A1 2020 jun 1

1. a) $x_1 \in (0, \pi)$ $\quad x_{n+1} = x_n - \sin x_n \quad n \geq 1$

$L = L - \sin L \quad \sin L = 0 \quad \boxed{L = 0} \quad x_{n+1} - x_n = -\sin x_n$

$0 < x_1 < \pi$

$0 < x_n < \pi \overset{?}{\Longrightarrow} 0 < x_{n+1} < \pi$

$\underset{>0}{x_n} - \sin \underset{\in(0,\pi)}{x_n} > 0$

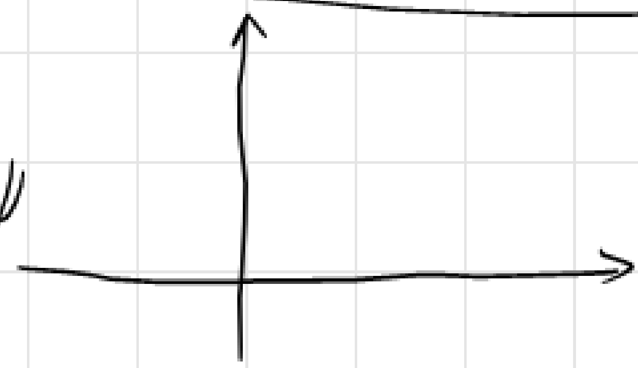
Definišimo f-ju $F(x) : (0, \pi) \to \mathbb{R}$

$F(x) = x - \sin x$

$F'(x) = \underset{\cos x \in [-1,1]}{1 - \cos x} \geq 0$ bi (F(x)) raste

$F(0) = 0 - 0 = 0 \Rightarrow x - \sin x > 0$

$\underbrace{\underset{<\pi}{x_n} - \underset{\in(0,1]}{\sin x_n}}_{\in(\pi, \pi-1]} < \pi$ ✓



$-\sin x_n \in (-1, 0) \Rightarrow$ Niz opada

b) $x_{n+1} = \frac{x_n^3}{6} + o(x_n^3)$ kad $n \to +\infty$

$x_n - \left(x_n - \frac{x_n^3}{6} + o(x_n)^3\right) = \frac{x_n^3}{6} + o(x_n^3)$ ✓

c) $\lim\limits_{n\to+\infty} \frac{x_n^3}{x_{n+1}} = \lim\limits_{n\to\infty} \frac{x_n^3}{\frac{x_n^3}{6} + o(x_n^3)} = 6$

2. $f(x) = \begin{cases} x^3 \cdot \cos\left(\frac{1}{x^2}\right), & x \neq 0 \\ b, & x = 0 \end{cases}$

$-1 \leq \cos \frac{1}{x^2} \leq 1 \quad / \cdot x^3$

$-x^3 \leq x^3 \cos \frac{1}{x^2} \leq x^3 \quad / \lim\limits_{x\to 0}$

$\underbrace{0 \quad\quad 0 \quad\quad 0}$

$\boxed{0 = b}$

$\lim\limits_{h\to 0^-} \frac{h^3 \cdot \cos\left(\frac{1}{h^2}\right) - \cancel{b}^{\,=0}}{h} = \lim\limits_{h\to 0^-} h^2 \cdot \cos\left(\frac{1}{h^2}\right) = 0$ ✓

$f'(x) = \begin{cases} 3x^2 \cdot \cos\left(\frac{1}{x^2}\right) + 2 \cdot \sin\left(\frac{1}{x^2}\right) \\ 0, & x = 0 \end{cases}$

Ne postoji $\lim\limits_{x\to 0} f(x)$

3. $f(x) = \frac{x^2}{x-1} \cdot e^{\frac{1}{x}}$ I $D_f: \mathbb{R} \setminus \{1, 0\}$

II $x > 1$ $f(x) > 0$ Nema nula, ni presek sa y-osom.

III Nije parna ni neparna. Nije periodična.

IV $\lim_{x \to 1^-_+} \frac{x^2}{x-1} \cdot e^{\frac{1}{x}} = \frac{1}{e} \cdot (-\infty) = \underset{+}{-}\infty$

$\lim_{x \to 0^+} \frac{x^2}{x-1} \cdot e^{\frac{1}{x} \to +\infty} = -\infty$ $\lim_{x \to 0^-} \frac{0}{-1} \cdot e^{\frac{1}{x} \to -\infty} \to 0 = 0$

K.A.

$a = \lim_{x \to \pm\infty} \frac{x^{\cancel{2}}}{\cancel{x}(x-1)} \cdot e^{\frac{1}{x}} = \lim_{x \to \pm\infty} \left(1 + \frac{1}{(x-1)}\right)^{\frac{x-1}{x-1}} \cdot e^{\frac{1}{x}} = e^{\frac{1}{x-1} + \frac{1}{x}} = x$

$\lim x \left(\frac{x}{x-1} \cdot e^{\frac{1}{x}} - 1\right) = \lim_{x \to \pm\infty} \frac{e^{\frac{2x-1}{x(x-1)}} - 1}{\frac{1}{x} \cdot \frac{2x-1}{2x-1} \cdot \frac{x-1}{x-1}} \to 1 = \frac{2x-1}{x-1} = 2$

$\boxed{x+2}$

V $e^{\frac{1}{x}} \cdot (-1) \cdot \frac{1}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{x-1} + \frac{2x^2 - 2x - x^2}{(x-1)^2} \cdot e^{\frac{1}{x}}$

$e^{\frac{1}{x}}\left(-\frac{1}{x-1} + \frac{x^2 - 2x}{(x-1)^2}\right) = e^{\frac{1}{x}} \cdot \left(\frac{1 - x + x^2 - 2x}{(x-1)^2}\right) \to x^2 - 3x + \frac{9}{4} - \frac{9}{4} + \frac{4}{4}$

$\left(x - \frac{3}{2}\right)^2 - \frac{\sqrt{5}}{2}$

	$-\infty$		$\frac{3}{2} + \frac{\sqrt{2}}{2}$	$+\infty$
$\left(x - \left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)\right)$	$-$	$-$	$+$	
$x - \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)$	$-$	$+$	$+$	
	$\nearrow$	$\searrow$ $x-3$	$\nearrow$ ≈ 6	

$2\frac{(2+3\sqrt{5})}{1+\sqrt{5}}$

$\frac{\frac{9 + 6\sqrt{5} + 5}{4}}{\frac{1+\sqrt{5}}{2}} \cdot e^{\frac{3+\sqrt{5}}{4}}$ $x_{2.4}$

VI $\frac{e^{1/x} \cdot \frac{1}{x^2} \cdot (-1) \cdot \left(x - \frac{3+\sqrt{5}}{2}\right)\left(3 - \frac{3-\sqrt{5}}{2}\right)}{(x^2-1)^2} + e^{\frac{1}{x}} \frac{(x-1)^2 \cdot (2x-3) - 2(x-1)(x^2 - 3x - 1)}{(x^2-1)^4}$

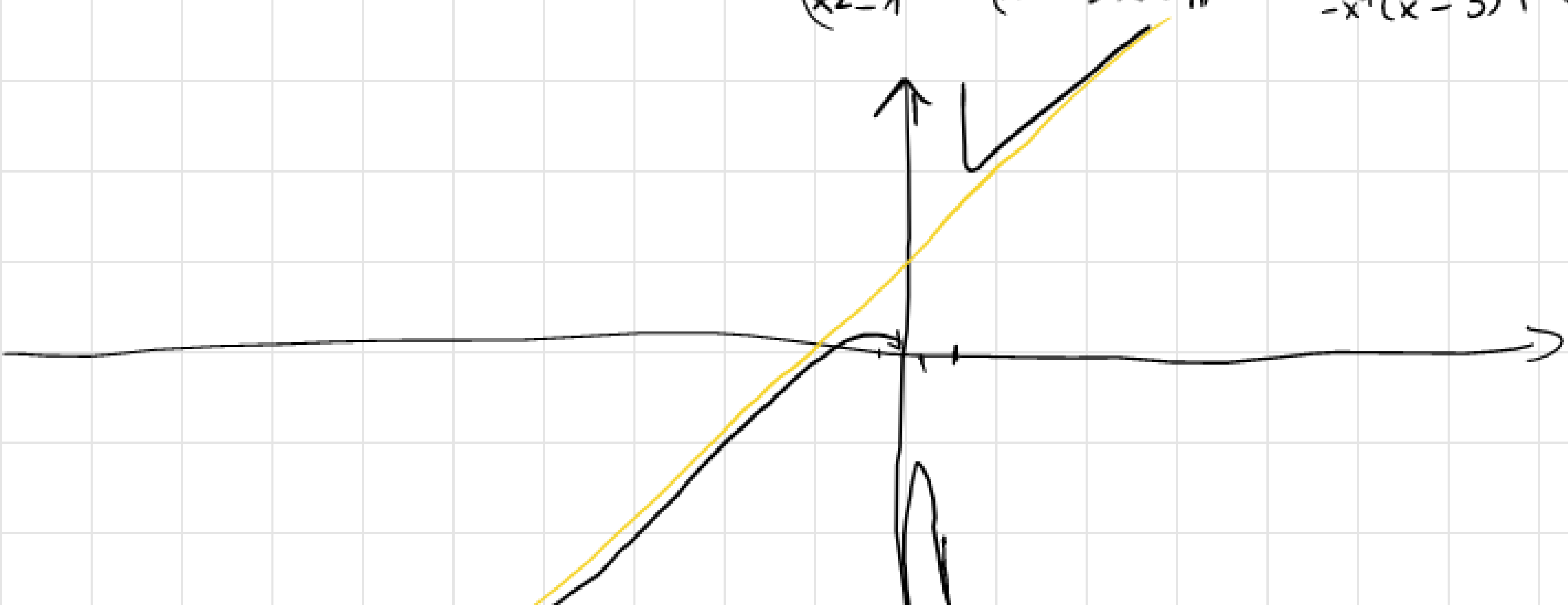
$\frac{e^{\frac{1}{x}}}{(x^2-1)^2} \left(\frac{2\cancel{x^2} - 5x + 3 - 2\cancel{x^2} + 6x + 2}{(x^2-1)} - x^2 - 3x + 1\right)$

$\frac{x+1}{x^2-1} - (x^3 - 3x + 1)$

$-x^5 + 3x^4 + x^3 - x^2 - 3x + 1 + x + 1$

$-x^5 + 3x^4 + x^3 - x^2 - 2x + 2$

$-x^4(x-3) + (x^2-2)(x-1)$



4. $16 \cdot 4 = 64$

a) $x^3 - 15x + 1 = 0 \quad (-4, 4)$

$3x^2 - 15 = 3(x^2-5) = 3(x-\sqrt{5})(x+\sqrt{5})$

	$-\infty$	$-\sqrt{5}$	$\sqrt{5}$	$+\infty$
$x-\sqrt{5}$	$-$	$-$	$+$	
$x+\sqrt{5}$	$-$	$+$	$+$	
	$\nearrow$	$\searrow$	$\nearrow$	

$f(-4): -64 + 60 + 1 = -3 < 0$

I rešenje

$f(-\sqrt{5}): -5\sqrt{5} + 15\sqrt{5} + 1 = 10\sqrt{5} + 1 > 0$

II rešenje

$f(\sqrt{5}): 5\sqrt{5} - 15\sqrt{5} + 1 = 1 - 10\sqrt{5} < 0$

III rešenje

$f(4): 64 - 60 + 1 = 5 > 0$

b) $-4 + \lambda$

$10\sqrt{5} + \lambda$

$\lambda - 10\sqrt{5}$

$4 + \lambda$

$\lambda < -10\sqrt{5}$ Nijedno

$\lambda = -10\sqrt{5}$ Jedno

$\lambda = -4$ Tri

$\lambda = 4$ Tri

$\lambda = 10\sqrt{5}$ Jedno

$\lambda > 10\sqrt{5}$ Nijedno

2020 jun 2

$$\left(\frac{n+1}{n-2}\right)^{(-1)^n \cdot n} + \sin\frac{(n+1)\pi}{2}$$

$$\lim_{n\to+\infty} \left(\left(1+\frac{3}{n-2}\right)^{\frac{n-2}{3}}\right)^{\frac{(-1)^n \cdot n}{n-2} \cdot \frac{3}{}} = e^{\frac{(-1)^n \cdot \cancel{n}}{\cancel{n}(1-\cancel{2/n})}} = e^{(-1)^n \cdot 3}$$

e

$\sin\frac{(n+1)\pi}{2}$

0: 1	2: -1
1: 0	3: 0

$\left\{ e^3 \pm 1,\ \frac{1}{e^3} \right\}$

2.

