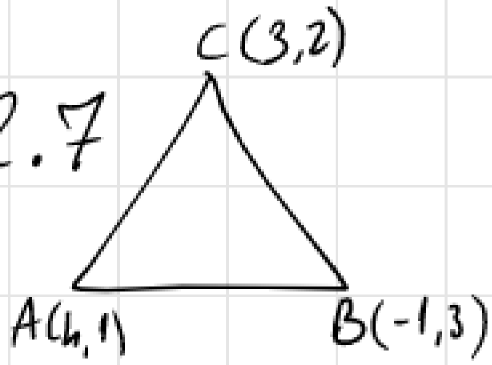
$\vec{CH} \circ (\vec{CB} - \vec{CA}) = 0$

$\vec{CH} \circ \vec{AB} = 0$

$\Rightarrow CH \perp AB$

$\Rightarrow H \in CC_1$ $\blacksquare$

2.7

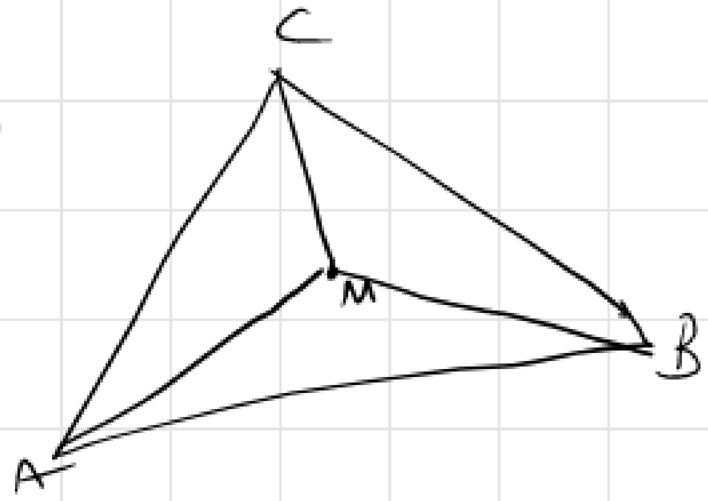


$\Delta_{ABC} = \begin{vmatrix} -5 & 2 \\ -1 & 1 \end{vmatrix} = -5 + 2 = -3 < 0$, „−" orijentacija

$\cdot e^3 = \vec{AB} \times \vec{AC}$

$P = \frac{1}{2} \left| 4(3-2) + (-1) \cdot (2-1) + 3(1-3) \right| = \frac{1}{2} \left| 4 - 1 - 6 \right| = \frac{3}{2}$

2.8 $M(-2,3)$, $A(1,4)$, $B(-1,3)$, $C(2,-3)$



$M \in$ unutrašnjost trougla ABC

$\iff ABM, BCM, CAM$ iste orijentacije

$$D_{ABM} = \begin{array}{c} \overrightarrow{AB} = \\ \overrightarrow{AM} = \end{array} \begin{vmatrix} 2 & -1 \\ -3 & 1 \end{vmatrix} = 2 - 3 = -1 < 0$$

$$D_{BCM} = \begin{array}{c} \overrightarrow{BC} = \\ \overrightarrow{BM} = \end{array} \begin{vmatrix} 3 & -6 \\ -1 & -3 \end{vmatrix} = -6 < 0 \quad D_{CAM} = \begin{vmatrix} -1 & 7 \\ -4 & 6 \end{vmatrix} = -6 + 28 > 0$$

M nije u unutrašnjosti trougla

2.9 $D(1,2)$, $E(4,-5)$, $F(-7,3)$

$A(2,-4)$, $B(1,-1)$, $C(1,1)$

$$\Delta_{ABC} = \begin{vmatrix} -1 & 3 \\ -1 & 5 \end{vmatrix} = -5 + 3 = -2 < 0$$

$\boxed{C}$, D i E su sa istih strana

$$\Delta_{ABD} = \begin{vmatrix} -1 & 3 \\ -1 & 6 \end{vmatrix} = -6 + 3 = -3 < 0$$

$$\Delta_{AB_E} = \begin{vmatrix} -1 & 3 \\ 2 & -1 \end{vmatrix} = 1 - 6 = -5 < 0 \qquad \Delta_{AB_F} = \begin{vmatrix} -1 & 3 \\ -9 & 7 \end{vmatrix} = -7 + 27 = 20 > 0$$

2.10 $[\vec{v}, \vec{u}, \vec{w}] = (\vec{v} \times \vec{u}) \circ \vec{w}$

$\vec{v} = (-2,1,4)$ $\vec{u} = (0,2,3)$ $\vec{w} = (5,1,-2)$

$$\vec{v} \times \vec{u} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -2 & 1 & 4 \\ 0 & 2 & 3 \end{vmatrix} = e_1 \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} - e_2 \begin{vmatrix} -2 & 4 \\ 0 & 3 \end{vmatrix} + e_3 \begin{vmatrix} -2 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= e_1(3-8) - e_2(-6+0) + e_3(-4+0) = (-5, 6, -4)$$

$$(\vec{v} \times \vec{u}) \circ \vec{w} = (-5, 6, -4) \circ (5, 1, -2) = -25 + 6 + 8 = -11$$

b) Jesu, jer nije ... $= 0$

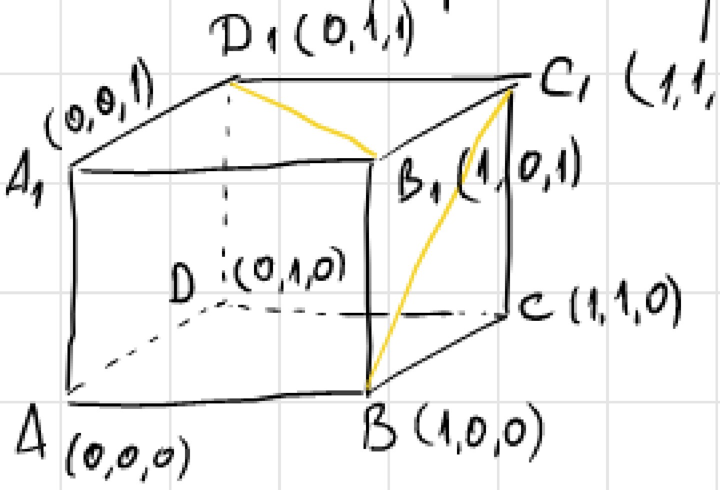
2.11 a) $A(-2,1)$ $B(-1,2)$ $C(4,5)$

$$AB \times AC = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & 0 \\ 6 & 4 & 0 \end{vmatrix} = e_3\,(4-6) \neq 0$$ Nisu

b) $A(1,4,2)$ $B(2,5,3)$ $C(7,-4,4)$, $D(5,-6,2)$

$$[AB, AC, AD] = \begin{vmatrix} 1 & 1 & 1 \\ 6 & -8 & 2 \\ 4 & -10 & 0 \end{vmatrix} = 20+8-60+32 = 0$$ Jesu

2.12

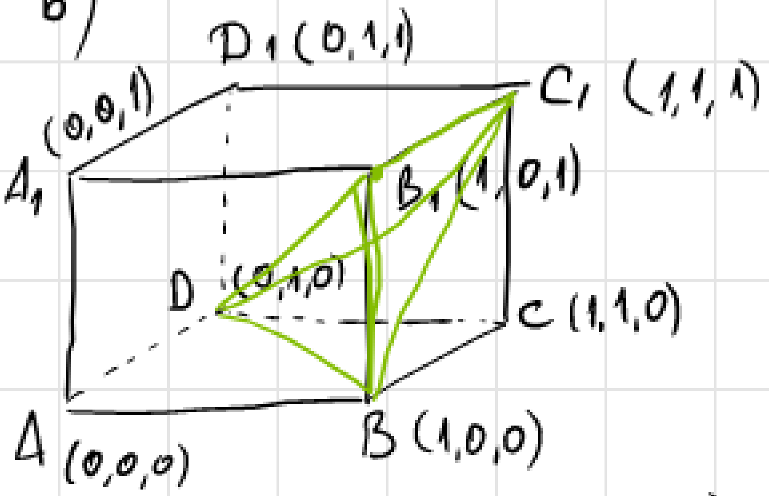


a) $\vec{BC_1} \circ \vec{DB_1} = |\vec{BC_1}| \cdot |\vec{DB_1}| \cdot \cos\varphi$

$(0,1,1) \circ (-1,1,0) = \sqrt{2} \cdot \sqrt{2} \cdot \cos\varphi$

$\frac{1}{2} = \cos\varphi \Rightarrow \varphi = 60^\circ$

b)



$BC_1 = (0,1,1)$ $BB_1 = (0,0,1)$ $BD = (-1,1,0)$

$BC_1 B_1 D$

$$\frac{1}{6}\left|[\vec{BC_1}, \vec{BB_1}, \vec{BD}]\right| = \frac{1}{6}\begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$= \lfloor 0 \ldots -1 \cdot (+1) + 1 \cdot (0+0) = -1 \rceil = \frac{1}{6} \cdot 1 = \frac{1}{6}$

1.13 $\vec{OT} = \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC})$

a) $\vec{MT} = \frac{1}{3}(\vec{MA} + \vec{MB} + \vec{MC})$

$\vec{MO} + \vec{OT} = \frac{1}{3}(\vec{MO} + \vec{OA} + \vec{MO} + \vec{OB} + \vec{MO} + \vec{OC})$

$\vec{MO} + \vec{OT} = \frac{1}{3} \cdot 3 \cdot \vec{MO} + \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC})$

$\vec{OT} = \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC})$

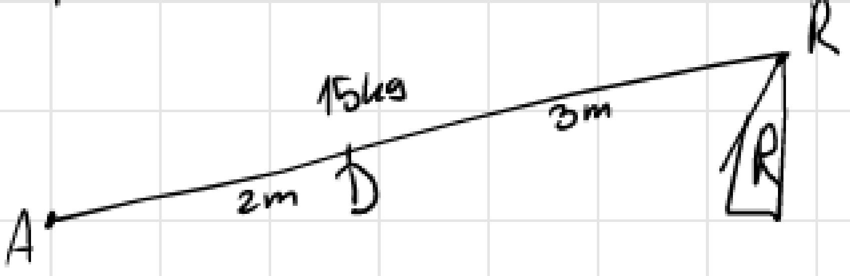
b) $\vec{OT} = \frac{1}{3}(\vec{OA} + \vec{OB} + \vec{OC}) \Rightarrow 3\vec{OT} = \vec{OA} + \vec{OB} + \vec{OC}$

$\vec{0} = TO + OA + TO + OB + TO + OC = \vec{TA} + \vec{TB} + \vec{TC}$ T je centar mase $A(1), B(1), C(1)$

$A_1 = \vec{AT} \cap \vec{BC}$ A_1 je centar mase tačaka B i C

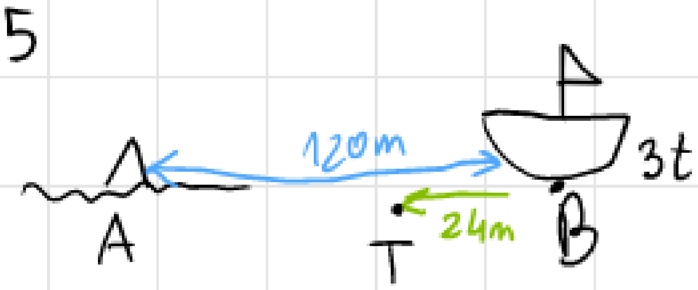
tj $A_1(2)$ —||— B_1 i C_1 $\Rightarrow |AT| : |TA_1| = 2 : 1$

1.14



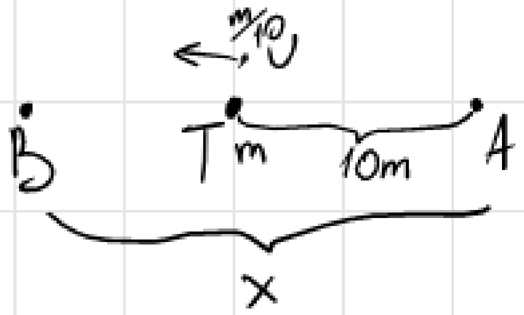
$$|\vec{AD}| : |\vec{AR}| = 2 : 5$$
$$|\vec{AR}| = \frac{5}{2}|\vec{AD}|$$
$$m_1 \cdot \vec{AD} + m_2 \cdot \vec{AR} = \vec{0}$$
$$15\,\vec{AD} + m \cdot \frac{5}{2}|\vec{AD}| = 0$$
$$m = -\frac{15}{5} \cdot 2 = -6\,kg$$

2.15



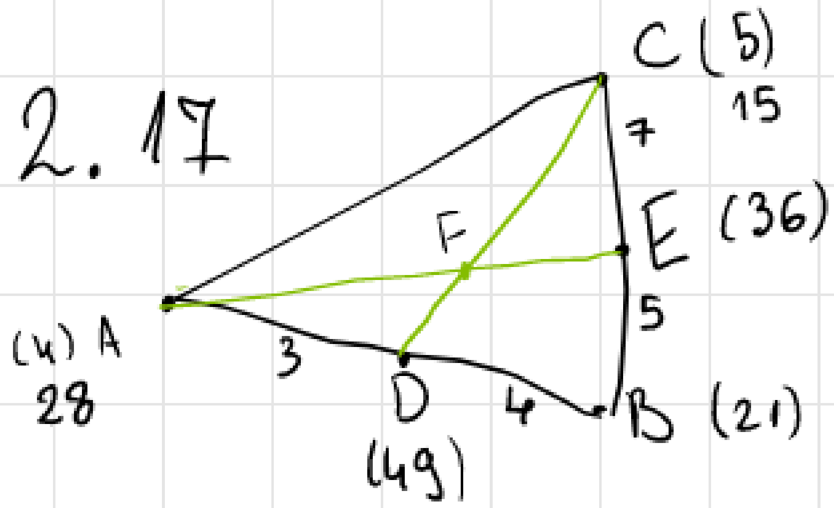
$$\vec{TB} = -\frac{24}{96}\vec{TA} = -\frac{1}{4}\vec{TA}$$
$$m \cdot \vec{TA} + 3t \cdot \vec{TB} = \vec{0}$$
$$m \cdot \vec{TA} - \frac{3}{4}t \cdot \vec{TA} = 0 \Rightarrow m = \frac{3}{4}t$$

1.16



$$10m\,\vec{TA} + m \cdot \vec{TB} = 0$$
$$\vec{TB} : \vec{AT} = x - 10 : 10 \Rightarrow \vec{TB} = \frac{x-10}{10} \cdot \vec{TA} \cdot (-1)$$
$$10m\,\vec{TA} - \frac{x-10}{10} \cdot m \cdot \vec{TA} = 0$$
$$\left(10 - \frac{x-10}{10}\right) = 0 \Rightarrow 100 - x + 10 = 0$$
$$x = 110\,m$$

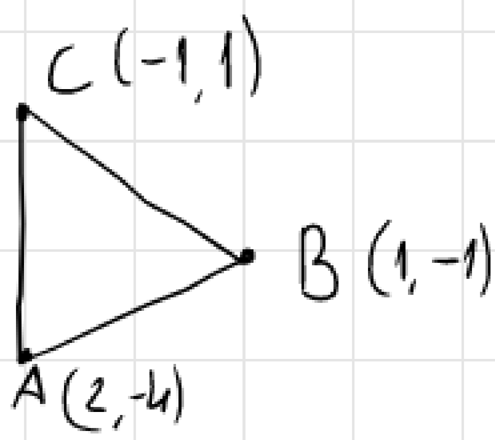
2.17



$$\frac{AE}{FE} = \frac{36}{28}$$
$$\frac{CF}{FD} = \frac{49}{15}$$

2.18 $F(28, 21, 15)$

2.19



$D(1,2), E(0,0), F(1,-2)$

$$\Delta_{ABD} = \begin{vmatrix} -1 & 3 \\ -1 & 6 \end{vmatrix} = -6 + 3 = -3$$

$$\Delta_{BCD} = \begin{vmatrix} -2 & 2 \\ 0 & 3 \end{vmatrix} = -6 \qquad \Delta_{CAD} = \begin{vmatrix} 3 & -5 \\ 2 & 1 \end{vmatrix} = 3 + 10 = 13$$

$D(-6 : 13 : -3)$ Van trougla

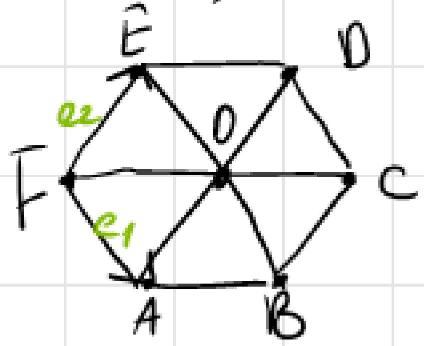
$\triangle ABE = \begin{vmatrix} -1 & 3 \\ -2 & 4 \end{vmatrix} = -4+6 = 2 \qquad \triangle BCE = \begin{vmatrix} -2 & 2 \\ -1 & 1 \end{vmatrix} = -2+2 = 0$

$\triangle CAE = \begin{vmatrix} 3 & -5 \\ 1 & -1 \end{vmatrix} = -3+5 = 2 \qquad E(0:2:2)$ Na stranici BC je.