$$f(0) = \begin{cases} a^{\frac{\sqrt{\sin(x^2) - x^3}}{x}}, & x<0 \\ b, & x=0 \\ x^{x^2}, & x>0 \end{cases}$$

a) $\lim_{x\to 0^-} a^{\frac{\sqrt{\sin(x^2)-x^3}}{x}} = \lim_{x\to 0^-} a \wedge \frac{1}{x} \cdot \sqrt{\frac{\sin(\cancel{x^2})}{\cancel{x^2}}^{1} \cdot x^2 - x^3} = a^{\frac{-x\sqrt{1-x}}{x}} = 1/a$

b) $\lim_{x\to 0^+} x^{x^2} = \lim_{x\to 0^+} e^{x^2 \cdot \ln x} = 1$

$f(0) = b$

$1/a = 1 = b$

$a = 1 = b$

3. $f(x) = (x+1)\, e^{\arctan(1/x)}$ I D_f: $\mathbb{R}\setminus\{0\}$

a) $\arctan(t)$

$g'(x) = \frac{1}{1+t^2}$

$g''(x) = -\frac{2t}{(1+t^2)^2}$ $g^{(3)}(x) = \frac{-2(1+t^2) + 2t\cdot 2\cdot 2t\,\cancel{(1+t^2)}}{(1+t^2)^3}$

$M_3^f = 0 + 1\cdot x + \frac{0\cdot x^2}{2} + \frac{-\cancel{2}x^3}{\cancel{6}\,3} = x - \frac{x^3}{3} + o(x^3)$ $[x = t]$

b) I D_f: $\mathbb{R}\setminus\{0\}$ II $x > -1$ $f > 0$ $x = -1$ nula, nema preseka sa y-osom

III Nije parna ni periodična ni neparna

IV $\lim_{x\to 0-} (x+1)\, e^{\arctan(1/x) \to -\infty \to -\pi/2} = e^{-\pi/2}$

$= e^{\pi/2}$

K.A. $\lim_{x\to\pm\infty} \frac{(x+1)\, e^{\arctan\left(\frac{1}{x}\right)}}{x} = 1$

$\lim_{x\to+\infty} (x+1)\, e^{\arctan\left(\frac{1}{x}\right)} - x = e^{\arctan\frac{1}{x}} + x\,\frac{\left(e^{\arctan\frac{1}{x}} - 1\right)}{\arctan\frac{1}{x}} \cdot \arctan\frac{1}{x}$ $\to 1$

$= 1 + \lim_{x\to+\infty} \frac{\arctan\frac{1}{x}}{\frac{1}{x}} = 1 + \frac{\frac{\cancel{x^2}}{1+x^2}\cdot\left(-\frac{1}{\cancel{x^2}}\right)}{-\frac{1}{\cancel{x^2}}} = 2$ $\boxed{y = x+2}$

V $e^{\arctan\left(\frac{1}{x}\right)} + (x+1)\cdot e^{\arctan\left(\frac{1}{x}\right)} \cdot \frac{\cancel{x^2}}{1+x^2}\cdot\left(-\frac{1}{\cancel{x^2}}\right) =$

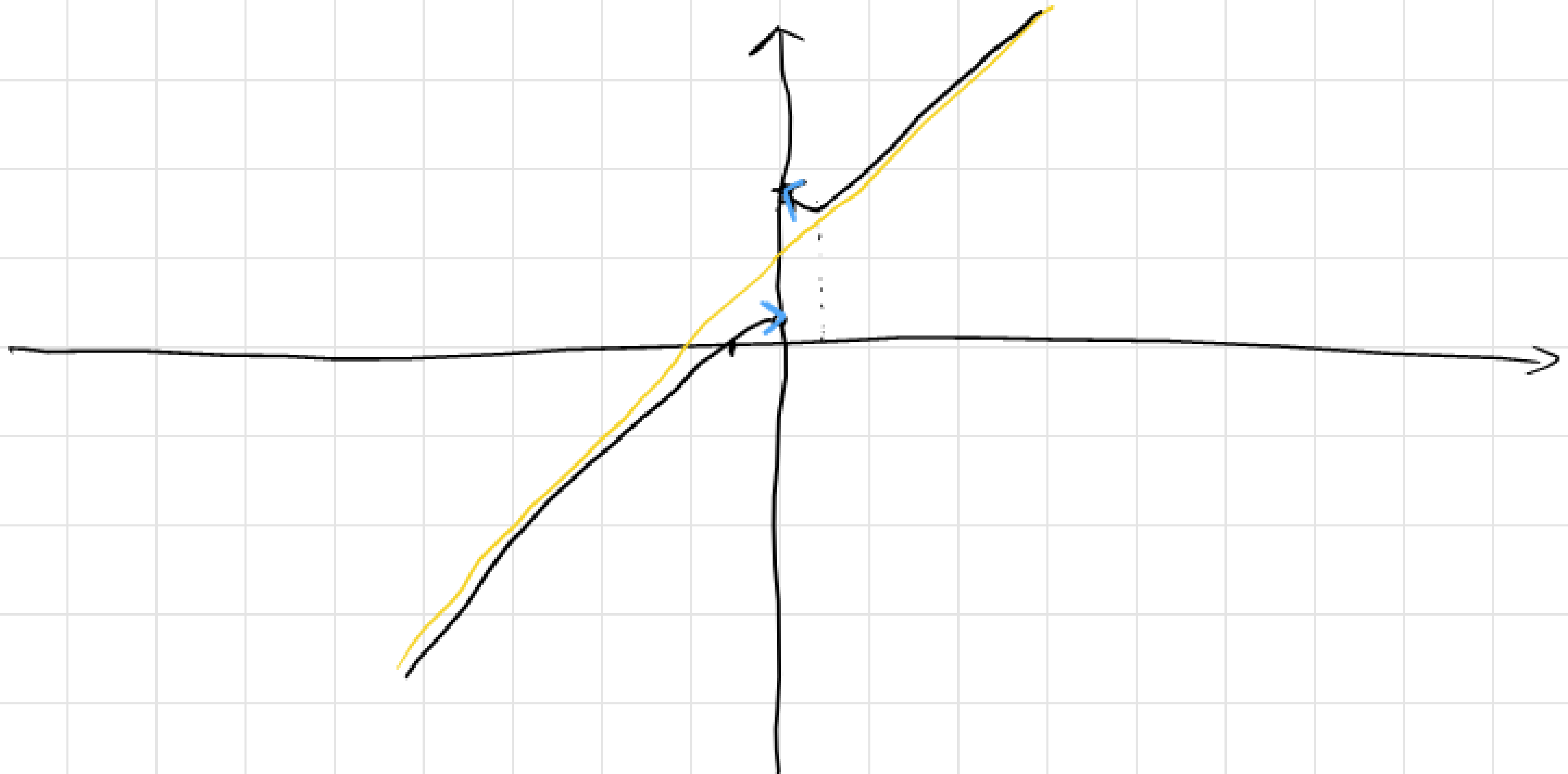
$\underbrace{e^{\arctan\frac{1}{x}}}_{>0}\left(1 - \frac{x+1}{1+x^2}\right) = \left(\frac{x(x-1)}{\underbrace{(1+x^2)}_{>0}}\right)$

	0		1	
x	−	+		+
$x-1$	−	−		+
	+	−		+
	↗	↘		↗

VI $e^{\arctan\frac{1}{x}}\left(\frac{-1}{1+x^2} + \frac{(2x-1)(1+x^2) - 2x(x^2-x)}{(1+x^2)^2}\right)$ $2\cdot e^{\pi/4}$

$-1 - \cancel{x^2} + \cancel{2x^3} - \cancel{x^2} + 2x - 1 - \cancel{2x^3} + \cancel{2x^2} = \boxed{2(x-1)}$

$x = 1$
Prevojna tačka



c) $(1, f(1))$ $\quad (1, 2e^{\pi/4})$

$y - y_0 = k \cdot (x - x_0)$

$k = f'(1) = e^{\arctan(\frac{1}{x})} + (x+1) \cdot e^{\arctan(\frac{1}{x})} \cdot \frac{\cancel{x^2}}{1+x^2} \cdot (-1) \cdot \frac{1}{\cancel{x^2}}$

$\pi/4 + \cancel{2} \cdot e^{\pi/4} \cdot (\frac{-1}{\cancel{2}}) = \pi/4 - e^{\pi/4}$

$y - 2e^{\pi/4} = (\pi/4 - e^{\pi/4})(x-1)$

$y = (\pi/4 - e^{\pi/4}) \cdot x - \pi/4 + e^{\pi/4}$

4. $x \in (0, +\infty)$ $\exists \gamma$ takvo da:

$\ln(x+1) - \ln x = \frac{1}{x+\gamma}$

a) Definisimo $F: \ln(x)$, neprekidna je i dif na D_f.

$f(b) - f(a) = f'(c)(b-a) \qquad b = x+1, \quad a = x$

$\ln(x+1) - \ln(x) = f'(c) \cdot (x+1-x) = f'(c)$

$\ln(x+1) - \ln(x) = \frac{1}{c} \qquad c \in (x, x+1)$

$\Rightarrow \gamma \in (0, 1)$

b) $g(x) = \ln(1+x) - \ln(x) - \frac{1}{x+1/2}$

$g'(x) = -\frac{1}{x(x+1)} + \frac{1}{(x+1/2)^2} = \frac{\cancel{x^2} + \cancel{x} - \cancel{x^2} - \cancel{x} - 1/4}{x(x+1)(x+1/2)^2} \Rightarrow$ Funkcija opada

$\lim_{x \to +\infty} \ln(1+x) - \ln(x) - \frac{1}{x+1/2} = \frac{\ln(\frac{1}{x} + 1)^{\nearrow 1} \cdot 1/x}{\frac{1}{x}} - \frac{1}{x+1/2} = 0$

$\Rightarrow$ g opada i ide u nulu $\Rightarrow$ g je poz.

c) $\ln(1+x) - \ln(x) = \frac{1}{x+\gamma} \qquad \gamma \in (0,1)$

$\ln(1+x) - \ln(x) - \frac{1}{x+\gamma} = 0$

$\ln(1+x) - \ln(x) - \frac{1}{x+1/2} > 0$

$\frac{1}{x+\gamma} - \frac{1}{x+1/2} > 0$

$\frac{1}{x+\gamma} > \frac{1}{x+1/2}$

$x + 1/2 > x + \gamma$

$\boxed{\gamma < 1/2}$ ✓

2020 sept 1

1. a) $x_0 \in (0,1)$

$$x_{n+1} = 1 - 2x_n + 3x_n^2 - x_n^3, \quad n \geq 0$$

$$x_0 \in (0,1), \quad x_n \in (0,1) \overset{?}{\Longrightarrow} x_{n+1} \in (0,1)$$

$$1 - 2x_n + 3x_n^2 - x_n^3 > 0$$

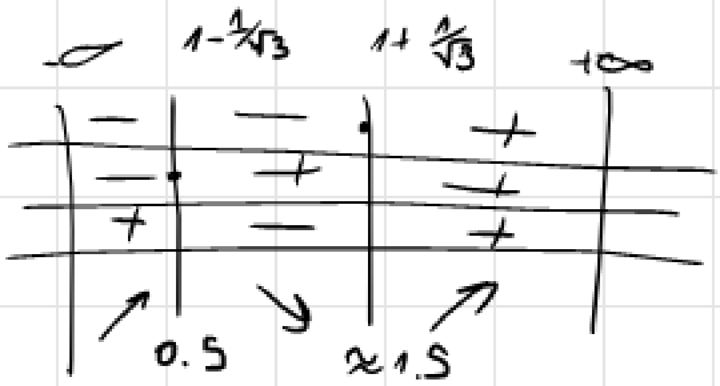
$$1 > x_n^3 - 3x_n^2 + 2x_n$$

$$\frac{3-\sqrt{3}}{3} = \frac{\sqrt{3}(\sqrt{3}-1)}{\sqrt{3} \approx 3} \approx 0.8$$

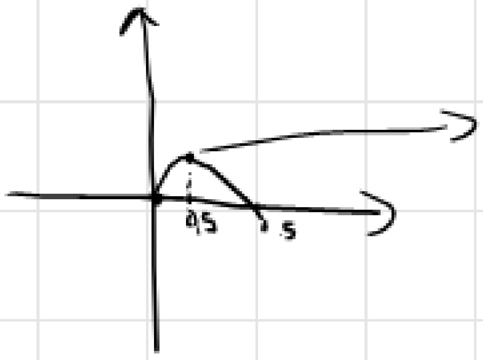
$$F(x) = x^3 - 3x^2 + 2x$$

$$F'(x) = 3x^2 - 6x + 2 = 3\left(x^2 - 2x + 1 - 1 + \tfrac{2}{3}\right)$$

$$3\left((x-1)^2 - \tfrac{1}{3}\right) = 3\left(x - \left(1 + \tfrac{1}{\sqrt{3}}\right)\right)\left(x - \left(1 - \tfrac{1}{\sqrt{3}}\right)\right)$$



$$2\sqrt{6}(2-\sqrt{3})$$



$$\frac{3-\sqrt{3}}{3}\left(\frac{9-6\sqrt{3}+3}{9 \cdot 3} - 3 + \sqrt{3} + 2\right)$$

$$\frac{3-\sqrt{3}}{3}\left(\frac{4-\sqrt{3}-3+3\sqrt{3}}{3}\right) = \frac{(3-\sqrt{3})(1+2\sqrt{3})}{9}$$

$$= \frac{-3+5\sqrt{3}}{9} \approx 0.35$$

$\Rightarrow$ Na intervalu $(0,1)$ x dostiže Max ≈ 0.35

$\Rightarrow 1 > x_n^3 - 3x_n^2 + 2x_n$ ✓

$$\cancel{1} - x_n^3 + 3x_n^2 - 2x_n < \cancel{1}$$

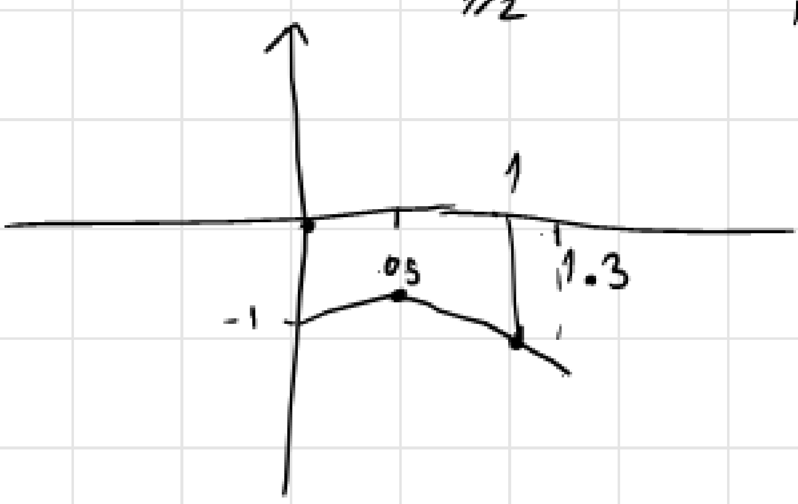
$x_n^3 - 3x_n^2 + 2x_n > 0$ Koristeći prethodno primetimo za $x_n \in (0,1)$ f-ja je veća od nule

b) $1 - x + 3x^2 - x^3 \geq x$

$x^3 - 3x^2 + 2x - 1 \leq 0$ za $x \in [0,1]$

$$\frac{6+2\sqrt{3}}{6} = \frac{3+\sqrt{3}}{3}$$

$f'(x) = 3x^2 - 6x + 2 \qquad x_{1/2} = \frac{6 \pm \sqrt{36-24}}{6} \quad \frac{3-\sqrt{3}}{3}$



$\frac{3-\sqrt{3}}{3} \approx 0.5 \qquad \frac{3+\sqrt{3}}{3} \approx 1.3$

$F(0.5) \approx -0.7$

$F(1) = -1$

$\Rightarrow$ Važi F-ja je negativna za sve $x \in [0,1]$

c) $L = 1 - 2L + 3L^2 - L^3$

$L^3 - 3L^2 + 3L - 1 = 0$

$(L^3 - 1) - 3L(L-1) = 0$

$(L-1)(L^2 + L + 1 - 3L) = (L-1)(L-1)^2 = (L-1)^3 = 0 \qquad \boxed{L = 1}$

Prethodno dokazali da je niz ograničen odozgo jedinicom

$x_{n+1} - x_n = 1 - 2x_n + 3x_n^2 - x_n^3 - x_n = 1 - 3x_n + 3x_n^2 - x_n^3$

$= (1 - x_n)^3 > 0$ jer je svako $x_n \in (0,1)$

Niz raste i konvergira ka 1.

2. $f(x) = \arctan\left(\frac{(x-1)^2}{x^2 - 2x}\right)$

$\begin{array}{c|c|c} & 0 & 2 \\ - & + & + \\ - & - & + \\ + & - & + \end{array}$

a) $D_f: \mathbb{R} \setminus \{0, 2\}$

II $x(x-2) \quad x \in (-\infty, 0) \cup (2, +\infty) \quad f(x) > 0$

$x \in (0,1) \cup (1,2) \quad f(x) < 0 \qquad (1,0)$ Nula

III $f(x) = \arctan\left(\frac{(x-1)^2}{(x-1)^2 - 1}\right)$

$g(x) = \arctan \frac{t^2}{t^2 - 1} \Rightarrow$ Jeste parna, nije periodična

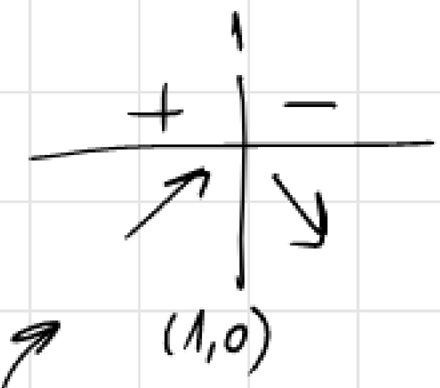
IV

$\lim_{x \to 2^+} \arctan\left(\frac{(x-1)^2}{x(x-2)}\right) = \pi/2 \qquad \lim_{x \to 0^+} \left(\frac{(x-1)^2}{\underbrace{x(x-2)}_{\to 0^-}}\right) = -\pi/2$

$\lim_{x \to 2^-} \arctan\left(\frac{(x-1)^2}{x(x-2)}\right) = -\pi/2 \qquad \lim_{x \to 0^-} \left(\frac{(x-1)^2}{\underbrace{x(x-2)}_{\to 0^+}}\right) = \pi/2$

$a = \lim_{x \to +\infty} \frac{\arctan\left(\frac{(x-1)^2}{x(x-2)}\right)}{x} = 0$

$b = \lim_{x \to \pm\infty} \arctan\left(\frac{(x-1)^2}{x(x-2)}\right) = \pi/4 \quad$ H.A.



V $\frac{1}{1 + \left(\frac{(x-1)^2}{x(x-1)}\right)^2} \cdot \frac{2(x-1)(x-2) \cdot x - (2x-2)(x-1)^2}{(x \cdot (x-2))^2}$

$\frac{\cancel{(x(x-2))^2}}{\underbrace{(x(x-2))^2 + (x-1)^2}_{>0}} \cdot \frac{(x-1)(\cancel{2x^2} - \cancel{4x} - \cancel{2x^2} + \cancel{4x} - 2)}{\cancel{(x(x-2))^2}} = \frac{\boxed{2(1-x)}}{((x-2)x)^2 + (x-1)^2}$

VI $-2x^4 + \cancel{8x^3} - 10x^2 + 4x - 2 - \cancel{8x^3} + 24x^2 - 20x + 4 - 8x^4 + 24x^3 - 20x^2 + 2x$

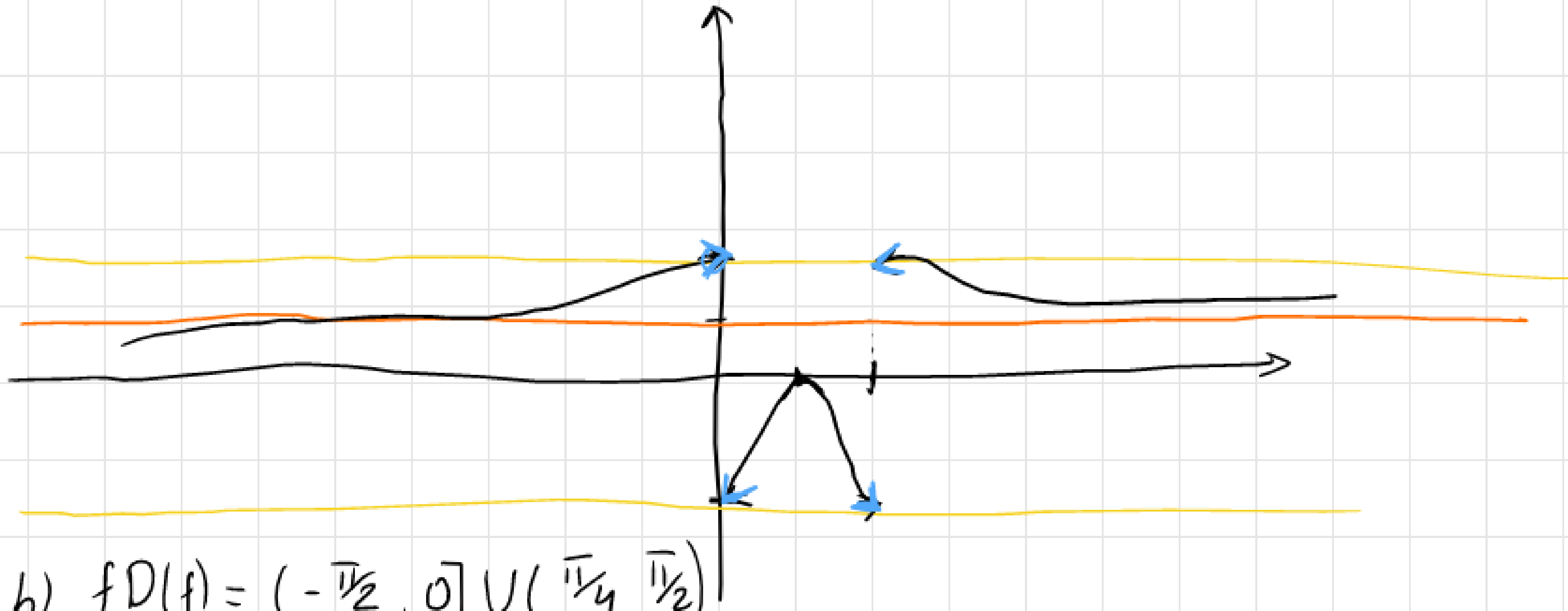
$= -10x^4 + 24x^3 - 6x^2 - 14x + 2 = -5x^4 + 12x^3 - 3x^2 - 7x + 1$

$-1: < 0$

$0: > 0$

$1: < 0$

$-1 \cap 0 \cup 1 \cap +\infty$



b) $f D(f) = (-\pi/2, 0] \cup (\pi/4, \pi/2)$

3.

$$f(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ \ln(\lambda + x^2), & x < 0 \end{cases}$$

a) $\lim_{x \to 0^-} \ln(\lambda + x^2) = \ln \lambda \quad \lim_{x \to 0^+} 1 - e^{-\lambda x} = 1 - 1 = 0$

$f(0) = 1 - e^{-\lambda \cdot 0} = 0 \quad \Rightarrow \ln \lambda = 0 \Leftrightarrow \lambda = 1$

b) $\lim_{h \to 0^+} \frac{(1 - e^{-(h+0)}) - 0}{h} = \lim_{h \to 0^+} \frac{e^{-h} - 1}{h} = 1$