$\triangle ABF = \begin{vmatrix} -1 & 3 \\ -1 & 2 \end{vmatrix} = -2+3 = 1 \qquad \triangle BCF = \begin{vmatrix} -2 & 2 \\ 0 & -1 \end{vmatrix} = 2$

$\triangle CAF = \begin{vmatrix} 3 & -5 \\ 2 & -3 \end{vmatrix} = -9+10 = 1 \qquad F(2:1:1)$ U trouglu je

3.1 $e = (\vec{e_1}, \vec{e_2}) \quad \vec{e_1} = \vec{FA} \quad e_2 = \vec{FE}$

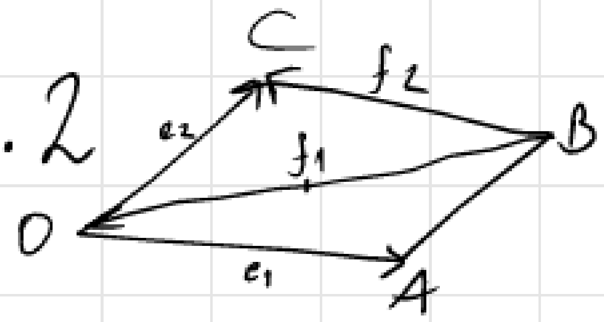


$[\vec{F}]_{Fe} = 0e_1 + 0e_2 \qquad [\vec{E}]_{Fe} = 0\cdot e_1 + 1\cdot e_2$

$[\vec{A}]_{Fe} = 1\cdot e_1 + 0\cdot e_2 \qquad [\vec{B}]_{Fe} = \vec{FO} + \vec{FA} = e_1 + e_2 + e_1 = 2e_1 + e_2\cdot 1$

$[\vec{C}]_{Fe} = 2\cdot\vec{FO} = 2e_1 + 2e_2 \qquad [\vec{D}]_{Fe} = \vec{FO} + e_2 = e_1 + e_2 + e_2 = 1e_1 + 2e_2$

3.2



$e = (\vec{OA}, \vec{OC}) \qquad f = (\tfrac{1}{2}\vec{BO}, \vec{BC})$

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \underbrace{(\ )}_{[\vec{f_1}]_e} & \underbrace{(\ )}_{[\vec{f_2}]_e} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \underbrace{\begin{bmatrix} \ \end{bmatrix}}_{[B]_{Oe}}$

$f_1 = \tfrac{1}{2}\vec{BO} = \tfrac{1}{2}(-(\vec{OA} + \vec{OC})) = -\tfrac{1}{2}e_1 - \tfrac{1}{2}e_2$

$f_2 = BC = -e_1 + 0\cdot e_2$

$\Big\} \; C_{e\to f} = \begin{bmatrix} -1/2 & -1 \\ -1/2 & 0 \end{bmatrix}$

$[B]_{Oe} = e_1 + e_2$

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1/2 & -1 \\ -1/2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$C^{-1} = \frac{1}{-1/2} \begin{bmatrix} 0 & 1 \\ 1/2 & -1/2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix}$

$C^{-1}\cdot \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + C^{-1}\cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \implies \begin{bmatrix} x \\ y \end{bmatrix} = C^{-1}\begin{bmatrix} x' \\ y' \end{bmatrix} - C^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

3.3 $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Pretpostavimo Oe ortonormirani

Ako je Oe ortonormirani reper ($\|\vec{e_1}\| = \|\vec{e_2}\| = 1$, $\vec{e_1} \perp \vec{e_2}$) tada:

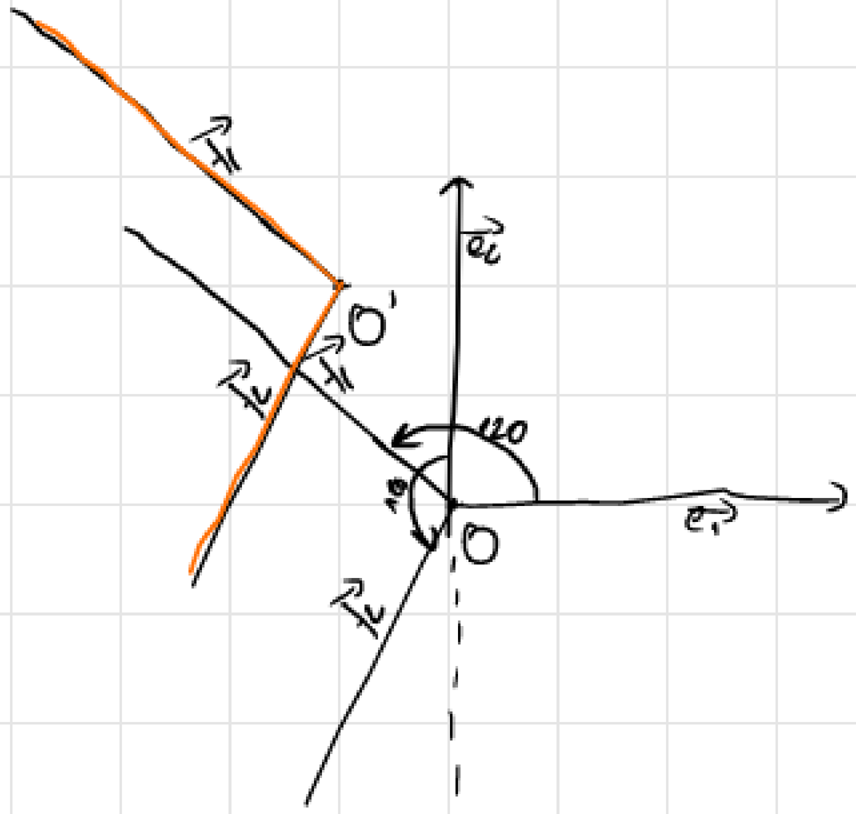
$\|\vec{f_1}\| = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = 1$

$\|\vec{f_2}\| = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = 1$

$\vec{f_1} \circ \vec{f_2} = (-\frac{1}{2}, \frac{\sqrt{3}}{2}) \circ (\frac{\sqrt{3}}{2}, \frac{1}{2}) = -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 0$

$\Leftarrow \vec{f_1} \perp \vec{f_2}$

Da



3.4 $A(-1,-1)$ $B(1,-1)$ $C(1,1)$ $D(-1,1)$

$A'(4,5)$ $B'(8,7)$ $C'(6,9)$ $D'(2,7)$

$x' = x \cdot a_1 + y \cdot a_2 + b_1$

$y' = x \cdot c_1 + y c_2 + b_2$

$4 = -a_1 - a_2 + b_1$ $\quad 6 = a_1 + a_2 + b_1$ $\quad 4 = 2a_1 \Rightarrow a_1 = 2$ $\quad c_1 = 1$

$5 = -c_1 - c_2 + b_2$ $\quad 9 = c_1 + c_2 + b_2$ $\quad 10 = 2b_1 \Rightarrow b_1 = 5$ $\quad b_2 = 7$

$8 = a_1 - a_2 + b_1$ $\overset{+}{\leftarrow}$ $2 = -a_1 + a_2 + b_1$ $\quad a_2 = -1$ $\quad c_2 = 1$

$7 = c_1 - c_2 + b_2$ $\quad 7 = -c_1 + c_2 + b_2$

a) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

b) $\det C = 2 + 1 = 3 = \frac{P(F')}{P(F)}$ $\quad P(F) = 2 \cdot 2 = 4 \Rightarrow 3 \cdot 4 = P(F') = 12$

3.5 $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 5 \\ 7 \end{bmatrix}$ $\quad C^{-1}/$

$$C^{-1}\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + C^{-1}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 5+7 \\ 14-5 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

3.6

$O(0,0)$ $A(1,0)$ $B(0,1)$

$O'(5,-4)$ $A'(7,-8)$ $B'(4,1)$

$\Rightarrow c_1 = 5 \quad c_2 = -4$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$7 = a_1 \cdot 1 + a_2 \cdot 0 + 5 \Rightarrow a_1 = 2 \qquad 4 = a_1 \cdot 0 + a_2 \cdot 1 + 5 \Rightarrow a_2 = -1$

$-8 = b_1 \cdot 1 + b_2 \cdot 0 - 4 \Rightarrow b_1 = -4 \qquad 1 = b_1 \cdot 0 + b_2 \cdot 1 - 4 \Rightarrow b_2 = 5$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

$\det C = 10 + 4 = 14 > 0$ čuva orijentaciju

3.7 $P(1,1)$ $Q(1,2)$ $R(4,4)$

$P'(5,-4)$ $Q'(7,-8)$ $R'(4,1)$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

x:

$5 = a_1 + a_2 + c_1$

$7 = a_1 + 2a_2 + c_1 \quad \cdot(-4)$

$4 = 4a_1 + 4a_2 + c_1$

$4 - 20 = c_1 - 4c_1 \quad c_1 = \frac{16}{3}$

$a_2 = 2$

$5 = a_1 + 2 + \frac{16}{3} \Rightarrow a_1 = \frac{-7}{3}$

y:

$-4 = b_1 + b_2 + c_2 \quad -4$

$-8 = b_1 + 2b_2 + c_2$

$1 = 4b_1 + 4b_2 + c_2$

$-4 = b_2$

$17 = -3c_2 \Rightarrow c_2 = \frac{-17}{3}$

$c_1 = 17/3$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -7/3 & 2 \\ 17/3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 16/3 \\ -17/3 \end{bmatrix}$$

$\det C = \frac{28}{3} - \frac{34}{3} = -\frac{6}{3} = -2 < 0$

Ne čuva orijentaciju

3.8 $P(0,0)$ $Q(5,5)$ $R(10,-15)$ $\quad \overrightarrow{PQ}(-5,-5), \overrightarrow{QR}(5,-20)\ \overrightarrow{RP}(-10,15)$

$P'(1,-7)$ $Q'(0,0)$ $R'(19,-8)$ $\quad \overrightarrow{P'Q'}(-1,7), \overrightarrow{Q'R'}(19,-8)\ \overrightarrow{R'P'}(-18,-1)$

$\|\overrightarrow{PQ}\| = \sqrt{25+25} = 5\sqrt{2} \quad \|\overrightarrow{QR}\| = \sqrt{25+400} = 5\sqrt{17} \quad \|\overrightarrow{RP}\| = \sqrt{325} = 5\sqrt{13}$

$\|P'Q'\| = \sqrt{1+49} = 5\sqrt{2} \quad \|Q'R'\| = \sqrt{19^2+8^2} = 5\sqrt{17} \quad \|R'P'\| = \sqrt{18^2+1} = 5\sqrt{13}$

$\Rightarrow SSS \Rightarrow$ Jesu podudarni

b) $P(0,0)$ $Q(5,5)$ $R(10,-15)$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$P'(1,-7)$ $Q'(0,0)$ $R'(19,-8)$ $b_1 = 1$ $b_2 = -7$

x: $0 = 5(a_1+a_2)+1$
$19 = 10a_1 - 15a_2 + 1$ $\cdot(-2)$

$19 = 10a_1 - 10a_1 - 15a_2 - 10a_2 + 1 - 2$
$20 = -25a_2 \Rightarrow a_2 = -4/5$

$-1 = -4 + 5a_1$
$a_1 = 3/5$

y: $0 = 5(b_1+b_2) - 7$
$-8 = 10b_1 - 15b_2 - 7$ $\cdot(-2)$

$-8 = -25b_2 - 7 + 14 \Rightarrow b_2 = 3/5$
$0 = 3 + 5b_1 - 7 \Rightarrow b_1 = 4/5$

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -7 \end{bmatrix}$ Rotacija + translacija ?

3.9 $C(1,2)$ $\lambda = -2$ $T_{OC} \circ H_\lambda \circ T_{CO}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \circ \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & 2 \\ 0 & -2 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \end{bmatrix}$ $O(0,0) \Rightarrow O'(3,6)$

3.10 $A(-2,3)$ $\varphi = 2\pi/3$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1/2 & -\sqrt{3}/2 & \frac{3\sqrt{3}-2}{2} \\ \sqrt{3}/2 & -1/2 & \frac{2\sqrt{3}+3}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & -\sqrt{3}/2 & \frac{3\sqrt{3}}{2} - 3 \\ \sqrt{3}/2 & -1/2 & \frac{2\sqrt{3}}{2} + \frac{9}{2} \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{matrix} x \\ y \end{matrix} = \begin{bmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} +\frac{3\sqrt{3}}{2} - 3 \\ \sqrt{3} + 9/2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/2 - \frac{3\sqrt{3}}{2} \\ \sqrt{3}/2 - 3/2 \end{bmatrix} + \begin{bmatrix} \frac{3\sqrt{3}}{2} - 3 \\ \sqrt{3} + 9/2 \end{bmatrix}$$

$\ulcorner M'\left(-7/2,\ 3 + \frac{3\sqrt{3}}{2}\right) \lrcorner$

3.11 $P(360, 420)$ $Q(520, 520)$ 800×600 Homotetijom

$520 - 360 = 160$ $520 - 420 = 100$ $800/160 = \boxed{5}$ $600/100 = 6$

λ

$C' = \frac{(800, 600)}{2} = (400, 300)$
$C = \frac{(360+520, 420+520)}{2} = (440, 470)$

$400 = 5 \cdot 440 + a$
$300 = 5 \cdot 470 + b$

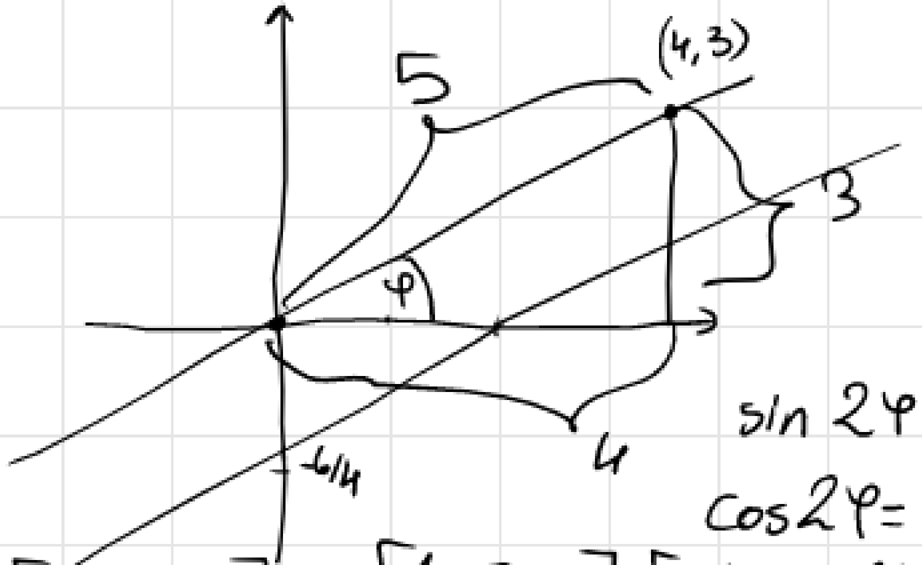
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 1800 \\ 2050 \end{bmatrix}$$

3.12 $3x-4y-6=0$

$y=\frac{3}{4}x-\frac{6}{4}$

$y_0=\frac{3}{4}x$

$x=4 \Rightarrow y_0(4)=3$



$\sin\varphi=\frac{3}{5}$

$\cos\varphi=4/5$

$\sin 2\varphi = 2\cdot 3/5 \cdot 4/5 = \frac{24}{25}$

$\cos 2\varphi = \cos^2\varphi-\sin^2\varphi = \frac{16-9}{25}=\frac{7}{25}$

$$\begin{bmatrix}1&0&2\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}7/25&-24/25&0\\24/25&7/25&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&-2\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&2\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}7/25&-24/25&-14/25\\24/25&7/25&-48/25\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}7/25&-24/25&36/25\\24/25&7/25&-48/25\\0&0&1\end{bmatrix}$$

3.13 $\varphi=\pi/2$ $p:(-1,0,0)$ $\vec{p}(2,1,2)$ $p=\left[\frac{2}{3},\frac{1}{3},\frac{2}{3}\right]$

$$[R_{p_0}(\varphi)_e] = pp^T+\underbrace{\cos\varphi}_{0}(E-pp^T)+\underbrace{\sin\varphi}_{1} p_x = pp^T+p_x$$

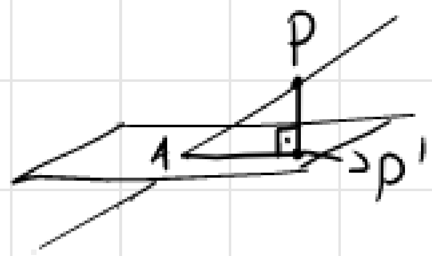
$$p_x=\begin{bmatrix}0&-2/3&1/3\\2/3&0&-2/3\\-1/3&2/3&0\end{bmatrix} \qquad pp^T=\begin{bmatrix}2/3\\1/3\\2/3\end{bmatrix}\begin{bmatrix}2/3&1/3&2/3\end{bmatrix}=\begin{bmatrix}4/9&2/9&4/9\\2/9&1/9&2/9\\4/9&2/9&4/9\end{bmatrix}$$

$$T_{\vec{op}}=\begin{bmatrix}1&0&0&-1\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \qquad T_{\vec{po}}=\begin{bmatrix}1&0&0&1\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$T_{\vec{op}}\circ R_{p_0}(\varphi)\circ T_{\vec{po}}=\left[\begin{array}{ccc|c}4/9&2/9&4/9&-5/9\\2/9&1/9&2/9&2/9\\4/9&2/9&4/9&4/9\end{array}\right]$$

lin → translatorni deo

3.14 $p: \frac{x-1}{1}=\frac{y}{2}=\frac{z+3}{2}$ $\alpha: 2x+y-4z+5=0$



$A\in p$ $A(t+1, 2t, 2t-3)$

$A\in\alpha \land A\in p$ $2t+2+2t-8t+12+5=0$ $4t=19 \Rightarrow t=19/4$

Proizvoljna tačka P npr $(1,0,-3)$

$P'\in n_\alpha \in \alpha \Rightarrow n_\alpha(2,1,-4)$, $P(1,0,-3)$ $\Big\}$ $P'(2t+1, t, -4t-3)$