$\lim_{h \to 0^-} \frac{\ln(1 + h^2)}{h} \cdot \frac{h}{h} = h = 0$ $\quad \to 1$

Nije diferencijabilna

4. $f: \mathbb{R} \to \mathbb{R}$

$g(x) = f(x) - x^{2n-1}$

$\lim_{x \to \pm\infty} f(x) - x^{2n-1} = \begin{matrix} -\infty \\ +\infty \end{matrix}$ Kosi Bolcano

$\Rightarrow \exists\, c$ takvo da $g(c) = 0$

tj. $f(c) = c^{2n-1}$

2020 sept 2

1. a) $\left(1+\frac{1}{x}\right)^{x} = e^{x\cdot\ln\left(1+\frac{1}{x}\right)}$

$\ln\left(1+\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{x^2\cdot 2} + \frac{1}{x^3\cdot 3} + o\left(\frac{1}{x^3}\right)$

$e^{1\left(1-\frac{1}{2x}+\frac{1}{x^2\cdot 3}+o\left(\frac{1}{x^2}\right)\right)}$

$e\cdot\left(1-\frac{1}{2x}+\frac{1}{x^2\cdot 3}+\frac{1}{2}\left(\frac{1}{3x^2}-\frac{1}{2x}\right)^2\right)$

$= e\cdot\left(1-\frac{1}{2x}+\frac{1}{3x^2}+\cancel{\frac{1}{9x^4}}-\cancel{\frac{1}{3x^4}}+\frac{1}{8x^2}+o\left(\frac{1}{x^4}\right)\right)$

$= e\left(1-\frac{1}{2x}+\frac{11}{24x^2}\right) \qquad e-\frac{e}{2}\cdot\frac{1}{x}+\frac{11e}{24}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)$

b) $\left(1+\frac{1}{x+1}\right)^{x+1} = e^{x+1\cdot\ln\left(1+\frac{1}{x+1}\right)}$

$\boxed{\frac{1}{x+1}} = \frac{1}{x}\cdot\left(1+\frac{1}{x}\right)^{-1} = \frac{1}{x}\cdot\left(1+\binom{-1}{1}\frac{1}{x}+\binom{-1}{2}\frac{1}{x^2}+\binom{-1}{3}\frac{1}{x^3}+o\left(\frac{1}{x^3}\right)\right)$

$= \frac{1}{x}\left(1-\frac{1}{x}+\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right) = \underbrace{\left(\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}+o\left(\frac{1}{x^3}\right)\right)}$

$e^{(x+1)\cdot\left(\frac{1}{x+1}-\frac{1}{(x+1)^2\cdot 2}+\frac{1}{3(x+1)^3}+o\left(\frac{1}{(x+1)^3}\right)\right)}$

$= e^{\left(1-\frac{1}{2(x+1)}+\frac{1}{3(x+1)^2}+o\left(\frac{1}{(x+1)^2}\right)\right)}$

$= e\cdot\left(-\frac{1}{2(x+1)}+\frac{1}{3(x+1)^2}+\frac{1}{2}\cdot\frac{1}{4(x+1)^2}+1\right)$

$= e\left(1-\frac{1}{2(x+1)}+\frac{1}{3(x+1)^2}+\frac{1}{8(x+1)^2}\right) = e\left(1-\frac{1}{2(x+1)}+\frac{11}{24}\cdot\frac{1}{(x+1)^2}\right)$

$= e\left(1-\frac{1}{2x}+\frac{1}{2x^2}+\frac{11}{24}\left(\frac{1}{x}-\frac{1}{x^2}\right)^2\right)$

$= e\left(1-\frac{1}{2x}+\frac{1}{2x^2}+\frac{11}{24x^2}\right) = e-\frac{e}{2x}+\frac{23}{24x^2}$

c)

$\lim_{n\to+\infty} n^2\left(\left(1+\frac{1}{n+1}\right)^{n+1}-\left(1+\frac{1}{n}\right)^n\right) = \lim_{n\to\infty} n^2\cdot\left(e-\frac{e}{2x}-\frac{11e}{24x^2}-e+\frac{e}{2x}+\frac{23e}{24x^2}\right)$

$= \lim_{n\to\infty} \cancel{n^2}\cdot+\frac{e}{2}\cdot\frac{1}{\cancel{n^2}} = \boxed{\frac{e}{2}}$

2. $f(x)=\frac{4}{x^2+3}$ i $g(x)=([f(x)]+1)\sqrt[3]{x^2-x}$

a) $[f(x)]$ $\quad x\in[-1,1]$ $\quad [f(x)]=1$

$x\in(-\infty,-1)\cup(1,+\infty)$ $\quad [f(x)]=0$

b) $\lim_{x\to 1+}(0+1)\sqrt[3]{1-1}=0$ $\qquad \lim_{x\to -1+} 2\cdot\sqrt[3]{1+1}=2\sqrt[3]{2}$

$\lim_{x\to 1-}(1+1)\sqrt[3]{1-1}=0$ $\qquad \lim_{x\to -1-} 1\cdot\sqrt[3]{2}=\sqrt[3]{2}$ $\Big\}$ Nije

$g(1)=(1+1)\sqrt[3]{1-1}=0$ ✓ $\qquad g(-1)=2\sqrt[3]{2}$

c)

$\left(\sqrt[3]{x^2-x}\right)'=1/3\cdot\frac{1\cdot(2x-1)}{\sqrt[3]{x^2-x}^{\,2}}$ $\quad$ Proverimo 0 i 1

$\lim_{h\to 0}\frac{g(h)-g(0)}{h}=\frac{2\sqrt[3]{h^2-h}-0}{h}\overset{LOP}{=}\frac{2(2h-1)}{3h\sqrt[3]{h-2+1/h}}=-\infty$

$\lim_{h\to 0^+}\frac{([f(1+h)]\overset{\to 0}{}+1)\sqrt[3]{(h+1)^2-h-1}}{h}=\frac{\sqrt[3]{h^2+h}}{h}=+\infty$ $\quad$ Dif na $\mathbb{R}\setminus\{-1,0,1\}$

3. $f(x)=\frac{x^2}{\sqrt[3]{x^3+8}}$ $\quad$ a) I $D_f:\mathbb{R}\setminus\{-2\}$ $\quad$ II $x>-2\ \ f(x)>0$, $\ x<-2\ \ f(x)<0$ $\quad (0,0)$ nula i presek

III Nije ni parna, ni neparna, ni periodična

IV Asimptote

$\lim_{\substack{x\to -2+\varepsilon\\ \varepsilon\to 0^+}}\frac{x^2}{\sqrt[3]{x^3+8}}=\frac{4-2\varepsilon+\varepsilon^2}{\sqrt[3]{\varepsilon^3-2\varepsilon^2+4\varepsilon-8+8}}=\frac{\varepsilon^2-2\varepsilon+4}{\varepsilon\sqrt[3]{1-2}}=+\infty$

$\ldots=-\infty$

K.A. $a=\lim_{x\to\pm\infty}\frac{x^2}{x\sqrt[3]{x^3+8}}=\lim_{x\to\pm\infty}\frac{\cancel{x^2}}{\cancel{x^2}\sqrt[3]{1+8/x^3}}=1$ $\quad -1/3$

$b=\lim_{x\to\pm\infty}\frac{x^2}{\sqrt[3]{x^3+8}}-x=\lim_{x\to\pm\infty}x^2\cdot\left(\frac{1-\sqrt[3]{1+8/x^3}}{\sqrt[3]{x^3+8}}\right)\cdot\frac{8/x^3}{8/x^3}=-\frac{8\cancel{x^2}}{3x^3\cdot x\sqrt[3]{1+8/x^3}}$

$=0$

V $\frac{2x\sqrt[3]{x^3+8}-1/3\cdot 3x^2\cdot\frac{1}{\sqrt[3]{x^3+8}^{\,2}}\cdot x^4}{\sqrt[3]{x^3+8}^{\,2}}$

$=\frac{2x^4+16x-x^4}{\sqrt[3]{x^3+8}^{\,4}}=x(x^3+16)$ $\quad x=0\ \vee\ x=-2\sqrt[3]{2}$

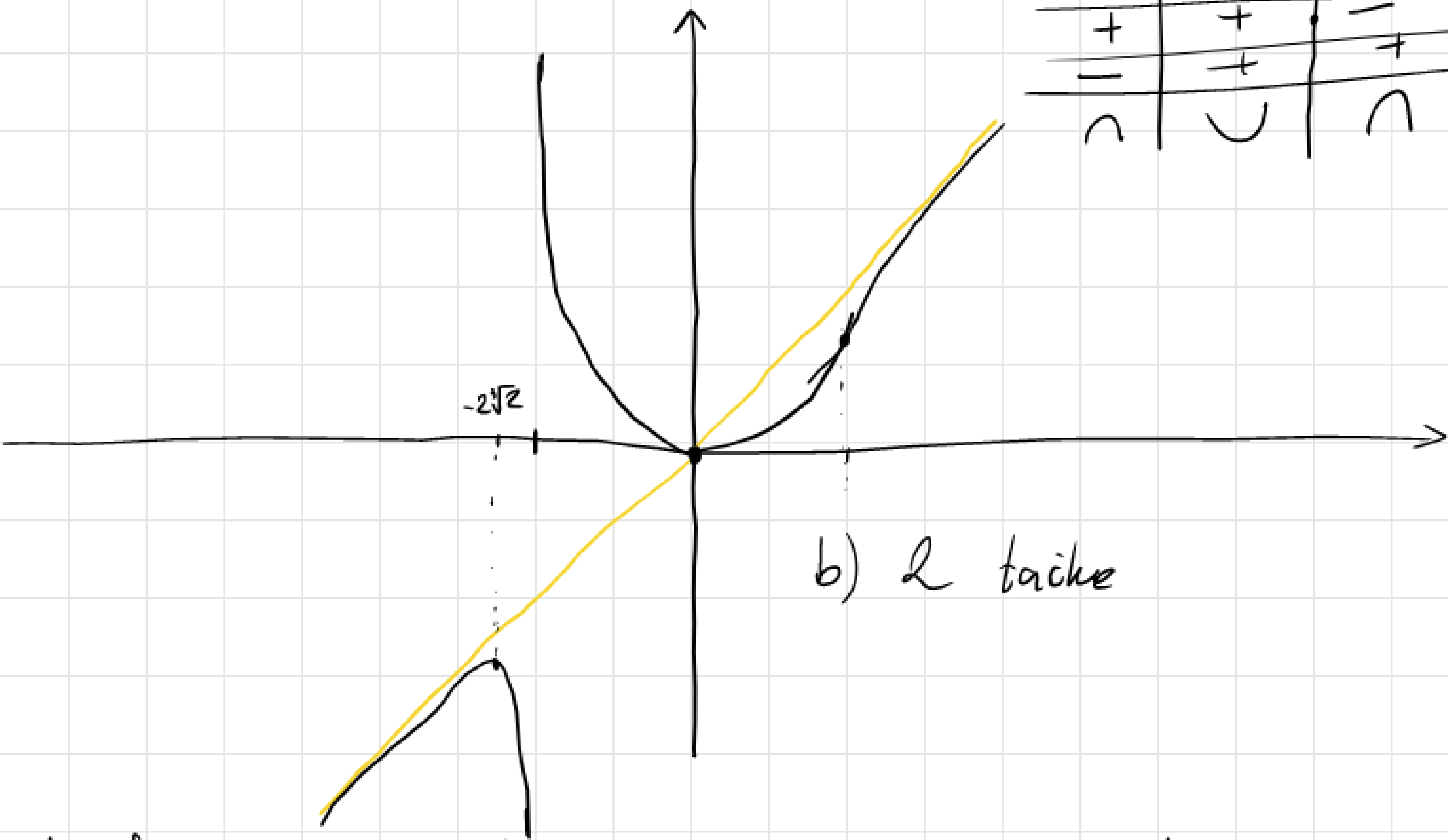
	$-2\sqrt[3]{2}$		0	
−		−		+
−		+		+
↗		↘		↗