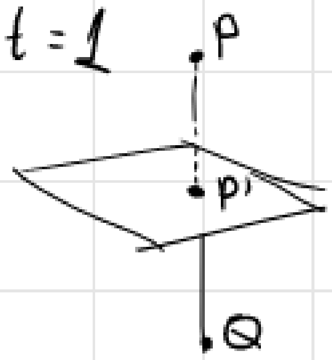
$P'\in\alpha \Rightarrow 4t+t+16t+2+12+5 \Rightarrow t=-\frac{19}{21}$

$$p'\left(-\frac{2\cdot 19}{21}+1, -\frac{19}{21}, -4\cdot\frac{-19}{21}-3\right) = p'\left(\frac{21-38}{21}, \frac{-19}{21}, \frac{76-63}{21}\right)$$
$$= p'\left(\frac{-17}{21}, -\frac{19}{21}, \frac{13}{21}\right)$$

$$\overrightarrow{AP'} = \vec{p} = \left(-\frac{17}{21}, -\frac{19}{21}, \frac{13}{21}\right) - \left(\frac{23}{4}, \frac{19}{2}, \frac{13}{2}\right)$$
$$= \frac{1}{84}\left(-68-23\cdot 21, -76-21\cdot 19\cdot 2, 52-2\cdot 21\cdot 13\right)$$
$$= \frac{1}{84}\left(-551, -874, -494\right)$$

$$p': \frac{x-\frac{23}{4}}{\frac{-551}{84}} = \frac{y-\frac{19}{2}}{\frac{-874}{84}} = \frac{z-\frac{13}{2}}{\frac{-494}{84}}$$

3.15 $Q=?$ $P=(3,-2,-4)$ $\alpha: 6x+2y-3z-75=0$



$P'=?$ $P\in n_\alpha$ tj. $n_\alpha=(6,2,-3) \Rightarrow P'=(6t+3, 2t-2, -3t-4)$

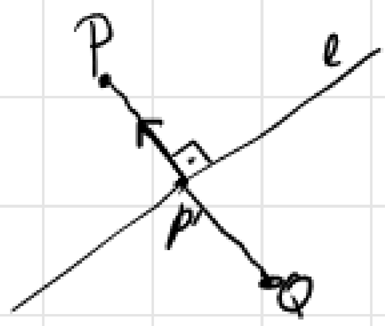
$P'\in\alpha$ $\underline{36t}+18+\underline{4t}-4+\underline{9t}+12-75=0$ $49t=49 \Rightarrow t=1$

$P'(9,0,-7)$

$P'=\frac{P+Q}{2} \Rightarrow (18,0,-14)=(3,-2,-4)+(x,y,z)$

$Q(15,2,-10)$

3.16 $l: \frac{x}{-2}=\frac{y-3}{4}=\frac{z-4}{1}$ $P(-1,-2,1)$



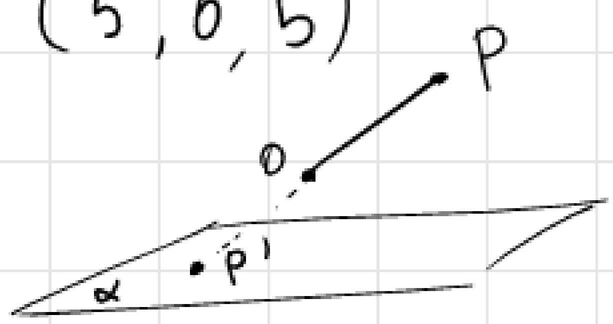
$P'\in l \Rightarrow P'(-2t, 4t+3, t+4)$

$\overrightarrow{PP'}\perp l \Rightarrow \overrightarrow{PP'}\circ l=0$ $PP'=(-2t+1, 4t+5, t+3)$

$\vec{l}(-2,4,1)$ $\underline{4t}-2+\underline{16t}+20+\underline{t}+3=0$ $21t=-21$ $t=-1$

$P'(2,-1,3)$

$P'=\frac{P+Q}{2}=(4,-2,6)-(-1,-2,1)=Q=(5,0,5)$



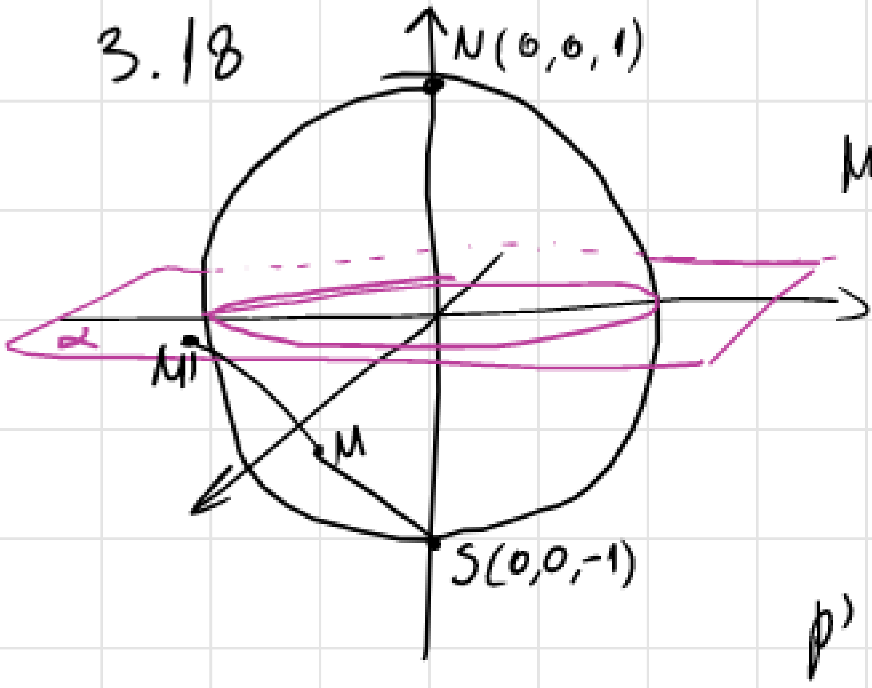
3.17 $P(1,2,3)$ $\alpha: z=-1$ $O(0,0)$

$P'=\overrightarrow{OP}\cap\alpha$ $P'\in\overrightarrow{OP} \Rightarrow P'(t,2t,3t)$

$P'\in\alpha \Rightarrow 3t=-1 \Rightarrow t=-1/3$

$\Big\}\ P'(-1/3, -2/3, -1)$

3.18



$\alpha: z=0$

$M' = \overrightarrow{SM} \cap \alpha \quad M' \in \overrightarrow{SM} \Rightarrow (x_0 \cdot t,\ y_0 \cdot t,\ -1+(z_0+1)t)$

$M' \in \alpha \quad -1+(z_0+1)\cdot t = 0$

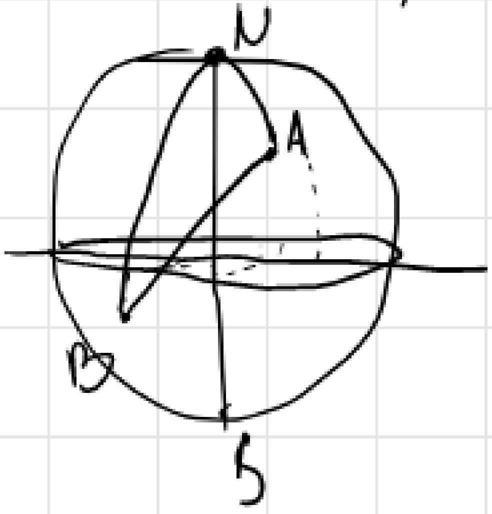
$t = \frac{1}{z_0+1}$

$\Rightarrow M'\left(\frac{x_0}{z_0+1}, \frac{y_0}{z+1}, 0\right)$

$p'\left(\frac{\frac{\sqrt6}{2}}{\frac{\sqrt3}{3}+1}, \frac{\frac{\sqrt2}{2}}{\frac{\sqrt3}{3}+1}, 0\right) = \left(\frac{\sqrt6}{2(\sqrt3+3)}, \frac{3\sqrt2}{2(\sqrt3+3)}, 0\right)$

3.19

A(60°N, 30°E) B(30°S, 60°W) Rastojanje



Od ekvatora 60 na gore, od Grinica 30° desno

$\widehat{ANB} = 90°$

$NA = 30° \quad NB = 120°$

$\cos AB = \cos NA \cos NB + \sin NA \cdot \sin NB \cdot \cancel{\cos ANB}$ →0

$= \frac{\sqrt3}{2} \cdot \left(-\frac12\right) = -\frac{\sqrt3}{4}$

$AB = r \cdot \arccos\left(-\frac{\sqrt3}{4}\right)$

4.1 $q: x = 3t-4 \Rightarrow t\ \frac{x+4}{3}$

$y = 2t+1 \Rightarrow t = \frac{y-1}{2}$

$\frac{x+4}{3} = \frac{y-1}{2} \quad / \cdot 6$

$2x+8-3y+3=0$

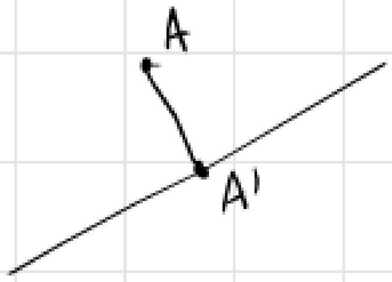
a) $2x-3y+11=0$

b) $\frac{2x+11}{3} = y \quad k = \frac23$

$y = k \cdot x + n$

$7 = 3 \cdot \frac23 + n \Rightarrow n = 5$

Prava je $y = \frac23 x + 5$



4.2 a) A(1,5) na $p: \begin{array}{l} x = t+4 \\ y = 3t-5 \end{array}$

$3x - 12 = y + 5$

$3x - y - 17 = 0 \quad \overrightarrow{n_p}(1,3) \rightarrow x + 3y + m = 0$

$1 + 15 + m = 0 \quad m = -16$

$x + 3y - 16 = 0$

b) $4x - \frac23 y + 7 = 0$

$\overrightarrow{n_p}\left(\frac23, 4\right) \quad \frac23 \cdot 1 + 5 \cdot 4 + m = 0 \Rightarrow m = -\frac{62}{3}$

4.3 $A(2,3)$, $B(-1,4)$

a) $AB(-3,1)$ $\qquad \frac{x-2}{-3} = \frac{y-3}{1} = t$

b) $C(14,-1)$ $\qquad \frac{14-2}{-3} = -4 \quad \frac{-1-3}{+1} = -4$ Ne

c) $\frac{1-2}{-3} = \frac{\frac{10}{3}-3}{1}$, $\frac{1}{3} = \frac{1}{3}$ ✓ Da $D(1, \frac{10}{3})$

$t_D = \frac{1}{3} \Rightarrow$ deli u $\boxed{1:2}$

4.4 $C(1,1)$ $D(-7,11)$ $A(2,-2)$ $B(1,3)$

$\vec{AB} = (1,-5) \Rightarrow \vec{n_p} = (5,1)$

$A \in p \Rightarrow 5 \cdot 2 - 2 \cdot 1 + m = 0 \qquad m = -8$

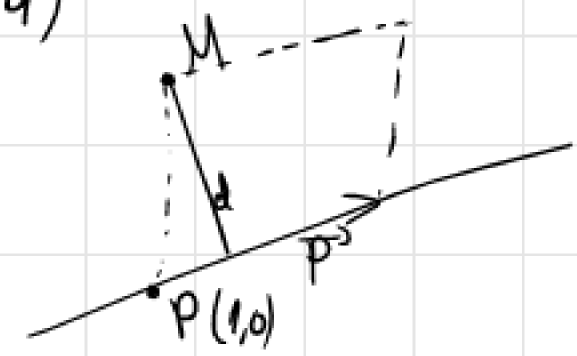
$$\left.\begin{array}{l} p(C) = 5 - 2 - 8 < 0 \\ p(D) = -35 + 11 - 8 < 0 \end{array}\right\} \text{Jesu}$$

4.5 $A(1,2)$ $B(3,7)$

$\vec{AB} = (2,5) \qquad A_i = (1,2) + \frac{i}{5}\vec{AB} \qquad i \in [1,5] \wedge i \in \mathbb{N}$

4.6 $M(1,-3)$ a) $p: 4x - 3y + 1 = 0 \qquad n = (3,4)$

$d(M,p) = \frac{|4 \cdot 1 + 9 + 1|}{\sqrt{9+16}} = \frac{14}{5}$



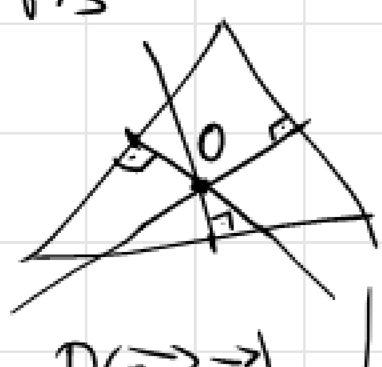
b) $\vec{p} = (3,-2)$ $P(1,0)$

$P = \|\vec{p} \times \vec{PM}\| = \|\vec{p}\| d(M,p)$

$\vec{PM} = (0,3) \qquad \vec{p} \times \vec{PM} = \begin{vmatrix} 3 & -2 \\ 0 & 3 \end{vmatrix} = 9 \qquad \|..\| = 9$

$\|\vec{p}\| = \sqrt{9+4} = \sqrt{13} \qquad d(M,p) = \frac{9}{\sqrt{13}}$

4.7 $A(0,3)$ $B(3,4)$ $C(5,1)$



$P = \frac{A+B}{2} = (\frac{3}{2}, \frac{7}{2}) \quad \vec{p} = (1,-3)$

$Q = \frac{B+C}{2} = (4, \frac{5}{2}) \quad \vec{q} = (3,-2)$

$p: x = +t_1 + \frac{3}{2} \qquad q: x = -3t_2 + 4$

$\quad y = -3t_1 + \frac{7}{2} \qquad \quad y = -2t_2 + \frac{5}{2}$

$t_2 = \frac{D(\vec{PQ}, \vec{p})}{D(\vec{p}, \vec{q})} = \frac{\begin{vmatrix} \frac{5}{2} & -1 \\ 1 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ -3 & -2 \end{vmatrix}} =$

$\vec{PQ}(\frac{5}{2}, -1)$

$= \frac{-15/2 + 1}{-2-9} = \frac{13}{22} \qquad O(\frac{49}{22}, \frac{29}{22})$

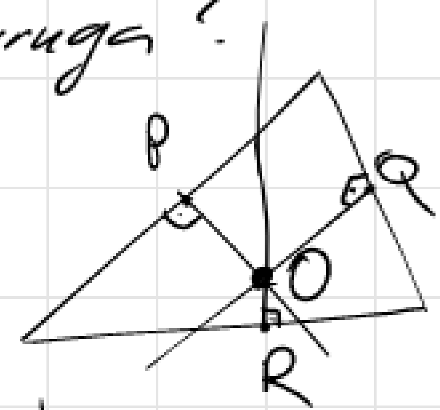
b) $AT = \frac{2}{3} AQ$ (Q je $S(BC) \Rightarrow AQ$ je tež duž $\triangle ABC$ iz A)

$$\left(4, \frac{5}{2} - \frac{6}{2}\right) = \left(4, -\frac{1}{2}\right) = AQ$$

$$AT = \left(\frac{8}{3}, -\frac{1}{3}\right) = [T] - [A] = [T] - (0,3)$$

$$\left(\frac{8}{3}, \frac{8}{3}\right) = T$$

4.8 težište, ortocentar, centar opisanog i upisanog kruga?



$A(-1,4)$, $B(2,3)$ $C(1,2)$

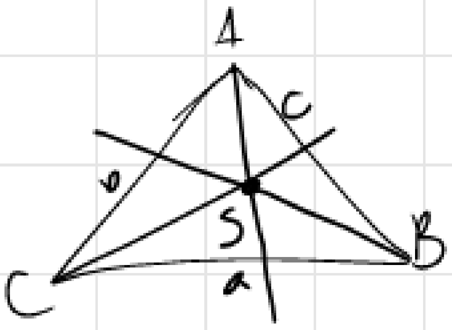
$$P = \frac{A+B}{2} = \left(\frac{1}{2}, \frac{7}{2}\right) \qquad \vec{p} = (1,3)$$

$$Q = \frac{B+C}{2} = \left(\frac{3}{2}, \frac{5}{2}\right) \qquad \vec{q} = (-1, 1)$$

p: $x = t + \frac{1}{2}$, $y = 3t + \frac{7}{2}$ q: $x = -1 + \frac{3}{2}$, $y = +s + \frac{5}{2}$

$$s = \frac{D(\vec{PQ}, p)}{D(p,q)} = \frac{\begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix}} = \frac{3+1}{1+3} = 1$$

$\boxed{O\left(\frac{1}{2}, \frac{7}{2}\right) \text{ - opisani krug}}$

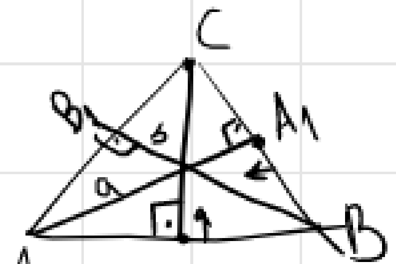


S=? a,b,c=?

$$\left.\begin{aligned} a &= \sqrt{1+1} = \sqrt{2} \\ b &= \sqrt{4+4} = 2\sqrt{2} \\ c &= \sqrt{9+1} = \sqrt{10} \end{aligned}\right\} + \sqrt{2}(3+\sqrt{5})$$

$$\left(\frac{-\sqrt{2} + 4\sqrt{2} + \sqrt{10}}{\sqrt{2}(3+\sqrt{5})}, \frac{4\sqrt{2} + 6\sqrt{2} + 2\sqrt{10}}{\sqrt{2}(3+\sqrt{5})}\right) = \left(1, \frac{10+2\sqrt{5}}{3+\sqrt{5}}\right)$$

$$T = \left(\frac{-1+2+1}{3}, \frac{4+3+2}{3}\right) = \boxed{\left(\frac{2}{3}, 3\right)}$$



a: $x = -t - 1$, $y = t + 4$

$\vec{BC}(-1,-1)$ $\vec{a} = (-1, 1)$ $A \in a \Rightarrow$

$\vec{AC}(-2,2)$ $\vec{b} = (2,2)$ $B \in \vec{b} \Rightarrow$

b: $x = 2s + 2$, $y = 2s + 3$

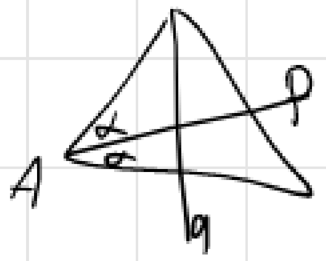
$$\left.\begin{aligned} O \in \vec{a} \quad x &= -t - 1 = 2s + 2 \\ O \in \vec{b} \quad y &= t + 4 = 2s + 3 \end{aligned}\right\} -$$

$$2t + 5 = 1 \Rightarrow t = -2$$

$$\boxed{O(1,2)}$$

Upisana kružnica

$A(-1,4)$, $B(2,3)$ $C(1,2)$



$$p: \vec{p} = \frac{\vec{AC}}{\|AC\|} + \frac{\vec{AB}}{\|AB\|} = \frac{(2,-2)}{2\sqrt{2}} + \frac{(3,-1)}{\sqrt{10}} = \frac{1}{\sqrt{10}}\left((\sqrt{5},-\sqrt{5})+(3,-1)\right) = \left(\frac{\sqrt{5}+3}{\sqrt{10}}, \frac{-1-\sqrt{5}}{\sqrt{10}}\right)$$

$$q: \vec{q} = \frac{\vec{CA}}{\|CA\|} + \frac{\vec{CB}}{\|CB\|}$$

$$\frac{(-2,2)}{2\sqrt{2}} + \frac{(+1,+1)}{\sqrt{2}} = \frac{1}{2\sqrt{2}}(0,2) = (0,\sqrt{2})$$

$q \ni C$, $A \ni p$ $\qquad q: x = 1 \quad y = \sqrt{2}t + 2$

$$p: x = \frac{(\sqrt{5}+3)s}{\sqrt{10}} - 1 \quad y = \frac{-1-\sqrt{5}}{\sqrt{10}} \cdot s + 4$$

$C \in p$ i $C \in q$

$$s = \frac{D(\vec{AC}, \vec{q})}{D(\vec{p}, \vec{q})} = \frac{\begin{vmatrix} 2 & -2 \\ 0 & \sqrt{2} \end{vmatrix}}{\begin{vmatrix} \frac{\sqrt{5}+3}{\sqrt{10}} & \frac{-1-\sqrt{5}}{\sqrt{10}} \\ 0 & \sqrt{2} \end{vmatrix}} = \frac{2\sqrt{2}}{\frac{\sqrt{5}+3}{\sqrt{5}}} = \frac{2\sqrt{10}}{\sqrt{5}+3}$$

$$\left(1, \frac{-2-2\sqrt{5}+4\sqrt{5}+12}{\sqrt{5}+3}\right) = \boxed{\left(1, \frac{10+2\sqrt{5}}{\sqrt{5}+3}\right)}$$

4.9 a) $P(1,-2)$, $\vec{p} = (1,2)$ $\quad Q(2,1)$ $\quad \vec{q} = (1,1)$

$p: x = t+1$, $y = 2t-2$ $\qquad q: x = s+2$, $y = s+1$

$$\begin{cases} t+1 = s+2 \\ 2t-2 = s+1 \end{cases} \quad t+1 = s+2 \quad t-3 = -1 \quad t = 2 \quad s = 1$$

$A(3,2)$

b) $P(1,-2)$ $\vec{p} = (0,1)$ $\quad Q(2,1)$ $\vec{q} = (0,-2)$ $\qquad D = \begin{vmatrix} 0 & 1 \\ 0 & -2 \end{vmatrix} = 0$

$p: x = 1$, $y = t-2$ $\qquad q: x = 2$, $y = -2t+1$ $\qquad \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1 \neq 0$ Paralelne su

c) $P(1,-2)$ $\vec{p} = (-1,-3)$, $Q(2,1)$ $\vec{q} = (1,3)$

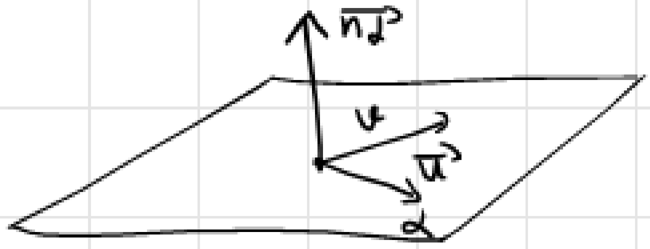
$p: x = -t+1$, $y = -3t-2$ $\qquad q: x = s+2$, $y = 3s+1$

$$D = \begin{vmatrix} -1 & -3 \\ 1 & 3 \end{vmatrix} = -3+3 = 0 \checkmark$$

$$D(\vec{PQ}, \vec{q}) = \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 3-3 = 0$$

Poklapaju se

$4.10 \quad x+2y-4z+3=0 \quad \vec{n_\alpha}(1,2,-4)$



$$\left.\begin{matrix} \vec{u},\vec{v} \parallel \alpha \\ \vec{n_\alpha} \perp \alpha \end{matrix}\right\} \Rightarrow \vec{u},\vec{v} \perp \vec{n_\alpha}$$

$\vec{u} \circ \vec{n_\alpha} = 0 \quad \vec{v} \circ \vec{n_\alpha} = 0$

$(x, 2y, -4z)$

$\vec{u} = (0, ?, ?) \Rightarrow 2y - 4z = 0 \Rightarrow y = 2z \Rightarrow \vec{u} = (0, 2, 1)$

$\vec{v} = (?, ?, 0) \Rightarrow x + 2y = 0 \Rightarrow x = -2y \Rightarrow \vec{v} = (2, -1, 0)$

Tačna sa α

$y = z = 0 \quad x = -3 \quad A(-3, 0, 0) \Rightarrow$

$\alpha: \ x = 2t - 3$
$\quad y = -t + 2s$
$\quad z = s$

11. $A(1,2,3) \quad B(1,-2,1) \quad C(0,0,1)$

$$\left.\begin{matrix} \vec{AB} = (0,-4,-2) \sim (0,2,1) & t \\ \vec{AC} = (1,2,2) & s \\ C \in \alpha & \end{matrix}\right\} \begin{matrix} x = s \\ y = 2t + 2s \\ z = t + 2s + 1 \end{matrix}$$

$$\vec{n_\alpha} \sim \vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ 0 & -4 & -2 \\ 1 & 2 & 2 \end{vmatrix} = e_1 \underbrace{(-8+4)}_{-4} + e_2 \cdot \underbrace{2}_{2} + e_3 \cdot \underbrace{4}_{4} = (-2, 1, 2)$$

$C \in \alpha \ (0,0,1)$

$\alpha: -2x + 1y + 2z - 2 = 0$

$4.12 \quad (x', y', z')$ u odnosu na ravan $\alpha: x + 2y - 2z + 5 = 0$

$z' = 0$

Vektor z' ose $\vec{f_3} = \dfrac{\vec{n_\alpha}}{\|\vec{n_\alpha}\|}$

vektor x' ose $\vec{f_1} \to$ bilo koji jedinični tkd. je $\vec{f_1} \perp \vec{f_3}$ tj.

$\vec{f_1} \circ \vec{f_3} = 0$

vektor y' ose $\vec{f_2} \to \vec{f_3} \times \vec{f_1}$

Koordinatni početak $O' \in \alpha$

$\vec{n_\alpha} = (1,2,-2) \quad \|\vec{n_\alpha}\| = 3 \ \} \ \vec{f_3} = \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)$

$\vec{f_1} = (x,y,z) \quad \vec{f_1} \circ \vec{f_3} = 0 = \underbrace{\frac{x}{3} + y\frac{2}{3} - \frac{2z}{3}}_{} = 0$

$\dfrac{(2,-1,0)}{\|\sqrt{4+1}\|} = \left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, 0\right)$

$$\vec{f_1}\times\vec{f_3} = \begin{vmatrix} \vec{e_1} & \vec{e_2} & \vec{e_3} \\ 1/3 & 2/3 & -2/3 \\ 2/\sqrt5 & -1/\sqrt5 & 0 \end{vmatrix} = e_1\cdot\left(-\frac{2}{3\sqrt5}\right) - e_2\cdot\frac{4}{3\sqrt5} + e_3\left(-\frac{1}{3\sqrt5}-\frac{4}{3\sqrt5}\right)$$
$$= \left(-\frac{2}{3\sqrt5}, -\frac{4}{3\sqrt5}, -\frac{5}{3\sqrt5}\right)$$

$O' \in \alpha \quad (-5,0,0)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2/\sqrt5 & -2/3\sqrt5 & 1/3 \\ -1/\sqrt5 & -4/3\sqrt5 & 2/3 \\ 0 & -5/3\sqrt5 & -2/3 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} - \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$
$\vec{f_1} \quad \vec{f_2}$

4.13 $p: x = 3t+4$, $t\in\mathbb{R}$
$y = -2t+1$
$z = t-2$

$$\underbrace{\frac{x-4}{3} = \frac{1-y}{2}}_{\alpha} = \frac{z+2}{1}, \quad \underbrace{\frac{1-y}{2} = \frac{z+2}{1}}_{\beta}$$

$\alpha: 2x-8 = 3-3y$
$2x+3y-11 = 0$

$\beta: 1-y-2z-4=0$
$y+2z+3=0$

4.14 α

$p: \frac{x-2}{1} = \frac{y-11}{5} = \frac{z-2}{1}$ $\quad \alpha\perp\beta \to \beta: y=0$

$\vec{p} = (1,5,1)$ $\quad \vec{n_\alpha}\perp\vec{p}$ $\quad \vec{n_\alpha} \sim \vec{p}\times\vec{n_\beta}$
$P = (2,11,2)$ $\quad \vec{n_\beta} = (0,1,0)$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 5 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -e_1 - e_2\cdot 0 + e_3 = (-1,0,1) \to -1(x-2)+1(z-2)=0$$
$-x+y=0$

4.15 $p: x+y-3=0$ $\quad x = 3-y$
$z-2x=0$ $\quad \frac{z}{2} = x$

$$\frac{x+0}{1} = \frac{3-y}{1} = \frac{z+0}{2}$$

4.16 $M(1,4,-3)$

a) $p: x-y-1=0 \quad z-2x=0$ $\quad \frac{x+0}{1} = \frac{y+1}{1} = \frac{z+0}{2}$ $\quad P(0,-1,0)$

$$\frac{\|\vec{PM}\times\vec{p}\|}{\|\vec{p}\|} = \frac{\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 5 & -3 \\ 1 & 1 & 2 \end{vmatrix}}{\sqrt6} = \frac{\|(13,-5,-4)\|}{\sqrt6} = \frac{\sqrt{169+25+16}}{\sqrt6} = \sqrt{35} \quad ?$$

b) $\alpha = 2x-y+4z=0$ $\quad d(M,\alpha) = \frac{|2-4-12|}{\sqrt{4+1+16}} = \frac{14}{\sqrt{21}}$
$\vec{n_\alpha} = (2,-1,4)$

4.17 $\alpha: x-2y+5z-1=0 \qquad n_\alpha=(1,-2,5)$

a) $\frac{x-2}{3}=\frac{y+4}{7}=\frac{z}{1}=t$

$\vec{p}=(3,7,1) \Rightarrow 3-14+5\neq 0 \Rightarrow \alpha\cap p=\{A\}$

$p: \left.\begin{array}{l} x=3t+2 \\ y=7t+4 \\ z=t \end{array}\right\}$ $\alpha: \underbrace{3t}+\bar{2}\ \underbrace{-14t}-\bar{8}+\underbrace{5t}-\bar{1}=0$

$-6t=7 \Rightarrow t=-7/6$

$A\left(\frac{-7+4}{2}, \frac{-49+24}{6}, -7/6\right) \quad \vec{p}\cap\alpha=\{A\}$

b) $q: \frac{x}{3}=\frac{y-2}{-1}=\frac{z-1}{-1}=t$

$Q=(0,2,1) \qquad \vec{n_\alpha}\circ\vec{q}=3+2-5=0$ ✓

$\vec{q}=(3,-1,-1) \rightarrow 0-4+5-1=0$ ✓ $\quad q\in\alpha$

c) $r: \frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{-1} \quad \alpha: x-2y+5z-1=0 \qquad n_\alpha=(1,-2,5)$

$\vec{r}=(3,-1,-1) \qquad \vec{n_\alpha}\circ\vec{r}=3+2-5=0$

$R(1,2,3) \rightarrow \alpha: 1-4+15-1\neq 0 \qquad r\parallel\alpha$

d) $s: x-y=1 \quad x+y-z+5=0$

$x-1=y \quad 1+y+y-z+5=0 \Rightarrow y=\frac{z-6}{2}$

$\frac{x-1}{1}=\frac{y}{1}=\frac{z-6}{2}$

$\alpha: x-2y+5z-1=0 \qquad n_\alpha=(1,-2,5) \qquad \vec{p}=(1,1,2) \rightarrow \begin{array}{l} x=t+1 \\ y=t \\ z=2t+6 \end{array}$

$\vec{n_\alpha}\circ\vec{p}=1-2+10\neq 0$

$t-2t+10t+1+30-1=0$

$9t=-30 \Rightarrow t=-30/9$

e) $t: x-z+2=0 \quad -y+3z+2=0$

$z=x+2 \qquad z=\frac{y-2}{3} \Rightarrow x+2=\frac{y-2}{3}=z$

$\alpha: x-2y+5z-1=0 \qquad \vec{n_\alpha}=(1,-2,5)$

$\vec{n_\alpha}\circ\vec{t}=1-6+5=0$ ✓

$T=(-2,2,0) \quad -2-4-1\neq 0 \quad \vec{t}\parallel\alpha$

4.18 $P(5,-2,1)$ $\vec{n_\alpha} \circ \vec{q} = 0$ $q: \frac{x+1}{-2} = \frac{y+4}{0} = \frac{z-2}{1}$

$-2a + 0b + c = 0 \Rightarrow c = 2a$

$a \cdot x + b \cdot y + 2a \cdot z + d = 0$

$5a - 2b + 2a + d = 0 \Rightarrow d = 2b - 7a$

$\alpha: ax + by + 2az + 2b - 7a = 0$

4.19 $p: x+y+z=0$ $\quad 2x - 2z + 3 = 0 \to f_2$

$f_1 \leftarrow$ $\beta: x - 4y - 8z + 12 = 0$ $\quad \pi/4$

$\alpha: f_1 + \lambda f_2 = (1+2\lambda)x + y + z(1-\lambda \cdot 2) + \lambda 3 = 0$

$\vec{n_\alpha} = (1+2\lambda, 1, 1-2\lambda)$ $\quad \vec{n_\beta} = (1,-4,-8) \to \|\vec{n_\beta}\| = 9$

$\vec{n_\alpha} \circ \vec{n_\beta} = \sqrt{1+4\lambda+4\lambda^2+1+1+4\lambda^2-4\lambda} \cdot 9 \cdot \frac{\sqrt{2}}{2} = 1+2\lambda-4-8+16\lambda$

$\sqrt{3+8\lambda^2} \cdot 9 \cdot \frac{1}{\sqrt{2}} = -11 + 18\lambda \; /^2$

$(3+8\lambda^2) \cdot 81/2 = 121 + \ldots$ Nađe se λ Ubacimo

4.20 $p: \frac{x-2}{1} = \frac{y-1}{3} = \frac{z+1}{1}$ $\quad p \in \alpha$ $\quad M(1,0,-1)$ $\sqrt{6}$ od α

$\vec{p} = (1,3,1)$ $\quad \vec{n_\alpha} \circ \vec{p} = 0 = 1a + 3b + 1c = 0 \to ax + 3by + cz + d = 0$

$P(2,1,-1)$ $\quad 2a + 3b - c + d = 0$?

$d(M,\alpha) = \frac{|a - c + d|}{\sqrt{a^2+9b^2+c^2}} = \sqrt{6}$ $\quad |a-c+d| = \sqrt{6} \cdot \sqrt{a^2+9b^2+c^2}$

$|\quad| = \ldots$

4.21 a) $p: \frac{x-2}{1} = \frac{y-2}{0} = \frac{z-2}{-1}$ $\quad \vec{p}(1,0,-1)$

$q: 2x = y \quad 3x = z$

$x = \frac{y}{2} = \frac{z}{3} \Rightarrow \vec{q}(1,2,3)$

$\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 0 & -1 \\ 1 & 2 & 3 \end{vmatrix} = e_1 \cdot 2 - e_2 4 + e_3 \cdot 2 = (2,-4,2) \neq \vec{0}$

$p: x = t+2$, $y = 2$, $z = -t+2$ $\quad q: x = s$, $y = 2s$, $z = 3s$

$\left.\right\}$ $s = t+2$, $s = 1$, $-t+2 = 3$, $t = -1$ $\quad (1,2,3)$

b) $p: \frac{x-2}{1} = \frac{y-2}{0} = \frac{z-2}{-1}$ $\quad \vec{p}(1,0,-1)$ $\quad q: \frac{x-1}{1} = \frac{y-2}{0} = \frac{z-3}{-1}$

$p: x = t+2$, $y = 2$, $z = -t+2$

$\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{vmatrix} = 0e_1 - e_2 \cdot 0 + e_3 \cdot 0 = 0$ ✓

$Q(1,2,3)$ $\quad 1 = t+2 \quad 3 = -t+2 \quad t = -1$ ✓

Poklapaju se

C) $q: \frac{x+1}{-2} = \frac{y-2}{0} = \frac{z+7}{2} \Rightarrow \vec{q}(-2,0,2)$, $Q(-1,2,-7)$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 0 & -1 \\ -2 & 0 & 2 \end{vmatrix} = \vec{0}$$

$p: \frac{x-2}{1} = \frac{y-2}{0} = \frac{z-2}{-1}$ $\vec{p}\ (1,0,-1)$

$x = t+2$

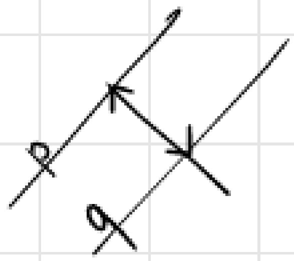
$y = 2$

$z = -t+2$

$-1 = t+2$

$-7 = -t+2$ ↯ Paralelne su

4.22.