VI $(4x^3+16)(x^3+8)\sqrt[3]{x^3+8}-(x^4+16x)\cdot 4/3\cdot 3x^2\cdot\sqrt[3]{x^3+8}\ \Big)\cdot\frac{1}{\sqrt[3]{x^3+8}^{\,8}}$

$$\frac{1}{\sqrt[3]{(x^3+8)^7}} \cdot (\cancel{4x^6} + 48x^3 + 128 - \cancel{4x^6} - 64x^3)$$

$$\frac{1}{(x^3+8)^2 \cdot \sqrt[3]{x^3+8}} \quad 16(8-x^3) = \frac{16}{(x^3+8)^2} \cdot \frac{(8-x^3)}{\sqrt[3]{x^3+8}}$$

	-2		2	
$+$		$+$		$-$
$-$		$+$		$+$
$\cap$		$\cup$		$\cap$



b) 2 tačke

4. f nepr na $[a,b]$, dif na (a,b) $f(a)=a$ i $f(b)=b$

a) $\exists c$ thd. $f(c) = \frac{a+b}{2}$ $\qquad f(c) - \frac{a+b}{2} = 0$

$g(x) = f(x) - \frac{a+b}{2}$

$$\left.\begin{aligned} g(a) &= \frac{a-b}{2} \\ g(b) &= \frac{b-a}{2} \end{aligned}\right\} \quad g(a)\cdot g(b) = -\frac{(a-b)^2}{4} < 0$$

$\Rightarrow \exists c \in (a,b)$ thd. $g(c) = 0$

tj. $f(c) = \frac{a+b}{2}$

b) $x < y < z$

$$\frac{y-x}{\frac{x+z}{2}-x} + \frac{z-y}{z-\frac{x+z}{2}} = 2 \qquad \frac{y-x}{\frac{x+z}{2}-x} + \frac{y-z}{\frac{x+z}{2}-z} = 2$$

$$\frac{y-x}{\frac{z-x}{2}} + \frac{y-z}{\frac{x-z}{2}} = \frac{\cancel{y}-x-\cancel{y}+z}{\frac{z-x}{2}} = \frac{\cancel{z-x}}{\frac{\cancel{z-x}}{2}} = 2$$

c) $a < s < c < t < b$

$\frac{1}{f'(s)} + \frac{1}{f'(t)} = 2$

$f(a) = a$
$f(b) = b$
$f(c) = \frac{a+b}{2}$

$\exists s \in (a,c)$ tkd.

$f'(s) = \frac{\frac{a+b}{2} - a}{c-a} = \frac{b-a}{2(c-a)}$

$\exists t \in (c,b)$ tkd.

$f'(t) = \frac{b - \frac{a+b}{2}}{b-c} = \frac{b-a}{2(b-c)}$

$\frac{2c-2a}{b-a} + \frac{2b-2c}{b-a} = \frac{2(b-a)}{b-a} = 2$ ✓

2020 sept 3

1. $x_1 = -1, \; x_{n+1} = e^{x_n} - 1 \quad n \geq 1$

a) $x \in \mathbb{R} \quad x \leq e^x - 1$

$e^x - 1 - x \geq 0$

$f(x) = e^x - x - 1$

$f'(x) = e^x - 1$

$x \geq 0$ f-ja raste ↗

$x < 0$ f-ja opada ↘

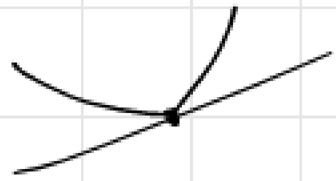
I $x \geq 0$ f ja raste

$e^0 - 1 - 0 = 1 - 1 = 0 \geq 0$ ✓

Važi

II Kako fja opada do $(0,0) \Rightarrow$

Sve vrednosti pre su veće.



b) $x_n < 0$ za svako n

B: $x_1 = -1 < 0$ IH: $x_n < 0 \Rightarrow x_{n+1} < 0$?

IK: $x_{n+1} = e^{x_n} - 1 < e^0 - 1 < 0$ ✓

c) $x_{n+1} - x_n = e^{x_n} - x - 1 \xrightarrow{\text{iz a)}} > 0$ tj. niz raste

$L = e^L - 1$

$L + 1 = e^L \Rightarrow L = 0 \quad \lim_{n \to +\infty} x_n = 0$

d) $\lim_{n \to +\infty} n \cdot x_n \quad (\infty \cdot 0)$ tražimo $\frac{1}{n \cdot x_n} = \frac{1/x_n}{n}$

$\lim_{n \to +\infty} \frac{\frac{1}{x_n}}{n} \overset{\text{štolc}}{=} \lim_{n \to +\infty} \frac{\frac{1}{e^{x_n} - 1} - \frac{1}{x_n}}{n+1-n}$ $\quad 1 + x + \frac{x^2}{2}$

$= \lim_{n \to +\infty} \frac{x_n - e^{x_n} + 1}{x_n (e^{x_n} - 1)} = \lim_{n \to +\infty} \frac{\cancel{x_n} + \cancel{1} - \cancel{1} - \cancel{x_n} - \frac{x_n^2}{2}}{x_n(\cancel{1} - \cancel{1} + x_n + \frac{x_n^2}{2})} = \frac{-\frac{x_n^2}{2}}{x_n^2} = -\frac{1}{2}$

$\Rightarrow \lim_{n \to +\infty} n \cdot x_n = -2$

2.

x^4

a) $tg(x) = f(0) + f'(0)\cdot x + \frac{f''(0)\cdot x^2}{2} + \frac{f^{(3)}(0)\cdot x^3}{6} + \frac{f^{(4)}(0)\cdot x^4}{24} + o(x^4)$

$f''(0) = 0$

$f'(x) = \frac{1}{\cos^2 x}$

$f''(x) = -2\frac{(-\sin x)}{\cos^4 x} = \frac{2\sin x}{\cos^4 x}$

$f^{(3)}(x) = \frac{2\cos^{\cancel{2}5} x - 4\cdot \cancel{\cos^3 x}\cdot(-\sin^2 x)}{\cos^{\cancel{8}5} x}$

$f^{(4)}(x) = \left(\frac{2(\cancel{\cos^2 x + \sin^2 x})^{1} + 2\sin^2 x}{\cos^3 x}\right)'$

$f^{(4)}(x) = \frac{4\sin x\cdot\cos^6 x + 5\cos^4 x\cdot 2\sin^3 x}{\cos^{10} x}$

$\Rightarrow M_4^f: \; x + \frac{2x^3}{\cancel{6}3} + o(x^3) = x + \frac{x^3}{3} + o(x^3)$

$(x - \frac{x^3}{6})^2$

b) $\lim_{x\to 0} \frac{\frac{tg x}{x} + \left(\frac{\sin x}{x}\right)^2 - 2}{x^2\cdot tg^2 x} = \lim_{x\to 0}\frac{x\cdot tg x + (\sin x)^2 - 2x^2}{x^4\, tg^2 x}$

$= \lim_{x\to 0} \frac{\cancel{x^2} + \cancel{\frac{x^4}{3}} + \cancel{x^2} - \cancel{\frac{x^4}{3}} + \frac{x^6}{36} - \cancel{2x^2}}{x^4\left(x^2 + \frac{2x^4}{3} + \frac{x^6}{3}\right)} = \frac{1}{36}\cdot\frac{\cancel{x^6}}{\cancel{x^6}\left(1 + \underbrace{\frac{2x^2}{3}}_{0} + \underbrace{\frac{x^4}{3}}_{0}\right)}$?

3. $\frac{\sin x}{2+\cos x}$ I $D_f: \mathbb{R}$ II $x \in (\pi + 2n\pi, 2(n+1)\pi), n\in\mathbb{N}$ $f(x) < 0$

$n\pi$, $n\in\mathbb{N}$ nule

III Periodična je

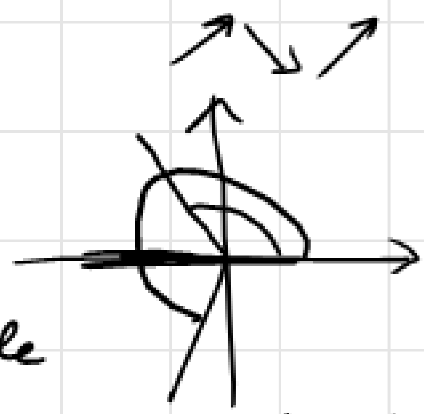
Presek sa y-osom $\Rightarrow (0,0)$

$f(x+2\pi) = \frac{\sin(x+2\pi)}{2+\cos(x+2\pi)} = f(x)$ ✓

$f(-x) = \frac{\sin(-x)}{2+\cos(-x)} = \frac{-\sin x}{2+\cos x} = -f(x)$ Neparna je

IV Asimptote Nema Vertikalne $\frac{\sin x \in [-1,1]}{2+\cos x \in [1,3]} \Rightarrow$ Nema kosu

Nema horizontalnu jer je periodična



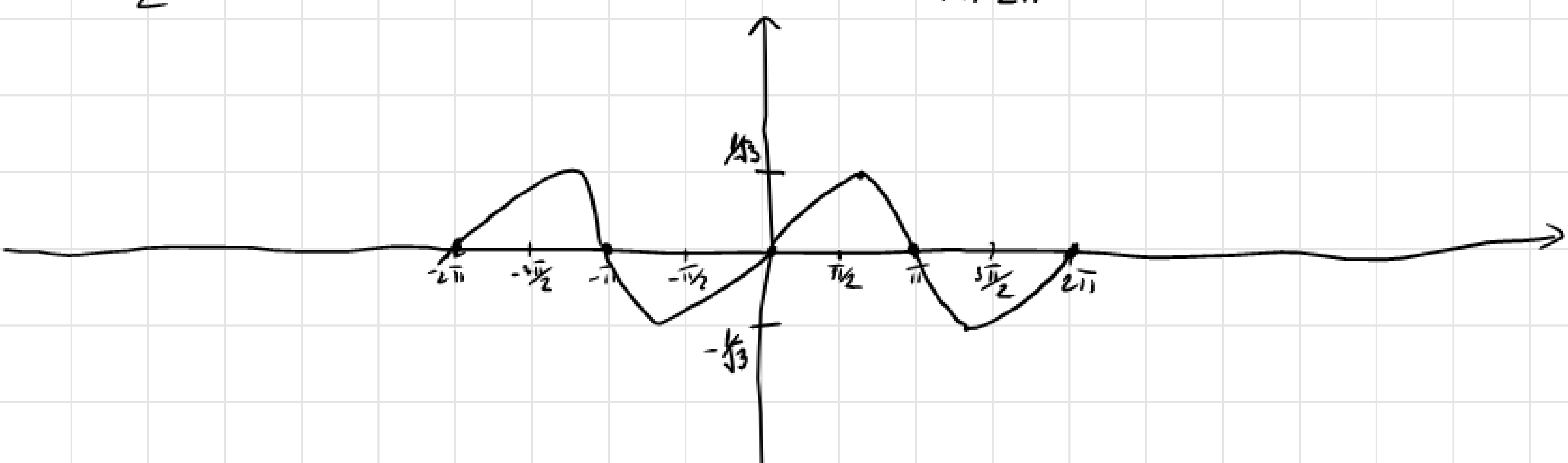
V $\frac{\cos x(2+\cos x) + \sin^2 x}{(2+\cos x)^2} = \frac{2\cos x + 1}{(2+\cos x)^2} = \cos x + \frac{1}{2}$

$\left.\begin{matrix}120 + n2\pi \\ 240 + n2\pi\end{matrix}\right\}$ Nule

izmedu njih opada, sunda drugo raste

VI $\frac{-2\sin x(2+\cos x) + (2\cos x + 1)\cdot 2\cdot\sin x}{(2+\cos x)^3} > 0$

$-2\sin x + 2\sin x\cos = 2\sin x(\cos x - 1)$ $0 \to$ tačke prevoja i $n\cdot 2\pi$



4. $x \cdot e^{\frac{x^2+1}{2x^2}} = 1555$, $x > 1$

$F(x): x \cdot e^{\frac{x^2+1}{2x^2}} - 1555$

$F'(x): e^{\frac{x^2+1}{2x^2}} + x \cdot e^{\frac{x^2+1}{2x^2}} \cdot \frac{4x^3 - 4x(x^2+1)}{4x^4}$