$p: \frac{x-4}{1} = \frac{y+3}{2} = \frac{z-12}{-1}$ $q: \frac{x-3}{-7} = \frac{y-1}{2} = \frac{z-1}{3}$

$P(t+4,\ 2t-3,\ -t+12)$ $Q(-7s+3,\ 2s+1,\ 3s+1)$ $\vec{p} = (1,2,-1)$

$\overrightarrow{PQ}\ (-7s-1-t,\ 2s+4-2t,\ 3s-11+t)$ $\vec{q} = (-7,2,3)$

$\overrightarrow{PQ} \circ \vec{p} = -7s-1-t+4s+8-4t-3s+11-t = 0$

$-6s+18-6t = 0 \ /:6$ $s-3+t = 0$

$\overrightarrow{PQ} \circ \vec{q} = +49s+7+7t+4s+8-4t+9s-33+3t = 62s+6t-$

Nađe se s i t $s=0, t=3$

$\overrightarrow{PQ}\ (-4,-2,-8)$

$\|\overrightarrow{PQ}\| = \sqrt{16+4+64} = \sqrt{84} = 2\sqrt{21}$

4.23 $\alpha: x+2y-3z-1=0$ $\beta: 2x-3y+4z+2=0$ $\vec{n_\alpha} = (1,2,-3)$ $\vec{n_\beta} = (2,-3,4)$

$\vec{n_\alpha} \circ \vec{n_\beta} = |\vec{n_\alpha}| \cdot |\vec{n_\beta}| \cdot \cos(\alpha,\beta) = \sqrt{1+4+9} \cdot \sqrt{4+9+16} \cdot \cos = 2-6-12 = -16$

$= \frac{-16}{\sqrt{14} \cdot \sqrt{29}} = \cos$

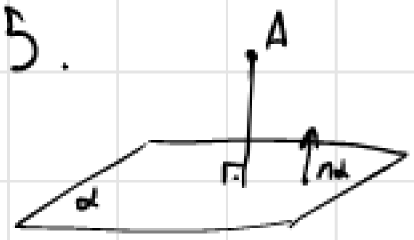
4.24 $p: x+2y-3z-1=0$ $x-z+2=0$ $\alpha: x-4y+2z+2=0$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 2 & -3 \\ 1 & 0 & -1 \end{vmatrix} = (-2,-2,-2) \Rightarrow \vec{p}(-2,-2,-2)$$

$-2+8-4 = 2 \Rightarrow \frac{2}{\sqrt{3 \cdot 4} \cdot \sqrt{1+16+4}} = \frac{1}{3\sqrt{7}} = \cos \measuredangle$

$\pi/2 -$ je konačno

4.25.



$A(2,1,-3) \quad \alpha: 2x-4y+z+5=0$

$\vec{n_\alpha}(2,-4,1) \qquad \frac{x-2}{2}=\frac{y-1}{-4}=\frac{z+3}{1}$

4.26. $p: \frac{x-2}{3}=\frac{y+4}{5}=\frac{z-1}{-2} \qquad q: \frac{x-\lambda}{2}=\frac{y-3}{1}=\frac{z+5}{0}$

$\{A\} \in p \quad x=3t+2 \qquad \{A\} \in q: x=2s+\lambda$

$y=5t-4 \qquad y=1s+3$

$z=-2t+1 \qquad z=-5$

$-2t+1=-5 \quad t=3$

$$\left.\begin{aligned} 11 &= 2s+\lambda \\ 11 &= 1s+3 \Rightarrow s=8 \\ -5 &= -5 \checkmark \end{aligned}\right\} \boxed{\lambda=-5}$$

4.27 $p \in L(2,-1,7) \qquad q: \frac{x-1}{2}=\frac{y-4}{-3}=\frac{z-3}{1} \qquad r: \frac{x-2}{-1}=\frac{y-11}{-3}=\frac{z+2}{0}$

$p: \frac{x-2}{a}=\frac{y+1}{b}=\frac{z-7}{c} \qquad \vec{LQ}=(-1,5,-4) \quad \vec{LR}=(0,12,-9)$

$$\begin{vmatrix} 2 & -3 & 1 \\ a & b & c \\ -1 & 5 & -4 \end{vmatrix} = 2(-4b-5c)+3(-4a+c)+1(5a+b) = -8b-10c-12a+3c+5a+b$$

$$=-7(b+c+a)=0$$

$a+b+c=0$

$$\begin{vmatrix} -1 & -3 & 0 \\ a & b & c \\ 0 & 12 & -9 \end{vmatrix} = 9b+12c+3(-9a) = 3(-9a+3b+4c)=0$$

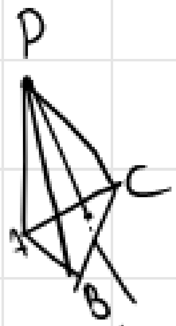
$a+b+c=0$

$-9a+3b+4c=0 \Rightarrow -9a-3a-3c+4c=0$

$c=12a \quad b=-13a$

$p: \frac{x-2}{1}=\frac{y+1}{-13}=\frac{z-7}{12}$

4.28 $p: \frac{x-1}{-1}=\frac{y}{3}=\frac{z+1}{2}$



$A(2,4,6) \quad B(-4,2,0) \quad C(6,4,-2)$

$P(-1,3,2)$ M

$\vec{PA}=(1,4,7)$

$\vec{PB}=(-5,2,1)$

$\vec{PC}=(5,4,-1)$

$$\begin{vmatrix} 1 & 4 & 7 \\ -5 & 2 & 1 \\ -1 & 3 & 2 \end{vmatrix} = (4-3)-4\cdot(-10)-4+7(-15)+14 = 1+32-91=-58$$

$$\begin{vmatrix} -5 & 2 & 1 \\ 5 & 4 & -1 \\ -1 & 3 & 2 \end{vmatrix} = -5(8+3)-2(10-1)+(15+4) = -55-18+19=-54$$

$$\begin{vmatrix} 5 & 4 & -1 \\ 1 & 4 & 7 \\ -1 & 3 & 2 \end{vmatrix} = 5(8-21)-4(2+7)-1(3+4) = -65-36-7=-108$$

$$\begin{vmatrix} 1 & 4 & 7 \\ -5 & 2 & 1 \\ 5 & 4 & -1 \end{vmatrix} = (-2-4)-4(5-5)+7(-20-10) = -6-210 = -216$$

$$t = \frac{216}{+216} = 1$$

$$M = (1,0,-1) + (-1,3,2) = (0,3,1)$$

4.29 $\alpha: -x+y+2z-3=0$ $A(-2,1,0)$ $B(2,-1,3)$ $C(1,2,3)$

Duž $[AB] = M = A + t\cdot\overrightarrow{AB}$ $t\in[0,1]$ $\overrightarrow{AB} = (4,-2,3)$

$$\begin{aligned} x &= 4t-2 \\ y &= -2t+1 \\ z &= 3t \end{aligned} \rightarrow \alpha : -4t+2+1-2t+6t-3=0$$

$0=0$ ✓ $[AB]\in\alpha$

Duž $[BC]$ $N = B + t\cdot\overrightarrow{BC}$ $\overrightarrow{BC} = (-1,3,0)$

$$\begin{aligned} x &= -t+2 \\ y &= 3t-1 \\ z &= 3 \end{aligned}$$

$\alpha: t-2+3t-1+6-3=0$

$4t=0 \Rightarrow t=0 \Rightarrow [BC]\cap\alpha = \{B\}$

Nema potrebe za $AC \Rightarrow \triangle ABC \cap \alpha = \{[AB]\}$

5.1 $x^2+y^2-2x+4y-11=0$

$(x-1)^2-1+(y+2)^2-4-11 \Rightarrow (x-1)^2+(y+2)^2 = 4^2$

$O(1,-2)$ $r=4$

$x = 1+4\cos\varphi$

$y = -2+4\sin\varphi$

$h'(\varphi) = (-4\sin\varphi, 4\cos\varphi)$

$\|h'(\varphi)\| = \sqrt{16\sin^2\varphi+16\cos^2\varphi} = 4 \neq 1$

$\gamma(t) = (x(t), y(t))$

$\gamma'(t) = (x'(t), y'(t))$

$s(t) = \int_{t_0}^{t} \|\gamma'(u)\|\,du$ $t_0 \to$ iz (a,b) proizvoljno $\|\gamma'(u)\| = 1$

$s(\varphi) = \int_0^{\varphi} \|h'(\varphi)\|\,du = 4\varphi \Rightarrow \varphi = s/4 \Rightarrow h(s) = (1+4\cos\frac{s}{4}, -2+4\sin(s/4))$

5. $(x-1)^2+(y+2)^2=4^2 \qquad (1,-2)$

a) $p: \vec{p}=(1,1) \quad P(2,-2)$

$$d(C,P)=\frac{\|\overrightarrow{CP}\times\vec{p}\|}{\|\vec{p}\|}=\frac{\left\|\begin{matrix}1&0\\1&1\end{matrix}\right\|}{\|\sqrt{1+1}\|}=\frac{\sqrt{1}}{\sqrt{2}}=\frac{\sqrt{2}}{2}<r$$

$\exists 2$ presečne tačne $\quad x=t+2 \quad y=t-2$

$(t+1)^2+(t)^2=16 \qquad 2t^2+2t+1-16=0$

$$t_{1/2}=\frac{-2\pm\sqrt{4+4\cdot2\cdot15}}{4} \rightarrow \frac{-2+2\sqrt{31}}{4}=\frac{-1+\sqrt{31}}{2}, \quad \rightarrow \frac{-1-\sqrt{31}}{2}$$

$S_1\left(\frac{3+\sqrt{31}}{2}, \frac{-5+\sqrt{31}}{2}\right)$

$S_2\left(\frac{3-\sqrt{31}}{2}, \frac{-5-\sqrt{31}}{2}\right)$

b) $q: x-y-4=0 \qquad C(1,-2) \quad r=4$

$\frac{x-4}{1}=\frac{y}{1}$

$$Q(4,0) \rightarrow d(C,Q)=\frac{\|(-3,-2)\times(1,1)\|}{\sqrt{2}}=\frac{1}{\sqrt{2}}<4$$

$\exists 2 \quad x=t+4 \quad y=t \qquad (t+3)^2+(t+2)^2=16$

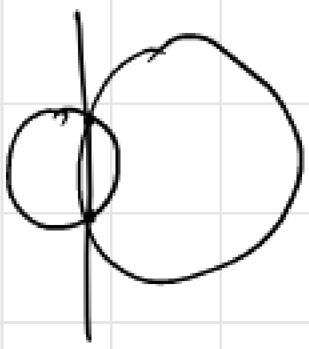
$2t^2+10t-3=0 \qquad t_{1/2}=\frac{-10\pm\sqrt{100+24}}{4}$

5.3 $\quad k: (x-4)^2+(y-3)^2=25 \qquad l: x=-1+4\cos t \Rightarrow \cos t=\frac{x+1}{4} \; /^2$

$y=-1+4\sin t \Rightarrow \sin t=\frac{y+1}{4} \; /^2 \;)+$

$1=\frac{(x+1)^2}{16}+\frac{(y+1)^2}{16}$

$(x+1)^2+(y+1)^2=16$



$k-l=(x-4)^2-(x+1)^2+(y-3)^2-(y+1)^2=9$

$-5(2x-3)+-4(2y-2)=9$

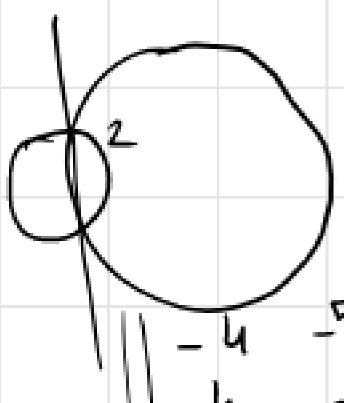
$-10x-15-8y+8-9=0 \Rightarrow 10x+8y+16=0 \;/:2$

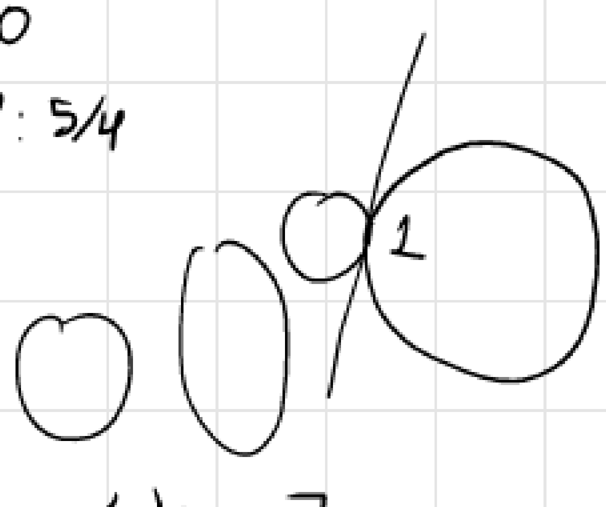
$5x+4y-7=0$

$5x=\frac{4y-7}{-1} \;/:5/4$

$\frac{x}{4}=\frac{y-7/4}{-5/4}$

$\vec{p}=(4,-5/4) \quad P(0,+7/4)$





$O(4,3)\,5 \quad O'(-1,-1)\,4$

$$d(O,P)=\frac{\|OP\times\vec{p}\|}{\|p\|}=\frac{\left\|\begin{matrix}-4&-5/4\\4&-5\end{matrix}\right\|}{\sqrt{16+25}}=\frac{25}{\sqrt{41}}<4 \quad \exists_2$$

..... Našli smo ih!

5.4 a) $3y^2+6y-x-1=0$

$(\sqrt{3}y)^2+2\cdot\sqrt{3}y\cdot\sqrt{3}+3-3-(\sqrt{x})^2-1=0$

$(\sqrt{3}y+\sqrt{3})^2-(\sqrt{x})^2=4 \Rightarrow \frac{(y+1)^2}{1/3}=4+x \rightarrow (y+1)^2=\left(\frac{4+x}{3}\right)$

$y+1=\bar{y} \quad 4+x=\bar{x}$

$\bar{y}^2=\frac{\bar{x}}{3}$

b) $x^2+25y^2-6x+50y+9=0$

$(5y)^2+5y\cdot2\cdot5+25-25+(x^2-6x+9)-9+9=0$

$(5y+5)^2+(x-3)^2=25$

$(y+5)^2+\frac{(x-3)^2}{25}=1$

c) $y^2-36x-2y-35=0$

$(y-1)^2=36(1+x)=\frac{x+1}{1/36} \rightarrow \bar{y}^2=\frac{\bar{x}}{1/36}$

$y-1=\bar{y} \quad 1+x=\bar{x}$

5.5 $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$xy=1 \rightarrow \left.\begin{array}{l} x=\bar{x}-\bar{y} \\ y=\bar{x}+\bar{y} \end{array}\right\} (\bar{x}-\bar{y})(\bar{x}+\bar{y})=1$

$\bar{x}^2-\bar{y}^2=1$ ✓

$\left(\frac{x}{a}-\frac{y}{b}\right)\left(\frac{x}{a}+\frac{y}{b}\right)=1$

$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$

$\begin{array}{cc} \uparrow & \uparrow \\ f_1 & f_2 \end{array}$

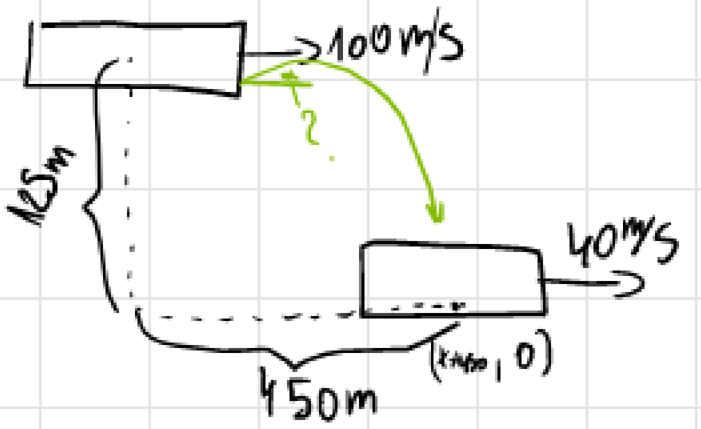
$-1+1=0 \quad f_1 \perp f_2$

$\sqrt{1+1}=\sqrt{2}\neq1$

$\frac{\pm1}{\sqrt{2}} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$

$\pi/4$

5.6



$x(t)=v_0\cdot\cos\varphi_0\cdot t$

$y(t)=-\frac{gt^2}{2}+h+v_0\cdot\sin\varphi_0\cdot t$

$\varphi_0=0 \Rightarrow x(t)=v_0\cdot t$

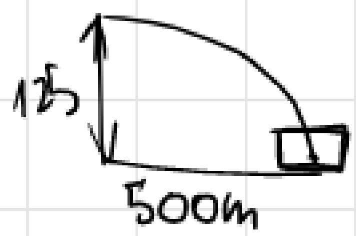
$y(t)=h-\frac{gt^2}{2}=0=y(t)$

$h=\frac{gt^2}{2}$

$125=5t^2 \Rightarrow \boxed{t=5\,s}$

$x(t)=100\,m/s\cdot t$

$x(5)=100\cdot5=500m$

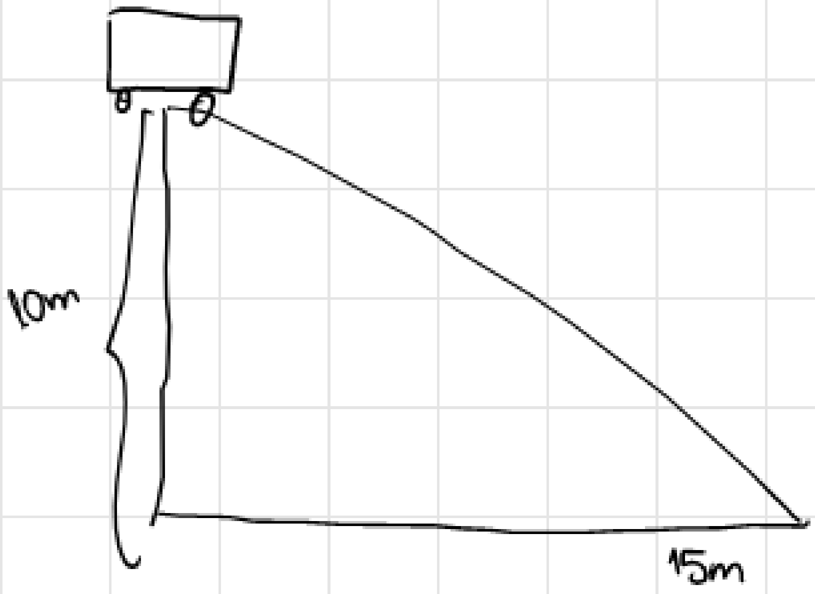


$\frac{500}{40}=12.5-5s=7.5s \rightarrow \cdot40=75\cdot4=150\cdot2=300m$

$450-300=150$

$\frac{150m}{60}=2.5s$

5.7



$v_0 = ?\ km/h$

$x(t) = v_0 \cdot t \quad y(t) = h - \frac{g t^2}{2}$

$y(t) = 0 = 10m - 5t^2 \Rightarrow t = \sqrt{2}\ s$

$(\sqrt{2}) = v_0 \cdot \sqrt{2} = 15$

$v_0 = \frac{15}{\sqrt{2}} \frac{m}{s} \cdot \frac{36\cancel{00}}{10\cancel{00}} = \frac{15 \cdot 36}{10\sqrt{2}} \frac{km}{h}$