$e^{\frac{x^2+1}{2x^2}} \left(1 + \frac{\cancel{4x^3} - \cancel{4x^3} - 4x}{4x^3}\right) \boxed{\left(1 - \frac{1}{x^2}\right)}$ Nas zanima na $x \in (1, +\infty)$ Uvek pozitivno $\Rightarrow$ F raste

$F(1) < 0$

$\lim\limits_{x \to +\infty} x \cdot e^{\frac{x^2+1}{2x^2} = 1/2} - 1555 = e^{1/2} \cdot x - 1555 = +\infty$ $\Big\}$ $\exists\, a \in (1, +\infty)$ $F(a) = 0$

b) Zanima nas presen $f(x) = x \cdot e^{\frac{x^2+1}{2x^2}}$ i $y = 1555$

$\Rightarrow$ Tražimo vertikalne asimptote:

$\lim\limits_{x \to 0+} x \cdot e^{\frac{x^2+1}{2x^2}} = \lim\limits_{x \to 0+} \frac{e^{\frac{1+\frac{1}{x^2}}{2}}}{\frac{1}{x}} \overset{LOP}{=} \frac{e^{\frac{1+\frac{1}{x^2}}{2}} \cdot \cancel{1/2} \cdot \cancel{(-2)} \cdot \frac{1}{x^3}}{\cancel{-}\frac{1}{x^2}} = e^{\frac{1+\frac{1}{x^2}}{2}} \cdot \frac{1}{x} = +\infty$

$\lim\limits_{x \to 0-} \ldots = -\infty$

+ Monotonost: $\left(1 - \frac{1}{x^2}\right)$ $x = \pm 1$

Na intervalu $(1, +\infty)$ znamo da postoji 1 rešenje, kao i u $x = 0+$

-1 je lokalni max $\Rightarrow -1 \cdot e^{\frac{2}{2}} = -e \Rightarrow$ na intervalu $(-\infty, 0)$ nema rešenja.

$\Rightarrow \exists$ 2 rešenja u intervalu $(1, +\infty)$ i tački 0^+.

2021 jan

1. $a_n = n^3 \left(\sin 1/n \cdot \ln\left(1 + 1/n + 1/n^2\right) - 1/n \cdot \ln\left(1 + 1/n\right)\right)$

a) $\lim\limits_{n \to +\infty} n^3 \left(\left(\frac{1}{n} - \frac{1}{n^3 \cdot 6}\right) \cdot \left(\frac{1}{n} + 1/n^2 - \frac{1}{2}\left(\frac{1}{n} + \frac{1}{n^2}\right)^2\right) - \frac{1}{n}\left(1/n - \frac{1}{n^2 \cdot 2} + \frac{1}{n^3 \cdot 3}\right)\right)$

$= \lim\limits_{n \to +\infty} n^3 \left(\cancel{1/n^2} + 1/n^3 - \cancel{\frac{1/3}{2n^3}} - \underbrace{\frac{1/4}{n}}_{\to 0} - \cancel{1/n^2} + \cancel{\frac{1}{n^3 \cdot 2}}\right) = \boxed{1}$

b) $\lim\limits_{n \to +\infty} \frac{\overbrace{1 \cdot a_1 + 2 \cdot a_2 + \ldots + n \cdot a_n}^{b_n}}{\underbrace{n^2}_{c_n \nearrow}} = \frac{1 \cdot a_1 + 2a_2 + \ldots + n \cdot a_n + (n+1)a_{n+1} - a_1 - 2a_2 - \ldots - n(a_n)}{(n+1)^2 - n^2}$

$= \lim\limits_{n \to +\infty} \frac{(n+1)\overbrace{a_{n+1}}^{\to 1}}{\cancel{n^2} + 2n + 1 - \cancel{n^2}} = \frac{(n+1) \cdot 1}{2n+1} = 1/2$

2. $f(x) = \begin{cases} \ln(1+3x^2) & , x \leq -1 \\ ax^2+bx+c & , -1 < x \leq 0 \\ \frac{\sin x}{\sqrt{x}} & , 0 > x \end{cases}$

a) $\lim_{x\to -1^-} \ln(1+3x^2) = 2\ln 2$

$\lim_{x\to -1^+} ax^2+bx+c = a-b+c$

$a-b = 2\ln 2$

$\lim_{x\to 0^-} ax^2+bx+c = c$

$\lim_{x\to 0^+} \frac{\sin x}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = 0 \quad \rightarrow 1$

$\Big\} c=0$

b) $f(x) = \begin{cases} \ln(1+3x^2) \\ ax^2+(a-2\ln 2)x \\ \frac{\sin x}{\sqrt{x}} \end{cases}$

$\lim_{h\to 0^-} \frac{\ln(1+3(h-1)^2) - 2\ln 2}{h}$

$= \lim_{h\to 0^-} \frac{\ln(1+3h^2-6h+3) - 2\ln 2}{h}$

$= \ln(4+3h^2-6h) - 2\ln 2$

$= \ln(4(1+\frac{3h^2-6h}{4})) - 2\ln 2$

$= \frac{\ln(1+\frac{3h^2-6h}{4})}{\frac{3h^2-6h}{4}} \cdot \frac{\frac{3h^2-6h}{4}}{h}$

$= \frac{(3h-6)\cancel{h}}{4\cdot \cancel{h}} = \boxed{-3/2}$

$\lim_{h\to 0^+} \frac{a(h-1)^2+(a-2\ln 2)(h-1)-2\ln 2}{h}$

$= \frac{ah^2-2ah+\cancel{a}+ah-\cancel{a}-2\ln 2\cdot h}{h}$

$ah - a - 2\ln 2 = -a-2\ln 2$

$\lim_{h\to 0^-} \frac{ah^2+(a-2\ln 2)h}{h} = a-2\ln 2$

$3/2 - 2\ln 2 = a$

$3/2 - 4\ln 2 = b$

!

$\lim_{h\to 0^+} \frac{\frac{\sin h}{\sqrt{h}}}{h} = \frac{\sin h}{\sqrt{h}\cdot h} = \frac{1}{\sqrt{h}} = +\infty$

3. a) $f(x) = \ln(x^2-1) + \frac{1}{x^2-1} - 2$

I $D_f: x \in (-\infty,-1) \cup (1,+\infty)$

II Kasnije ćemo pronaći intervale gde je f-ja poz/neg kao i nule

Nema presek sa y-osom

III $f(-x) = f(x) \Rightarrow$ Parna je; Nije periodična

IV $\lim_{x\to 1^-} \ln(x^2-1) + \frac{1}{(x-1)(x+1)} - 2 = \lim_{x\to 1^-} \ln(x-1) + \ln(x+1) + \frac{1}{(x-1)(x+1)} - 2$

$= \lim \ln 2 - 2 + \ln(x-1) + \frac{1}{2(x-1)} = \lim_{t\to 0^-} \frac{2t\ln t + 1}{2t} = +\infty$

F-ja je parna pa $\lim_{x\to -1^+} f(x) = +\infty$

$$a = \lim_{x\to\pm\infty} \frac{1}{x}\left(\ln(x^2-1) + \frac{1}{x^2-1} - 2\right) = \frac{1}{x}\left(\ln x^2\left(1-\frac{1}{x^2}\right) + \frac{1}{x^2-1} - 2\right)$$

$$= \lim_{x\to\pm\infty} \frac{1}{x}\left(2\ln x + \frac{\ln\left(1-\frac{1}{x^2}\right)}{-\frac{1}{x^2}} - \frac{1}{x^2} + \frac{1}{x^2-1} - 2\right)$$

$$\frac{1}{x}\left(2\ln x + \frac{\cancel{x^2} - \cancel{x^2} + 1}{(x^2-1)\,x^2} - 2\right) \underset{=}{LOP} \quad \frac{\frac{2}{x}}{1} = \frac{2}{x} = 0$$

$$b = \lim_{x\to\pm\infty} (2\ln x - 2) = \pm\infty \quad \Rightarrow \text{Nema}$$

V. $\frac{2x}{x^2-1} - \frac{2x}{(x^2-1)^2} = \frac{2}{(x^2-1)^2}_{>0}\left(x(x^2-1) - x\right) = x(x^2-2) = (x^3-2x)$

	$-\infty$	$-\sqrt{2}$	0	$\sqrt{2}$	$+\infty$
$x+\sqrt{2}$	$-$	$+$	$+$	$+$	
x	$-$	$-$	$+$	$+$	
$x-\sqrt{2}$	$-$	$-$	$-$	$+$	
$f'(x)$	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$	

$f(-\sqrt{2}) = 0 + 1 - 2 = -1$

$f(\sqrt{2}) = -1$

$\Rightarrow \lim_{x\to\pm\infty} f(x) > 0 \Rightarrow \exists$ nule između $-\infty$ i $-\sqrt{2}$ i $\sqrt{2}$ i $+\infty$

dodatno $\exists$ nule između $-\sqrt{2}$ i -1, i 1 i $\sqrt{2}$

VI $\left(2(3x^2-2)(x^2-1)^{\cancel{2}} - 2\cancel{(x^2-1)}\cdot 2x\cdot 2(x^3-2x)\right)\cdot \frac{1}{(x^2-1)^3}$

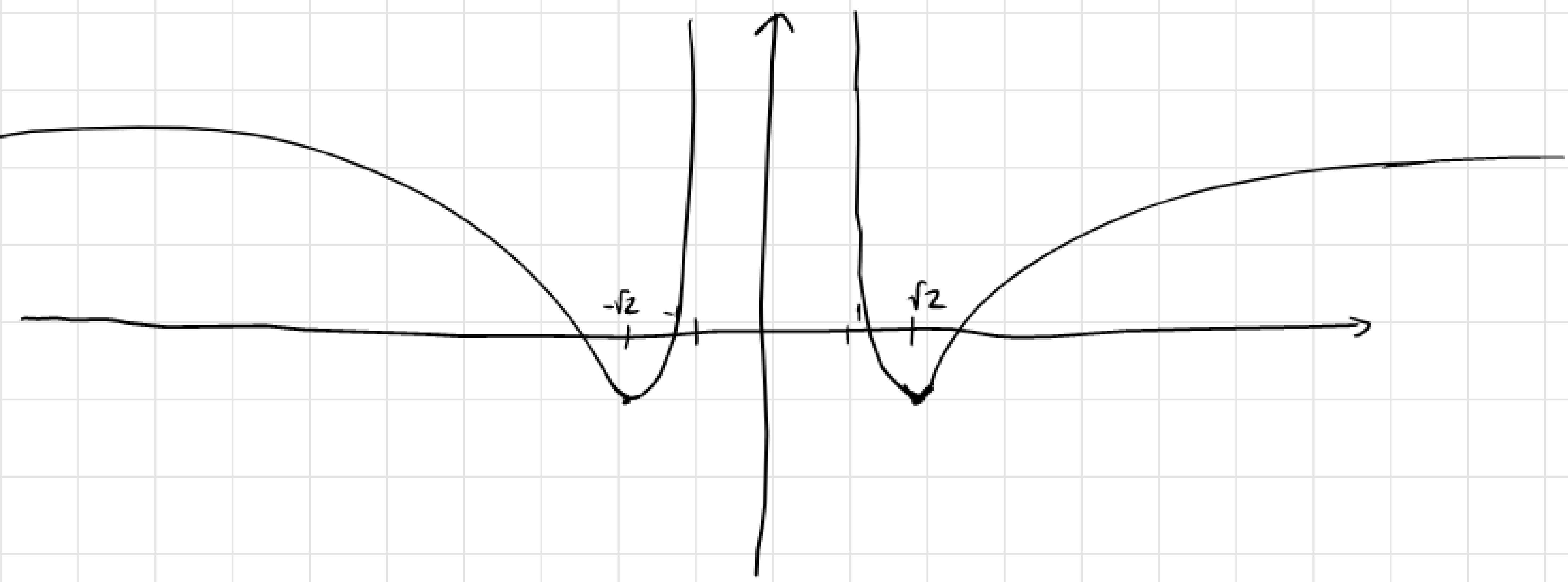
$= \left(6x^4 - 6x^2 - 4x^2 + 4 - 8x^4 + 16x^2\right)\cdot\frac{1}{\cancel{(x^2-1)^2}}\cdot\frac{1}{x^2-1}$