5.8 $e = 0.05 \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad c = \sqrt{a^2 - b^2}$

$a = 5$ AJ

$e = \frac{c}{a} \quad 0.05 \cdot 5 = c \quad a + c$ - najmanje $= 5.25$

$0.25 = c \quad a - c$ - najveće $= 4.75$

$\sqrt{125} \sim 11.18$ godina

5.9

$\alpha_2(t) = \sum_{i=0}^{2} \binom{2}{i} t^i (1-t)^{-i} \cdot P_i =$

$(1-t)^2 \cdot (1,1) + 2t(1-t)(2,2) + t^2(4,-1)$

$(1-2t+t^2)\begin{bmatrix}1\\1\end{bmatrix} + (2t-2t^2)\begin{bmatrix}2\\2\end{bmatrix} + t^2\begin{bmatrix}4\\-1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} + 2t\begin{bmatrix}2-1\\2-1\end{bmatrix} + t^2\begin{bmatrix}4+1-4\\-1+1-4\end{bmatrix}$

$\alpha_2(t) = (\underbrace{1+2t+t^2}_{x}), (\underbrace{1+2t-4t^2}_{y}))$

$= \begin{bmatrix}1\\1\end{bmatrix} + 2t\begin{bmatrix}1\\1\end{bmatrix} + t^2\begin{bmatrix}1\\-4\end{bmatrix}$

5.10 $P_0 = (2,3) \quad P_1 = (-1,4) \quad P_2(3,0) \quad P_3 = (1,-2)$

a) $\alpha_3(t) = \sum_{i=0}^{3} \binom{3}{i} t^i (1-t)^{3-i} \cdot P_i = (1-t)^3 \cdot \begin{bmatrix}2\\3\end{bmatrix} + 3 \cdot t(1-t)^2\begin{bmatrix}-1\\4\end{bmatrix} + 3t^2(1-t)\begin{bmatrix}3\\0\end{bmatrix} + t^3\begin{bmatrix}1\\-2\end{bmatrix}$

$= (1-3t+3t^2-t^3)\begin{bmatrix}2\\3\end{bmatrix} + (3t-6t^2-3t^3)\begin{bmatrix}-1\\4\end{bmatrix} + (3t^2-3t^3)\begin{bmatrix}3\\0\end{bmatrix} + t^3\begin{bmatrix}1\\-2\end{bmatrix}$

$= \begin{bmatrix}2\\3\end{bmatrix} + 3t\begin{bmatrix}-2-1\\-3+4\end{bmatrix} + 3t^2\begin{bmatrix}2+2+3\\3-8\end{bmatrix} - t^3\begin{bmatrix}2-3+9-1\\3+12+2\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix} + \begin{bmatrix}-9\\3\end{bmatrix}t + \begin{bmatrix}21\\-15\end{bmatrix}t^2 + t^3\begin{bmatrix}-13\\7\end{bmatrix}$

b) $\alpha_3'(t) = \begin{bmatrix}-9\\3\end{bmatrix} + \begin{bmatrix}42\\-30\end{bmatrix}t + \begin{bmatrix}-39\\21\end{bmatrix}t^2$

$P_0 \Rightarrow \alpha_3'(0) = \begin{bmatrix}-9\\3\end{bmatrix} \Rightarrow \frac{x-2}{-3} = \frac{y-3}{1} \Rightarrow P_1 \in t_{P_0}$ tj $P_0P_1 = t_{P_0}$

$P_3 \Rightarrow \alpha_3'(1) = \begin{bmatrix}-6\\-6\end{bmatrix} \Rightarrow \frac{x-1}{1} = \frac{y+2}{1} \Rightarrow P_2 \in t_{P_3}$ tj. $P_2P_3 = t_{P_3}$

c) $\alpha(1/2)$ P_0P_3

$\rightarrow \left(-9+42/2-39\frac{1}{4},\ 3-30/2+21/4\right) = (9/4, -27/4) \sim (1,-3)$

$P_0P_3 = (1-2, -2-3) = (-1,-5) \Rightarrow \vec{p}$ $\quad \frac{x-1}{-1} = \frac{y+2}{-5}$

$\begin{vmatrix} -1 & -5 \\ 1 & -3 \end{vmatrix} = 3+5 = 8 > 0$ Nisu paralelne

d)

$P_2 = (3,0) \Rightarrow \tilde{P}_2' = (-4,-11)$

$\rightarrow \binom{3}{2} t^2(1-t)(-7,-11) = (3t^2-3t^3)\begin{pmatrix}-7\\-11\end{pmatrix}$

$\alpha_3(t) = \begin{bmatrix}2\\3\end{bmatrix} + \begin{bmatrix}-9\\3\end{bmatrix}t + \begin{bmatrix}21\\-15\end{bmatrix}t^2 + t^3\begin{bmatrix}-13\\7\end{bmatrix}$ $\quad \tilde{\alpha}_3(t) = \begin{bmatrix}2\\3\end{bmatrix} + \begin{bmatrix}-9\\3\end{bmatrix}t + \begin{bmatrix}0\\-48\end{bmatrix}t^2 + \begin{bmatrix}8\\40\end{bmatrix}t^3$

5.11 $\quad P_0 = (2,1) \quad P_1 = (6,13) \quad P_2 = (14,-7) \quad p: y = 5$

$r: x = 2+3s$
$\quad y = 12-2s \quad s \in \mathbb{R}$

a) $\alpha_2(t) = \underbrace{(1-t)^2}_{1-2t+t^2}\begin{bmatrix}2\\1\end{bmatrix} + \underbrace{2(1-t)\cdot t}_{2t-2t^2}\begin{bmatrix}6\\13\end{bmatrix} + t^2\begin{bmatrix}14\\-7\end{bmatrix}$

$\alpha_2(t) = \begin{bmatrix}2\\1\end{bmatrix} + \begin{bmatrix}8\\24\end{bmatrix}t + \begin{bmatrix}4\\-32\end{bmatrix}t^2 \quad \alpha_2(t) = (2+8t+4t^2,\ 1+24t-32t^2)$

b) Sa $p: y=5$

$P_0P_1 = (4,12) \rightarrow \begin{matrix} x = 4t+2 \\ y = 12t+1 \end{matrix} \rightarrow \begin{matrix} 12t = 4 \\ t = 1/3 \end{matrix} \quad (10/3, 5)$

$P_1P_2 = (8,-20) \rightarrow \begin{matrix} x = 8t+6 \\ y = -20t+13 \end{matrix} \quad \begin{matrix} -20t = -8 \\ t = \frac{2}{5} \end{matrix} = (46/5, 5)$

Sa r: $x = 2+3s$
$y = 12-2s \quad R(2,12)$

$\begin{vmatrix} 3 & -2 \\ 4 & 12 \end{vmatrix} = 36+8 \neq 0$

$P_0P_1 = (4,12) \rightarrow \begin{matrix} x = 4t+2 \\ y = 12t+1 \end{matrix}$ $\quad t = \frac{D(\overrightarrow{P_0R}, \vec{r})}{(\overrightarrow{P_0P_1}, \vec{r})} = \frac{\begin{vmatrix} 0 & 11 \\ 3 & -2 \end{vmatrix}}{\begin{vmatrix} 4 & 12 \\ 3 & -2 \end{vmatrix}} = \frac{-33}{-8-36} = \frac{33}{44} = 3/4$

$A(5,10)$

$P_1P_2 = (8,-20) \rightarrow \begin{matrix} x = 8t+6 \\ y = -20t+13 \end{matrix}$ $\quad \begin{vmatrix} 3 & -2 \\ 8 & -20 \end{vmatrix} = -60+16 \neq 0$

$P_1(6,13)$ $\quad t = \frac{|\overrightarrow{P_1R}, \vec{r}|}{\begin{vmatrix} 8 & -20 \\ 3 & -2 \end{vmatrix}} = \frac{\begin{vmatrix} -4 & -1 \\ 3 & -2 \end{vmatrix}}{44} = \frac{8+3}{44} = \frac{11}{44} = 1/4$

$B(8,8)$

c) $p: y = 5$

$$\alpha_2(t) = (2+8t+4t^2,\ 1+24t-32t^2)$$

$$5 = 1+24t-32t^2$$

$$32t-24t+4=0 \rightarrow t_1 = 1/4 \quad t_2 = 1/2$$

$t_2 \rightarrow (7,5)$, $t_1 \rightarrow (17/4,\ 5)$

$r:\ x = 2+3s$, $y = 12-2s$

$$3s = 8t+4t^2 \rightarrow s = \frac{8t+4t^2}{3}$$

$$12-2s = 1+24t-32t^2$$

$$36-16t-8t^2 = 3+72t-96t^2$$

$$33-88t+88t^2 = 0 \quad |:11 \Rightarrow 8t^2-8t+3=0$$

$$D = 64-4\cdot3\cdot8 < 0 \quad \text{Ne postoji } t\in[0,1]$$

d) $y^2 = 4x \quad P_0 = (2,1) \quad P_1 = (6,13) \quad P_2 = (14,-7)$

$\hookrightarrow \beta(t) = (t^2, 2t)$

$Q_0 = \beta(0) = (0,0)$ $\quad$ Q_1 na presecu tangenti

$Q_2 = \beta(1) = (1,2)$ $\quad$ $\beta'(t) = (2t, 2)$

$\beta'(0) = (0,2) \sim (0,1)$ $\quad$ $x = 0$, $y = t+0$

$\beta'(1) = (2,2) \sim (1,1)$ $\quad$ $x = s+1$, $y = s+2$ $\Rightarrow s = -1$ $\Big\}$ $(0,1)$ Q_1

$$\begin{array}{ccccc} (2,1) & \xrightarrow{g^{-1}} & \overset{O}{(0,0)} & \xrightarrow{f} & (0,0) \\ (6,13) & \xleftarrow{g} & A(1,0) & & (0,1) \\ (14,-7) & & B(0,1) & & (1,2) \end{array}$$

$g: OAB \rightarrow P_0P_1P_2$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 4 & 12 \\ 12 & -8 \end{bmatrix} \\ \uparrow P_0P_1 \quad \uparrow P_0P_2 \end{array} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C^{-1} = \frac{1}{-32-144}\begin{bmatrix} -8 & -12 \\ -12 & 4 \end{bmatrix} = \frac{1}{44}\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}$$

$\rightarrow -176 = -4\cdot(44)$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{44}\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix} - \begin{bmatrix} 7/44 \\ 5/44 \end{bmatrix}$$

f:

$(0,0) \rightarrow (0,0)$

$(1,0) \rightarrow (0,1)$

$(0,1) \rightarrow (1,2)$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$f\circ g^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{44} & \frac{3}{44} & -\frac{7}{44} \\ \frac{3}{44} & -\frac{1}{44} & -\frac{5}{44} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{44} & -\frac{1}{44} & -\frac{5}{44} \\ \frac{8}{44} & \frac{1}{44} & -\frac{17}{44} \\ 0 & 0 & 1 \end{bmatrix}$$

Finally: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \frac{1}{44}\begin{bmatrix} 3 & -1 \\ 8 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} -5/44 \\ -17/44 \end{bmatrix}$

5.12 $\alpha_4(t)$ $\quad P_0(5,-4)\quad P_1(-13,14)\quad P_2(5,50)\quad P_3(32,41)\quad P_4(14,15)$

$t=1/3$

$$\begin{tikzcd}[row sep=tiny, column sep=small]
P_{00} \arrow[dr] & & & & \\
 & P_{10} \arrow[dr] & & & \\
P_{01} \arrow[ur] \arrow[dr] & & P_{20} \arrow[dr] & & \\
 & P_{11} \arrow[ur] \arrow[dr] & & P_{30} \arrow[dr] & \\
P_{02} \arrow[ur] \arrow[dr] & & P_{21} \arrow[ur] \arrow[dr] & & P_{40} \\
 & P_{12} \arrow[ur] \arrow[dr] & & P_{31} \arrow[ur] & \\
P_{03} \arrow[ur] \arrow[dr] & & P_{22} \arrow[ur] & & \\
 & P_{13} \arrow[ur] & & & \\
P_{04} \arrow[ur] & & & &
\end{tikzcd}$$

$t=1/3$

$P_{10} = (1-t)P_{00} + tP_{01} = \left(\frac{10}{3}, \frac{-8}{3}\right)+\left(-\frac{13}{3}, \frac{14}{3}\right) = (-1,2)$

$P_{11} = \frac{2}{3}P_{01} + \frac{1}{3}P_{02} = \left(\frac{-26}{3}, \frac{28}{3}\right)+\left(\frac{5}{3}, \frac{50}{3}\right) = (-7,26)$

$P_{12} = \frac{2}{3}P_{02} + 1/3\, P_{03} = \left(\frac{10}{3}, \frac{100}{3}\right)+\left(\frac{32}{3}, \frac{41}{3}\right) = (14, 47)$

$P_{13} = \frac{2}{3}P_{03} + 1/3\, P_{04} = \left(\frac{14}{3}, \frac{15}{3}\right)+\left(\frac{64}{3}, \frac{82}{3}\right) = (26, 32\tfrac{1}{3})$

$P_{20} = \frac{2}{3}(-1,2) + 1/3(-7,26) = \left(-\frac{2}{3}, 4/3\right)+\left(-7/3, 26/3\right) = (-3, 10)$

$P_{21} = \left(-\frac{14}{3}, \frac{52}{3}\right)+\left(\frac{14}{3}, \frac{47}{3}\right) = (0,33)$

$P_{22} = \left(\frac{28}{3}, \frac{94}{3}\right)+\left(\frac{26}{3}, 10\tfrac{7}{9}\right) = (18, 42\tfrac{1}{9})$

$P_{30} = \left(-2, \frac{20}{3}\right)+(0,11) = (-2, 17\tfrac{2}{3})$

$P_{31} = (0,22)+(6, 14\tfrac{1}{27}) = (6, 36\tfrac{1}{27})$

$P_{40} = (-4/3, 34/3 + 4/9)+(2, 12\tfrac{1}{81}) = \left(2/3, \frac{106\cdot 9 + 12\cdot 81 + 1}{81}\right)$

Ne znam da li je tačno i ne pada mi na pamet opet da radim

5.13 $P_0=(0,-2)\quad P_1=(-2,4)\quad P_2=(0,10)\quad P_3=(6,8)\qquad t=1/2$

$$\begin{tikzcd}[row sep=tiny, column sep=small]
P_{00} \arrow[dr] & & & \\
 & P_{10} \arrow[dr] & & \\
P_{01} \arrow[ur] \arrow[dr] & & P_{20} \arrow[dr] & \\
 & P_{11} \arrow[ur] \arrow[dr] & & P_{30} \\
P_{02} \arrow[ur] \arrow[dr] & & P_{21} \arrow[ur] & \\
 & P_{12} \arrow[ur] & & \\
P_{03} \arrow[ur] & & &
\end{tikzcd}$$

$P_{10} = \frac{1}{2}(-2,2) = (-1,1)$

$P_{11} = 1/2(-2,14) = (-1,7)$

$P_{12} = 1/2(6,18) = (3,9)$

$P_{20} = 1/2(-2,8) = (-1,4)$

$\boxed{P_{30} = (0,6)}$

$P_{21} = 1/2(2,16) = (1,8)$

$\tilde{\alpha}_3(t) : \overset{0}{(0,-2)}, \overset{1}{(-1,1)}, \overset{2}{(-1,4)}, \overset{3}{(0,6)}$