$= \frac{(-2x^4 + 6x^2 + 4)}{x^2-1} = \frac{2(-x^4+3x^2+4)}{x^2-1}$

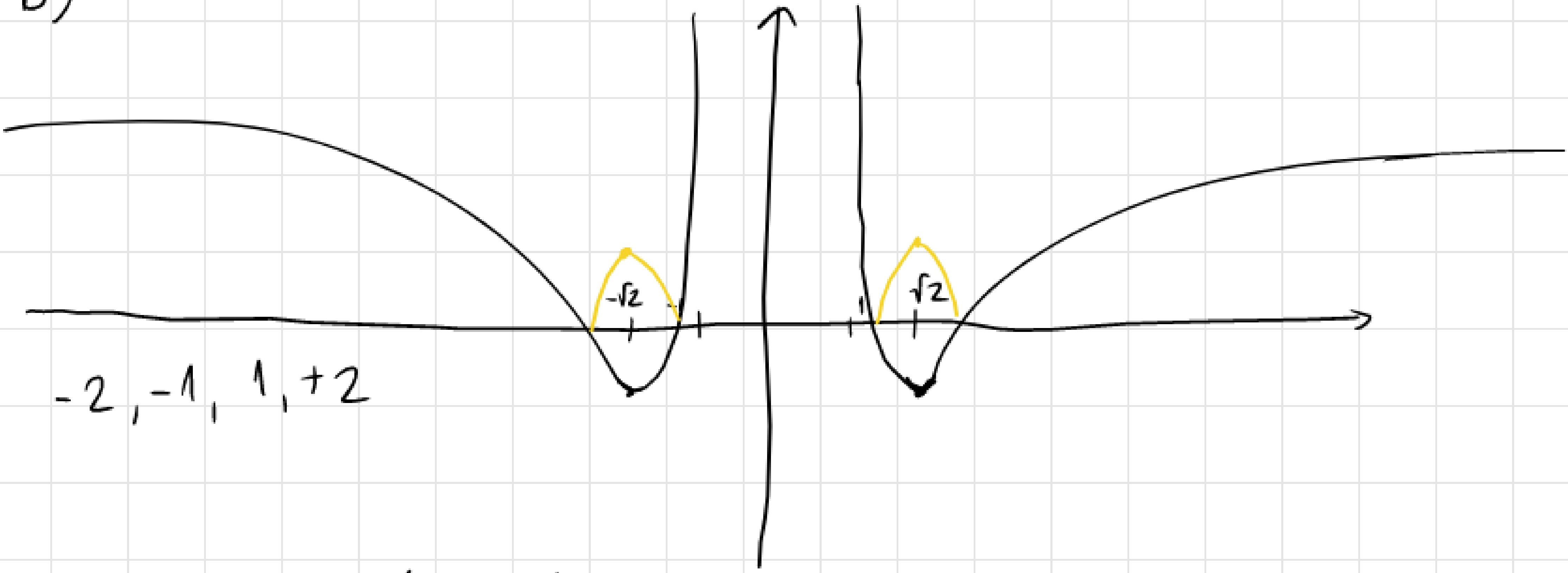
$\frac{-3 \pm \sqrt{9+16}}{-2}$ → $\frac{-3-5}{-2} = 4$, → $\frac{-3+5}{-2} = \cancel{-1}$

$x^2 = 4 \quad \boxed{x = \pm 2}$

	$-\infty$	-2	-1	1	2	$+\infty$
$-x^4+3x^2+4$	$-$	$+$		$+$	$-$	
x^2-1	$+$	$+$		$+$	$+$	
	$\cap$	$\cup$		$\cup$	$\cap$	



b)



$-2, -1, 1, +2$

2021 jun 1 drugi tok

1b)

$$X_n = \left(\frac{2n-3}{1+2n}\right)^{(-1)^n \cdot n} + \arctan\left((-1)^{n+1} \cdot 2n\right) \cdot \sin\left(\frac{2n\pi}{3}\right)$$

$$\left(1 - \frac{4}{2n+1}\right)^{\frac{-4}{2n+1} (2n+1)} \qquad e^{\frac{(2n+1)}{(-4)}(-1)^n n}$$

$n \equiv_2 0 \quad -\pi/2$

$n \equiv_2 1 \quad \pi/2$

$0: 0 \qquad 3: 0$

$1: \frac{\sqrt{3}}{2} \qquad 4: \frac{\sqrt{3}}{2}$

$2: -\frac{\sqrt{3}}{2} \qquad 5: -\frac{\sqrt{3}}{2}$

$$\lim_{n\to\infty} e^{\frac{-4\cancel{n}(-1)^n}{2\cancel{n}}} = e^{2\cdot(-1)^{n+1}} \to e^2, \quad \to \frac{1}{e^2}$$

$e^2 + \pi/2 \cdot \sqrt{3}/2, \quad e^2, \quad e^2 - \frac{\pi}{2} \cdot \frac{\sqrt{3}}{2}$

$1/e^2 - \pi/2 \cdot \frac{\sqrt{3}}{2}, \quad 1/e^2, \quad 1/e^2 + \pi/2 \cdot \sqrt{3}/2$

2.

$$f(x) = \begin{cases} A, & x \le 0 \\ \frac{x^x - 1}{x \ln x} + B, & 0 < x < 1 \\ C, & x = 1 \\ \frac{x^x - x}{\ln x - x + 1}, & x > 1 \end{cases}$$

$\lim_{x\to 0^-} A = A$

$\lim_{x\to 0^+} \frac{x^x - 1}{x \ln x} + B = \lim_{x\to 0^+} \frac{e^{x\ln x} - 1}{x \ln x} + B = 1 + B$

$$\lim_{x\to 1^-} \frac{x^x - 1}{x\ln x} + B = \left[\begin{matrix} t = x-1 \\ t \to 0^- \end{matrix}\right] = \lim_{t\to 0^-} \frac{e^{(t+1)\cdot\ln(t+1)} - 1}{(t+1)\ln(t+1)} + B$$

$\lim_{x\to 1} C = C$

$$= B + \lim_{t\to 0^-} \frac{e^{(t+1)\ln(t+1)} \cdot \cancel{(\ln(t+1)+1)}}{\cancel{(\ln(t+1)+1)}} = B + 1$$

$$\lim_{x\to 1^+} \frac{e^{x\ln x} - x}{\ln x - x + 1} \overset{LOP}{=} \lim_{x\to 1^+} \frac{e^{x\ln x}(\ln x + 1) - 1}{\frac{1}{x}} \overset{LOP}{=} \lim_{x\to 1^+} \frac{e^{x\ln x}\left((\ln x + 1)^2 + \frac{1}{x}\right)}{-1/x^2}$$

$$= \frac{2}{-1} = \boxed{-2}$$

$A = 1 + B$

$C = -2 = 1 + B \Rightarrow B = -3, \quad A = -2$

3. $f(x) = e^{-1/x}\sqrt{x^2+x} = e^{-1/x}\sqrt{x(x+1)}$

	$-\infty$ to -1	-1 to 0	0 to $+\infty$
x	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$
	$+$	$-$	$+$

I $D_f: (-\infty, -1] \cup (0, +\infty)$

II -1 je nula. Ne postoji presek sa y-osom. Fja je + na D_f.

III $f(-x) = e^{1/x}\sqrt{x^2-x}$ Ni parna, ni neparna. Nije periodična.

IV $\lim\limits_{x \to 0^+} e^{\underbrace{-1/x}_{-\infty}}\sqrt{x^2+x} = 0$

K.A. $a = \lim\limits_{x \to +\infty} \dfrac{e^{-1/x}\sqrt{x^2+x}}{x} = +1$

$b = \lim\limits_{x \to +\infty} e^{-1/x} \cdot \sqrt{x^2+x} - x = \lim\limits_{x \to +\infty} (e^{-1/x} - 1 + 1) \cdot x \cdot \left(\sqrt{1 + \tfrac{1}{x}} - 1 + 1\right) - x$

$= \lim\limits_{x \to +\infty} \left(\dfrac{(e^{-1/x} - 1) \cdot (-1/x)}{-1/x} + 1\right) \cdot x \cdot \left(\dfrac{\left((1+\frac{1}{x})^{1/2} - 1\right)}{\frac{1}{x}} \cdot 1/x + 1\right) - x$

$= \lim\limits_{x \to +\infty} \left(-\frac{1}{x} + 1\right) \cdot x \cdot \left(\frac{1}{2x} + 1\right) - x = \lim\limits_{x \to +\infty} (x-1)\left(\frac{1}{2x} + 1\right) - x$

$= \lim\limits_{x \to +\infty} 1/2 - \underbrace{\frac{1}{2x}}_{\to 0} + \cancel{x} - \cancel{x} - 1 = \boxed{-1/2}$

$b_2 = \lim\limits_{x \to -\infty} e^{-1/x}\sqrt{x^2+x} + x = \begin{bmatrix} t = -x \\ t \to +\infty \end{bmatrix} = \lim\limits_{t \to +\infty} e^{1/t}\sqrt{t^2 - t} - t$

$= \lim\limits_{t \to +\infty} \left(\dfrac{(e^{1/t} - 1) \cdot 1/t}{1/t} + 1\right) \cdot t \cdot \left(\dfrac{((1 - 1/t)^{1/2} - 1) \cdot (-1/t)}{-1/t} + 1\right) - t$

$(1/t + 1) \cdot t \cdot \left(1 - \frac{1}{2t}\right) - t = (1 + t)\left(1 - \frac{1}{2t}\right) - t = 1 + t - t - \frac{1}{2t} - 1/2 = 1/2$

V $\left(e^{-1/x} \cdot \sqrt{x^2+x}\right)' = e^{-1/x}\left(\dfrac{\sqrt{x^2+x}}{x^2} + \dfrac{2x+1}{\sqrt{x^2+x}}\right) = \dfrac{e^{-1/x}}{x^2\sqrt{x^2+x}}\left(x^2 + x + 2x^3 + x^2\right)$

$= x\,(\underbrace{2x^2 + 2x + 1}_{>0}) \Rightarrow$ $x > 0$ raste, $x < 0$ opada. $f'(0) = 0 = \operatorname{tg}\alpha$ $\alpha \to$ ugao u odnosu na x-osu gde ulazi

VI Nepotrebno (Nemojte ovamo na ispitu)

4. b) $x, y \in [0,2]$ $\quad |f(x) - f(y)| \leq |x-y|$

Za svako $x \in (0,2)$ važi $|f'(x)| \leq 1$

$$f(x) - f(y) = f'(c) \cdot (x-y) \quad /|\ |$$

$$|f(x) - f(y)| = |f'(c) \cdot (x-y)|$$

$$|f(x) - f(y)| = |x-y| \cdot |f'(c)| \quad c \in (0,2) \quad 0 \leq |f'(c)| \leq 1$$

$$\Rightarrow |f(x) - f(y)| \leq |x-y|$$

c)

$f(0) = f(2) = 1 \quad x \in [0,2] \quad f(x) \geq 0 \quad \forall f'(x) \in [-1,1]$

$$\frac{f(x)-1}{x} = f'(a), \ 0<a<x \qquad \frac{f(x)-1}{x-2} = f'(b), \ x<b<2$$

$$-1 \leq \frac{f(x)-1}{x} \leq 1 \qquad -1 \leq \frac{f(x)-1}{x-2} \leq 1$$

$$\boxed{1-x \leq f(x)} \leq 1+x \qquad 2-x \geq f(x)-1 \geq x-2$$

$$\boxed{x-1 \leq f(x)} \leq 3-x$$

$$f(x) \geq |1-x| \Longrightarrow f(x) \geq 0$$

2021 jun 1 tok 1. Isto kao ovaj gore

2021 jun 2 tok 2.