$\tilde{\tilde{\alpha}}_3(t) : (6,8), (3,9), (1,8), (0,6)$

b) $Q_0=(0,-2)\quad Q_4=(0,6)\qquad Q_1 = \frac{1}{4}(0,-2) + (1+\frac{1}{4})(-1,1) = (-5/4, 3/4)$ $\quad 5/4$

$Q_2 = \frac{1}{2}(-1,1) + \frac{3}{2}(-1,4) = (-2, \frac{13}{2})\qquad Q_3 = \frac{3}{4}(-1,4)+\frac{7}{4}(0,6) = (-3/4, \frac{54}{4})$

$Q_4=(0,6)$

14. $\sqrt{2}\cos\varphi \quad \sqrt{2}\sin\varphi \quad \varphi \in [-\pi/4, \pi/4]$

$$t = tg\frac{\varphi}{2} = \frac{\sin\varphi/2}{\cos\varphi/2} = \frac{\sqrt{1-\cos\varphi}}{\sqrt{1+\cos\varphi}} \longrightarrow t^2(1+\cos\varphi) = 1-\cos\varphi$$

$$\cos\varphi(t^2+1) = 1-t^2$$

$$\cos\varphi = \frac{1-t^2}{1+t^2}$$

$$\sin\varphi/2 = \sqrt{1-\cos^2\varphi/2} = \sqrt{1-\frac{(1-t^2)^2}{(1+t^2)^2}}$$

$$= \sqrt{\frac{2t^2 \cdot 2}{(1+t^2)^2}} = \frac{2t}{1+t^2}$$

$$k: x = \sqrt{2}\cdot\frac{1-t^2}{1+t^2} \qquad \varphi \in [0, \pi/2]$$

$$y = \sqrt{2}\cdot\frac{2t}{1+t^2}$$

$$\frac{\sqrt{2}}{1+t^2}(1-t^2, 2t) \Rightarrow \quad 1+t^2 = w_0(1-t)^2 + 2w_1(1-t)t + w_2t^2$$

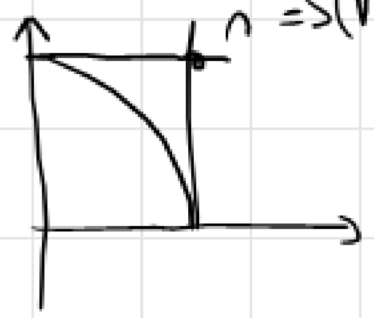
$$= w_0 + t(-2w_0+2w_1) + t^2(w_0-2w_1+w_2)$$

$$\left(\frac{\sqrt{2}(1-t^2)}{1+t^2}, \frac{2\sqrt{2}t}{1+t^2}\right) = \frac{w_0P_0(1-t)^2 + 2P_1w_1(1-t)t + w_2P_2t^2}{w_0 + t(-2w_0+2w_1) + t^2(w_0-2w_1+w_2)}$$

$1+t^2$: $w_0 = 1$

$-2w_0 + 2w_1 = 0 \Rightarrow w_1 = 1$

$w_0 - 2w_1 + w_2 = 1 \qquad w_2 = 2 \longrightarrow \boxed{\begin{matrix} w_0 = 1 = w_1 \\ w_2 = 2 \end{matrix}}$



$t = 0 \qquad\qquad t_1 = 1$

$(\sqrt{2}, 0) = P_0 \qquad (0, \sqrt{2}) = P_2 \qquad P_1 \in t_{P_0} \cap t_{P_2}$

$k: x^2 + y^2 = 2$

tangenta $u\,(x_0, y_0) = x x_0 + y y_0 = 2$

$u\ P_0 \Rightarrow \sqrt{2}x = 2 \Rightarrow x = \sqrt{2}$

$u\ P_2 = y\sqrt{2} = 2 \Rightarrow y = \sqrt{2}$

$\Big\} \ P_1 = (\sqrt{2}, \sqrt{2})$

Rotacija za $-\pi/4$ na $P_0, P_1, P_2 \longrightarrow Q_0, Q_1, Q_2$

težine ostaju

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos -\pi/4 & -\sin -\pi/4 \\ \sin -\pi/4 & \cos -\pi/4 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \qquad Q_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad Q_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\left(\frac{\sqrt{2}(1-t^2)}{1+t^2}, \frac{2\sqrt{2}t}{1+t^2}\right) = \frac{1Q_0(1-t)^2 + 2Q_1 1(1-t)t + 2Q_2t^2}{1 + t(-2\cdot 1 + 2\cdot 1) + t^2(1 - 2\cdot 1 + 2)} = \frac{Q_0 + t(-2Q_0 + 2Q_1) + t^3(Q_0 - 2Q_1 + 2Q_2)}{1+t^2}$$

$$= \frac{(1,-1) + t\begin{bmatrix} -2+4 \\ 2 \end{bmatrix} + t^3\begin{bmatrix} 1-4+2 \\ -1+2 \end{bmatrix}}{1+t^2} = \frac{(1+2t-t^3, -1+2t+t^3)}{1+t^2} = r_2(t)$$

5.15 $x^2-y^2=1 \quad x=\frac{1}{\cos\varphi} \quad y=\operatorname{tg}\varphi \quad \varphi\in[0,2\pi]$

$t=\operatorname{tg}\frac{\varphi}{2}$

$[0,\frac{\pi}{3})\ \cos\varphi = \frac{1-t^2}{1+t^2} \Rightarrow x=\frac{1+t^2}{1-t^2}$

$\sin\varphi=\frac{2t}{1+t^2} \Rightarrow y=\frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}}=\frac{2t}{1-t^2}$

$\left.\right\} \left(\frac{1+t^2}{1-t^2}, \frac{2t}{1-t^2}\right) \Rightarrow t\in[0,\frac{1}{\sqrt3}] \quad a=\sqrt3 t$

$=\left(\frac{1+\frac{a^2}{3}}{1-\frac{a^2}{3}}, \frac{2\sqrt3\ a}{1-\frac{a^2}{3}}\right)=\left(\frac{3+a^2}{3-a^2}, \frac{a2\sqrt3}{3-a^2}\right) \quad a\in[0,1]$

$3-a^2= w_0+a(-2w_0+2w_1)+a^2(w_0-2w_1+w_2)$

$3=w_0$

$-1=3-2w_1+w_2$

$0=-6+2w_1 \Rightarrow w_1=3 \Rightarrow w_2=2 \qquad w_0=3=w_1, \quad w_2=2$

kontrolne tačke: $a=0 \quad P_0=(1,0) \quad a=1 \quad P_2=(2,\sqrt3)$

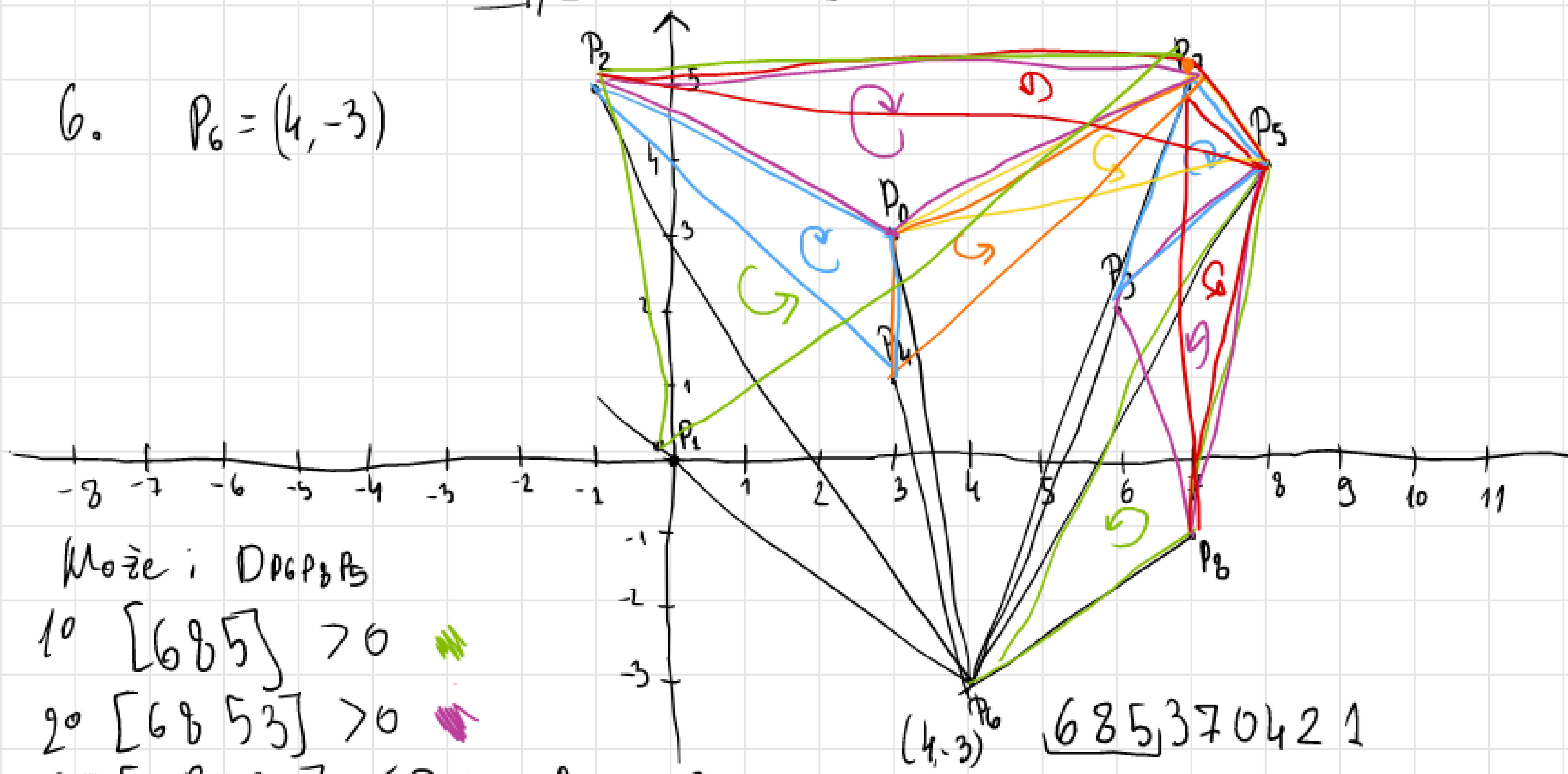
$H: x^2-y^2=1$

$t: xx_0-yy_0=1 \qquad t_{P_0}= x=1 \qquad t_{P_1}=2x-\sqrt3 y=1 \quad \cap$ je $(1,\frac{\sqrt3}{3})$

$$r_2(t)=\frac{3(1-t)^2\begin{bmatrix}1\\0\end{bmatrix}+2\cdot3\cdot(1-t)t\begin{bmatrix}1\\ \sqrt3/3\end{bmatrix}+2\cdot t^2\begin{bmatrix}2\\ \sqrt3\end{bmatrix}}{3-t^2}=$$

$$=\frac{\begin{bmatrix}3\\0\end{bmatrix}+t\begin{bmatrix}-6+6\\+2\sqrt3\end{bmatrix}+t^2\begin{bmatrix}3-6+4\\-2\sqrt3+2\sqrt3\end{bmatrix}}{-\|-}=\left(\frac{3+t^2}{3-t^2}, \frac{2\sqrt3 t}{3-t^2}\right)$$

6. $P_6=(4,-3)$



$685,370421$

Može i $D_{P_6P_8P_5}$

1° $[685] > 0$

2° $[6853] > 0$

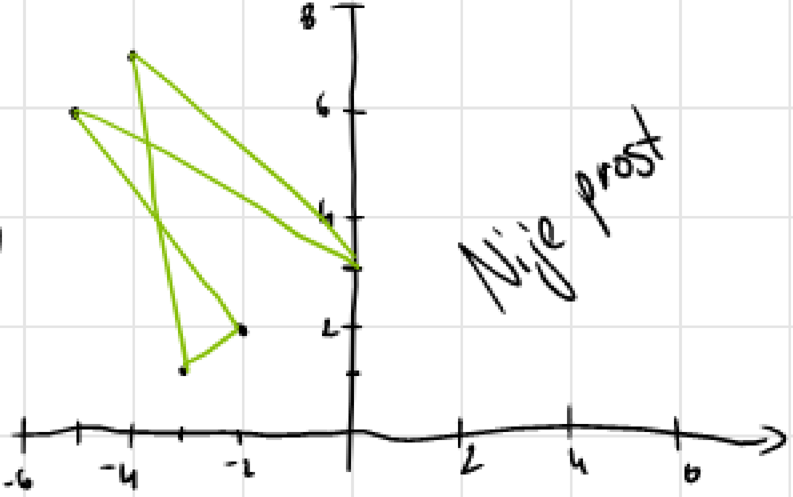
3° $[68537] < 0$ Brišem 3

4° $[6857] > 0 \rightarrow$ 5° $[68570]$ 6° $[685704]$ 7° $[68570\cancel{4}2]$

8° $[6857\cancel{0}2]$ 9° $[68572]$ 10° $[685721]$ 11° $[6857216]$ W

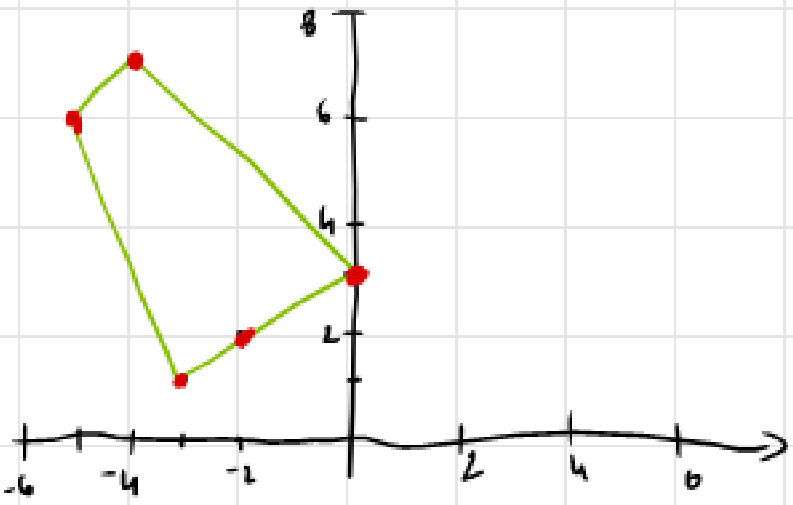
Nacrta se na kraju

6.2 $P_0=(-3,1)$
$P_1=(-2,2)$
$P_2=(-5,6)$
$P_3=(0,3)$
$P_4=(-4,7)$

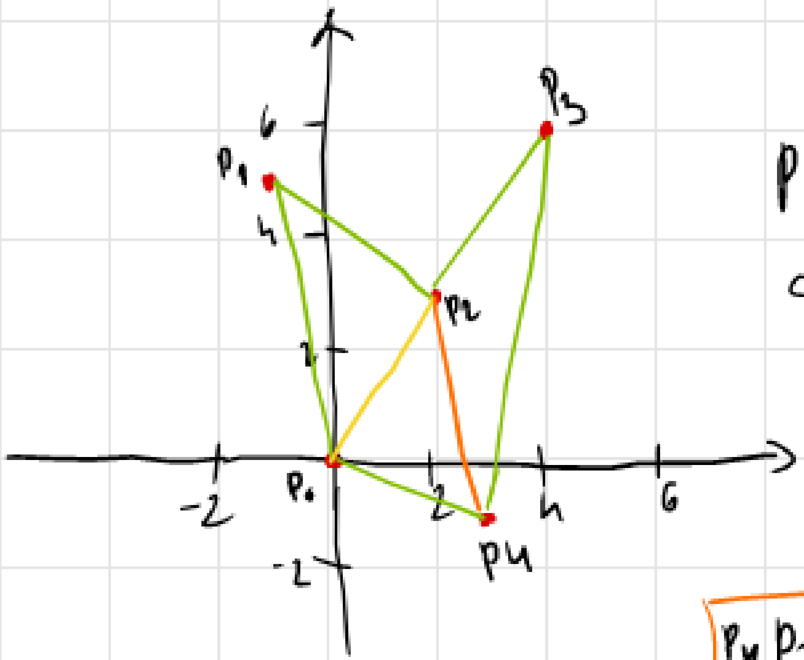


Nije prost

0 3 1 4 2



6.3 a) $P_0=(0,0)$
$P_1=(-1,5)$
$P_2=(2,3)$
$P_3=(4,6)$
$P_4=(3,-1)$



P_1 ima najmanju x koordinatu
Susedna su P_0 i P_2

$\boxed{P_0P_2}$

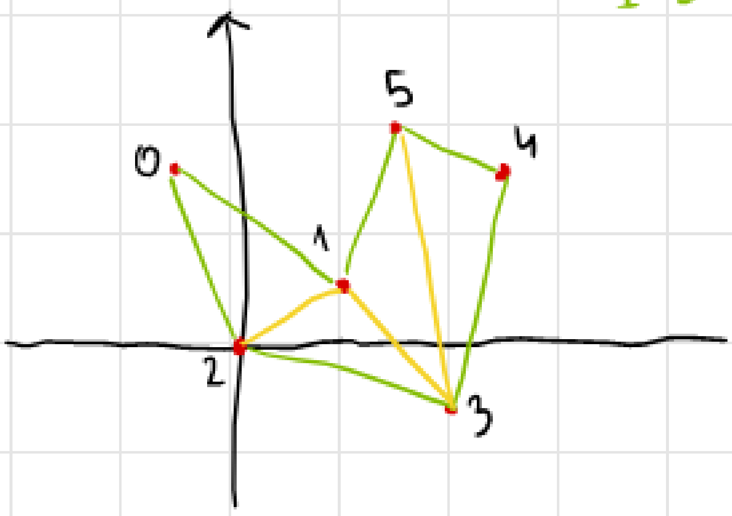
$P_0P_4P_3P_2 \Rightarrow P_0$ ima najmanju x koord

$\boxed{P_4P_2}$

6.4 $P_0=(-1,3)$
$P_1=(2,1)$
$P_2=(0,0)$
$P_3=(4,-1)$
$P_4=(5,3)$
$P_5=(3,4)$

$P_0P_1P_5P_4P_3P_2$

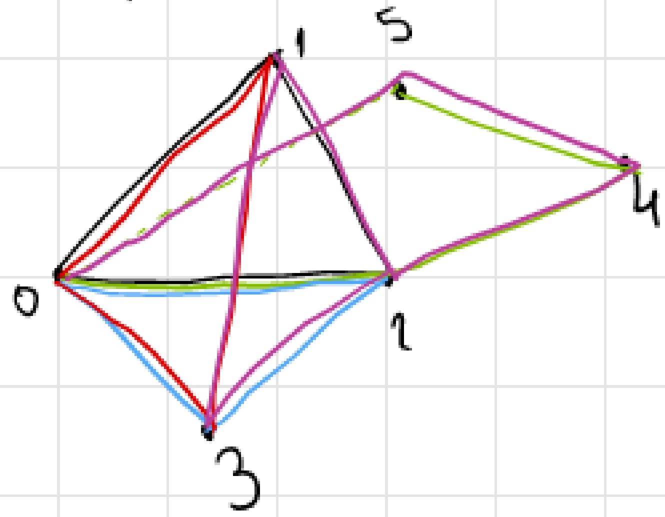
Biramo P_0 i P_1P_2
Zatim P_2 i P_1P_3
Pa P_1 P_3P_5



7.1 i) $T=\{T_0,T_1,\dots T_5\}$ $P=\{p_0,p_1,p_2,p_3\}$
$p_0=\langle 0,1,2\rangle$ $p_1=\langle 0,2,4,5\rangle$ $p_2=\langle 3,2,0\rangle$
$p_3=\langle 1,0,3\rangle$

p_0: 01 12 20 p_3= 10, 03, 31
p_1: 02 24 45 50
p_2: 32 20 03
$I=\{01,12,20,24,45,50,32,03,31\}$

b) Tamo nešto



c) Ne 20 pripada 3-ma pljosnama
e) $Rub=\{12,24,45,50,32,31\}$ Komponente ruba: 1 2 3 1

ii) $T = \{T_0, \dots, T_{10}\}$ $P = \{p_0, p_1, \dots, p_4\}$

$\boxed{p_0} = \underline{01}, \boxed{17}, \underline{74}, 40$

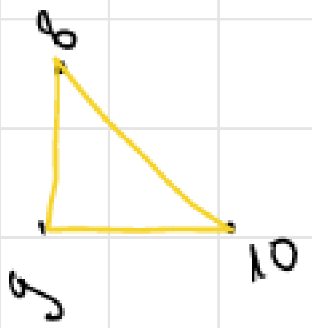
$\boxed{p_1} = \underline{12}, 26, \underline{67}, \boxed{71}$

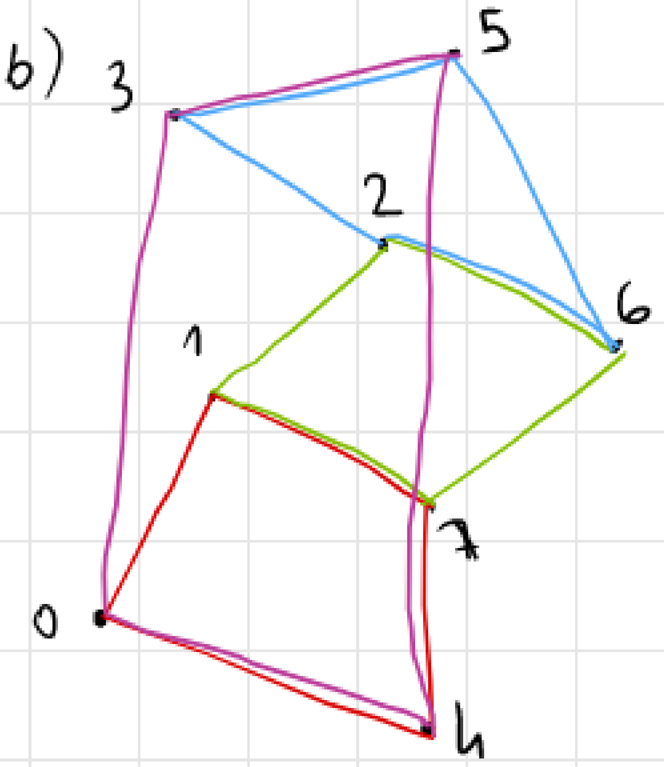
$\boxed{p_2} = \underline{23}, 35, \underline{56}, 62$

$\boxed{p_3} = \underline{30}, 04, \underline{45}, 53$

$\boxed{p_4} = 89, 910, 108$

b)





a) $I = \{01, 17, 74, 40, 12, 26, 67, 23, 35, 56, 30, 45, 89, 910, 108\}$

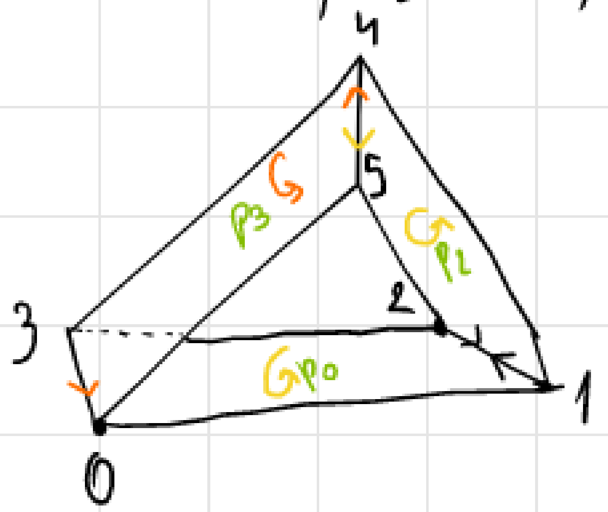
c) Jeste

d) Nije povezana jer p_4 nije povezana sa ostalim pljosnima

e) $R = \{01, 74, 12, 67, 23, 56, 30, 45, 89, 910, 108\}$

Komponente: 89108, 45674, 01230

7.2 a)



$T = \{T_0, T_1, T_2, T_3, T_4, T_5\}$ $P = \{p_0, p_1, p_2\}$

$p_0 = \langle 0,1,2,3 \rangle$ $p_1 = \langle 1,2,5,4 \rangle$ $p_2 = \langle 0,3,4,5 \rangle$

b) Fiksirajmo orijentaciju pljosni p_0 0123

Uskladimo orj. pljosni p_1 sa p_0 preko ivice $\boxed{12}$ pa će p_1 imati orj. $\langle 2145 \rangle$ Zatim uskladimo orj p_2 sa pljosni p_1 preko ivice $\boxed{45}$ pa će p_2 biti $\langle 5430 \rangle$

Proverimo orijentacije p_0 i p_2 nakon usklađivanja i vidimo

p_0: 01 12 23 $\boxed{30}$

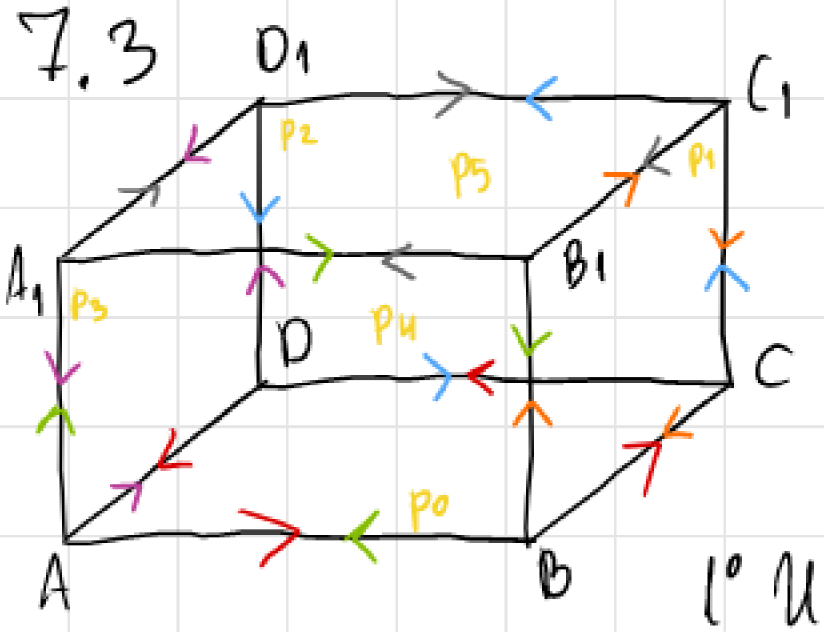
p_1: 21 14 45 52

p_2: 54 43 $\boxed{30}$ 05

Imamo unutrašnju 30 i njih nije moguće uskladiti (tj ako ih uskladimo neka druga će biti problem)

$\Rightarrow$ Mebijusova traka je neorijentabilna.

7.3



Prema $\boxed{p_0} = \langle ABCD \rangle$

$p_1 = \langle BB_1C_1C \rangle$ $p_2 = \langle CDD_1C_1 \rangle$

$p_3 = \langle D_1A_1AD \rangle$ $p_4 = \langle ABB_1A_1 \rangle$

$p_5 = \langle A_1B_1C_1D_1 \rangle$

1° Uskladimo p_0 i p_1 Zajednička ivica je BC i $\boxed{p_1} = \langle CBB_1C_1 \rangle$

2° p_0 i p_2 Zaj. ivica je CD pa će $\boxed{p_2} = \langle DCC_1D_1 \rangle$

3° p_0 i p_3 Zaj. ivica je DA pa će $\boxed{p_3} = \langle ADD_1A_1 \rangle$

4° p_0 i p_4 Zaj AB $\boxed{p_4} = \langle BAA_1B_1 \rangle$

5° Odredićemo orj p_5 na osnovu p_1, p_2, p_3 i p_4 jer nema zaj ivica sa p_5:

p_1:	CB	BB_1	B_1C_1	C_1C
p_2:	DC	CC_1	C_1D_1	D_1D
p_3:	AD	DD_1	D_1A_1	A_1A
p_4:	BA	AA_1	A_1B_1	B_1B

Svi su usklađeni još p_5

$p_5 = \langle A_1B_1C_1D_1 \rangle$

Sa p_4 A_1B_1 $\Rightarrow$ $p_5 = \langle D_1C_1B_1A_1 \rangle$

Sa p_3 D_1A_1 dobro je

Sa p_2 C_1D_1 dobro je

Sa p_1 B_1C_1 dobro je

7.4 $p_0 = \langle 0,1,4,3 \rangle$ $p_1 = \langle 1,2,5,4 \rangle$ $p_2 = \langle 2,0,3,5 \rangle$ $p_3 = \langle 6,8,5,3 \rangle$

$p_4 = \langle 6,3,4,7 \rangle$ $p_5 = \langle 4,5,8,7 \rangle$ $p_6 = \langle 8,7,1,2 \rangle$ $p_7 = \langle 0,1,7,6 \rangle$ $p_8 = \langle 0,6,8,2 \rangle$

a)

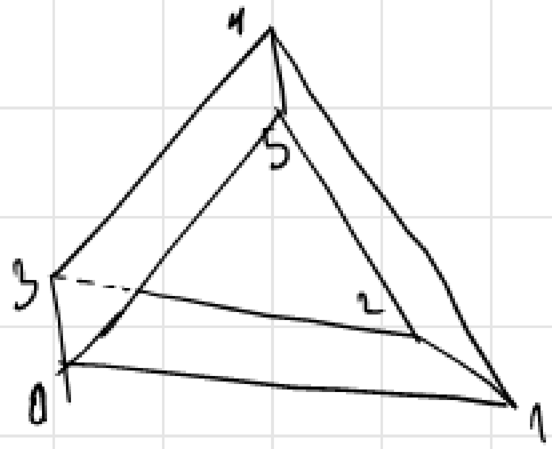
p_0:	01	14	43	30
p_1:	12	25	54	41
p_2:	20	03	35	52
p_3:	68	85	53	36
p_4:	63	34	47	76
p_5:	45	58	87	74
p_6:	87	71	12	28
p_7:	01	17	76	60
p_8:	06	68	82	20

Svaka ivica se pojavljuje 2 puta pa nema rubova.

b) $I = \frac{4 \cdot 9}{2} = 18$ $P = 9$ $T = 9$ $\chi(M) = T - I + P = 9 - 18 + 9 = 0$

$\chi(M) = 2 - 2 \cdot r \Rightarrow r = 1$

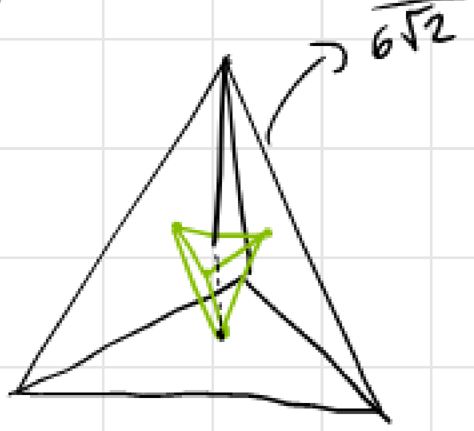
7.5

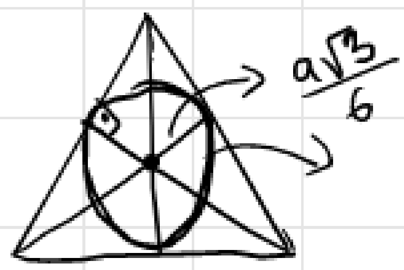


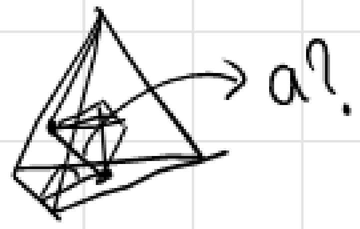
$T = 6 \quad p = 3 \quad I = 9$

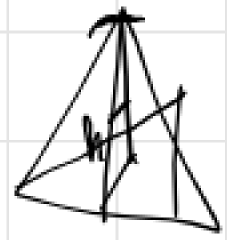
$\chi(M) = 6 + 3 - 9 = 0$

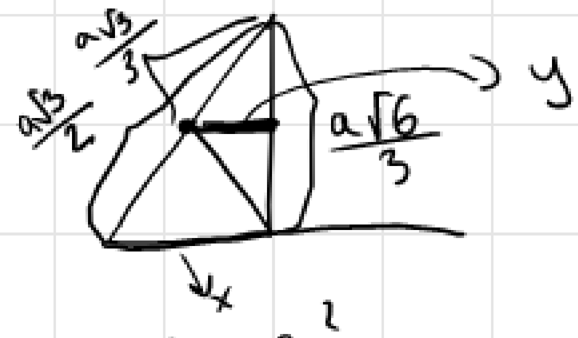
7.6











$\frac{3a^2}{4} - \frac{6a^2}{9} = x$

$\frac{27-24a^2}{36} \Rightarrow x = \frac{a\sqrt{3}}{6}$

$\frac{a\sqrt{3}}{3} : y = \frac{a\sqrt{3}}{2} : \frac{a\sqrt{3}}{6}$

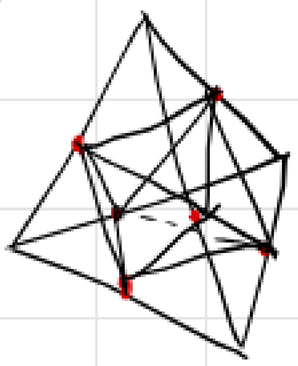
$\frac{a\sqrt{3}}{3} \cdot \frac{a\cancel{\sqrt{3}}}{6 \cdot 3} = \frac{a\cancel{\sqrt{3}}}{\cancel{2}} \cdot y$

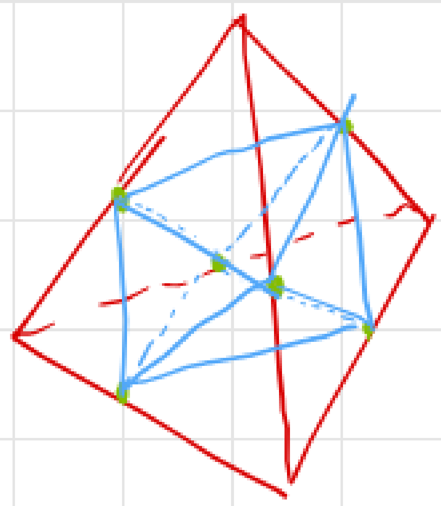
$\frac{a\sqrt{3}}{9} = y$

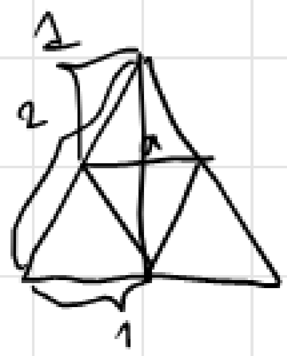
$\frac{a\sqrt{3}}{9} = r_m \Rightarrow \frac{a\sqrt{3}}{6}$ je h a $\frac{a}{3}$ stranica

$\frac{\frac{a^3}{6\sqrt{2}}}{\frac{a^3}{27 \cdot 6\sqrt{2}}} = 27 : 1$ je odnos

7. 8

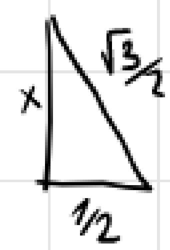






$1 : \frac{a}{2} = 2 : 1$

$\boxed{a = 1}$



$\frac{3}{4} - \frac{1}{4} = x^2$

$\frac{\sqrt{2}}{2} = x$

$V = \left(1 \cdot 1 \cdot \frac{\sqrt{2}}{2}\right) \cdot \frac{1}{3} \cdot 2$

$\boxed{V = \frac{\sqrt{2}}{3}}$