1. $x_1 = a > 0$ i $x_{n+1} = \sqrt[4]{1 + x_n \cdot 4} - 1$

a) B: $x_1 = a > 0$ ✓ IH $x_n \geq 0 \Rightarrow x_{n+1} \geq 0$?

$$\sqrt[4]{1+4x_n} \geq 1 \quad /^4$$

$$1 + 4x_n \geq 1 \Rightarrow 4x_n \geq 0 \text{ tj } x_n \geq 0 \text{ što jeste tačno.}$$

b) $L = \sqrt[4]{1+4L} - 1 \qquad L_1 = 0$, ako $\exists \lim x_n$

1. Dokazali smo da $x_n \geq 0$ za $\forall x_n$

2. $\sqrt[4]{1+4x_n} - 1 - x_n \ \square\ 0$

$$\sqrt[4]{1+4x_n} \ \square\ x_n + 1 \quad /^4$$

$$\cancel{1} + \cancel{4x_n} \ \square\ x_n^4 + 4x_n^3 + 6x_n^2 \cancel{+4x_n} \cancel{+1}$$

$$0 \leq$$

$$\Rightarrow \text{Niz opada} \Rightarrow \lim_{n \to +\infty} x_n = 0$$

2. $f(x) = \ln\left|\frac{x-1}{x}\right| + |x+1|$

I $x \in \mathbb{R}\setminus\{0,1\}$ II Teško nam je da nađemo intervale kad je poz i nule. (nasmiješ)

III $f(-x) \neq f(x)$, $\neq -f(x)$ nije periodična.

IV $\lim_{x\to 0^-} \ln\left|\frac{x-1}{x}\right| + |x+1| = 1 + \lim_{x\to 0^-} \ln\left|\frac{x-1}{x}\right| = +\infty$

$\lim_{x\to 0^+} \ln \ldots = +\infty$

$\lim_{x\to 1^-} \ln|1-x| - \ln|x| + 2 = 2 + \lim_{x\to 1^-} \ln|-\varepsilon| = -\infty$, $\varepsilon \to 0$

$\lim_{x\to 1^+} \ldots = -\infty$

$\lim_{x\to+\infty} \frac{\ln\left|\frac{x-1}{x}\right| + |1+x|}{x} \overset{LOP}{=} \frac{\cancel{x}}{x-1}\cdot\frac{(x-x+1)}{x^{\cancel{2}}} + 1 = 1$

$\lim_{x\to-\infty} \ldots = \left\lceil \begin{matrix} t=-x \\ t\to+\infty \end{matrix}\right\rfloor = \lim_{t\to+\infty} \frac{1}{t}\left(\ln\left|\frac{t+1}{t}\right| + |1-t|\right) \overset{LOP}{=} \frac{\cancel{t}}{t+1}\cdot\frac{t-t-1}{t^2} - 1 = -1$

$\lim_{x\to+\infty} \ln(1-1/x) + 1 + x - x = -\frac{1}{x} + 1 = 1$

$\lim_{x\to-\infty} \ln(1-1/x) - 1 - \cancel{x} + \cancel{x} = -\frac{1}{x} - 1 = -1$

	$-\infty$	0	1	$+\infty$
$x-1$	$-$	$-$	$+$	
x	$-$	$+$	$+$	
	$+$	$-$	$+$	

V $\left(\ln\left|\frac{x-1}{x}\right| + |1+x|\right)' = \begin{cases} \text{I } x\in(-\infty,-1) \\ \text{II } x\in(-1,0)\cup(1,+\infty) \\ \text{III } x\in(0,1) \end{cases}$

	$-\infty$ … -1	-1 … 0	0 … 1	1 … $+\infty$
$\frac{x-1}{x}$	$+$	$+$	$-$	$+$
$1+x$	$-$	$+$	$+$	$+$

I $\frac{\cancel{x}}{x-1}\cdot\frac{x-x+1}{x} - 1 = \frac{1}{x(x-1)} - 1 = \frac{1+x-x^2}{x(x-1)}$, $x(x-1)>0$

$-(x^2 - \frac{1}{2}\cdot 2\cdot x + \frac{1}{4} - \frac{5}{4})$

$-((x-\frac{1}{2})^2 - (\frac{\sqrt{5}}{2})^2)$

$= -(x - \frac{1+\sqrt{5}}{2})(x - \frac{1-\sqrt{5}}{2})$, <0

$-(\frac{1-\sqrt{5}}{2} - x)$

$-\infty$	$\frac{1-\sqrt{5}}{2}$	-1
$-$ ↘	$+$ ↗	

II $\frac{1}{x(x-1)} + 1 = \frac{x^2-x+1}{x(x-1)}$, $x(x-1)>0$ $D = 1-4 < 0$ Uvek pozitivna

-1	0	1	$+\infty$
↗		↗	

III $\frac{\cancel{x}}{1-x}\cdot\frac{1-x+x}{x^{\cancel{2}}} + 1 = \frac{x - x^2 + 1}{x(1-x)}$, $0 < x(1-x) > 0$ $= -(x^2-x-1)$

$-(x - \frac{1+\sqrt{5}}{2})(x - \frac{1-\sqrt{5}}{2})$, >0

0	1
↘	

$x - \frac{1+\sqrt{5}}{2} < 0$

IV Nema str.

3. b) $\operatorname{arctg} x$ do x^3

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$f''(x) = -\frac{1}{(1+x^2)^2} \cdot 2x$

$f'''(x) = \frac{-2(1+x^2)^2 + 2x \cdot 2 \cdot 2x\,(1+x^2)}{(1+x^2)^4}$

$\rightarrow 0 + x \cdot 1 + \frac{x^2 \cdot 0}{2} + \frac{x^3}{6} \cdot (-2) + o(x^3)$

$= x - \frac{x^3}{3} + o(x^3)$

4. $a, b \in \mathbb{R}$ $a < b$ $f, g : [a,b] \to \mathbb{R}$ nepr i dif na (a,b)

$f(a)\,g(b) = f(b)\,g(a)$ $f(x)\,g(x) \neq 0$ $x \in [a,b]$

$\exists c$ tud. $\frac{f'(c)}{f(c)} = \frac{g'(c)}{g(c)}$

$h(x) = \frac{f(x)}{g(x)}$ $h(b) - h(a) = h'(c) \cdot (b-a)$

$\frac{f(a)g(b) - f(b)g(a)}{(b-a) \cdot g(b)^2} = \frac{f'(c)g(c) - g'(c)f(c)}{g^2(c)}$

$0 = \left(f'(c)\,g(c) - g'(c)\,f(c)\right) \frac{1}{g^2(c)} \; / \cdot g(c)^2$

$f'(c)\,g(c) = g'(c)\,f(c) \quad / \; f(c) \cdot g(c)$

$\frac{f'(c)}{f(c)} = \frac{g'(c)}{g(c)}$

2021 jun 2 1 tok (Samo oni što nisu bili drugom toku)

1. v) $x_{n+1} = \sqrt[4]{1 + 4x_n} - 1 = (1 + (4x_n))^{1/4} - 1$ $\qquad \frac{1/4 \cdot -3/4}{2} = -\frac{3}{16 \cdot 2}$

$= 1 + \binom{1/4}{1} \cdot x_n \cdot 4 + \binom{1/4}{2} \cdot 16 x_n^2 + o(x_n^2) - 1$

$= x_n - 3/2\, x_n^2 + o(x_n^2)$

g) $\lim_{n \to \infty} n \cdot x_n$ tražićemo $\lim_{n \to \infty} \frac{\frac{1}{x_n}}{n}$

Štolc odmah $\lim_{n\to\infty} \frac{\frac{1}{\sqrt[4]{1+4x_n} - 1} - \frac{1}{x_n}}{1} = \lim_{n\to\infty} \frac{1}{x_n(1 - \frac{3}{2}x_n)} - \frac{1}{x_n} = \lim_{n\to\infty} \frac{\cancel{1} - \cancel{1} + 3/2\, \cancel{x_n}}{\cancel{x_n}\,(1 - 3/2\, x_n)}$

$= 3/2$

$\Rightarrow \lim_{n\to\infty} n \cdot x_n = 2/3$

2. b) $x \neq 0$ $e^{2x^2} > 1 + 2x^2 \Rightarrow e^{2x^2} - 2x^2 - 1 > 0$

$F(x) = e^{2x^2} - 2x^2 - 1$ $\quad F'(x) = 4x\,(e^{2x^2} - 1)$

	$-\infty$	0	$+\infty$
x	$-$		$+$
$e^{2x^2}-1$	$+$		$+$
	$\searrow$		$\nearrow$

$\lim_{x \to 0^-} F(x)$ $\quad e^{2x^2} - 2x^2 - 1 = \frac{e^{2x^2} - 1}{2x^2} \cdot 2x^2 - 2x^2 = 0$

$\lim_{x \to 0^+}$ ..... $= 0$

$\Big\}$ min dostiže u okolini tačke 0

$\Longrightarrow$ Svuda drugo je veća od nula.

v)

$$\lim_{x\to 0\pm} \frac{x\sin(\operatorname{arctg} x)-x^2}{e^{2x^2}-2x^2-1} \overset{LOP}{=} \lim_{x\to 0\pm} \frac{x\sin\left(x-\frac{x^3}{3}\right)-x^2}{1+2x^2+\frac{4x^4}{2}-2x^2-1}$$

$$=\lim_{x\to 0\pm} \frac{\cancel{x}\left(x-\frac{x^3}{3}-\frac{(x-\frac{x^3}{3})^3}{6}-x\right)}{2x^{\cancel{4}\,3}} = \frac{-\cancel{x^3}/3-\cancel{x^3}/6}{2\cancel{x^3}} = -\frac{1/2}{2} = \boxed{-1/4}$$

$$\Rightarrow L=-1/4$$

3. b) Pogledajmo grafik. Ima 4 rešenja 0^+; 0^- asimptote u $+\infty$ i dve kose asimptote koje idu u $+\infty$.

2021 sept0 1. tok

1. $a_{n+1}=\sqrt{\frac{1+a_n}{1-a_n}}-1 \quad n\geq 1 \quad a_1\in(-1,0)$

a) Ako $\exists \lim_{n\to\infty} a_n$: $L=\sqrt{\frac{1+L}{1-L}}-1$

	$-\infty$	-1	1	$+\infty$
$1+a_n$	$-$	$+$	$+$	
$1-a_n$	$+$	$+$	$-$	

$a_n\in[-1,1)$

$$L^2+2L+1=\frac{1+L}{1-L}$$

$$L^2-L^3+\cancel{2L}-2L^2+\cancel{1}-\cancel{L}=\cancel{1}+\cancel{L}$$

$$-L^3-L^2=0 \qquad L^2(L+1)=0$$

$$L_1=0 \quad L_1=-1$$

$$a_{n+1}-a_n=\sqrt{\frac{1+a_n}{1-a_n}}-1-a_n \;\square\; 0$$

$\in[-1,1)$

$$\sqrt{\frac{1+a_n}{1-a_n}} \;\square\; a_n+1 \quad /^2$$

$$\frac{1+a_n}{1-a_n} \;\square\; a_n^2+2a_n+1$$

$$\cancel{1+a_n} \;\square\; a_n^2+\cancel{2a_n}+\cancel{1}-a_n^3-2a_n^2-\cancel{a_n}$$

$$a_n^3+a_n^2 \;\square\; 0$$

$$\underbrace{a_n^2}_{\geq 0}\underbrace{(a_n+1)}_{\geq 0} \;\square\; 0$$

$\geq 0 \Longrightarrow a_{n+1}-a_n\geq 0 \quad a_n\uparrow$

I o B: $a_1<0$, $a_n<0 \Rightarrow a_{n+1}<0$?

$$\sqrt{\frac{1+a_n}{1-a_n}}-1<0$$

$$\frac{1+a_n}{1-a_n}<1$$

$$\cancel{1}+a_n<\cancel{1}-a_n$$

$$2a_n<0 \Rightarrow a_n<0 \ \checkmark$$

II B: $a_1>-1 \quad a_n>-1 \Rightarrow a_{n+1}>-1$?

$$\sqrt{\frac{1+a_n}{1-a_n}}-1>-1$$

$$\sqrt{\frac{1+a_n}{1-a_n}}>0 \quad \text{važi} \ \checkmark$$

$$\lim_{n\to+\infty} a_n=0$$

b) $\lim_{n\to+\infty} n\cdot a_n \qquad \lim_{n\to\infty}\frac{1}{n\cdot a_n}=\lim_{n\to\infty}\frac{1}{a_{n+1}}-\frac{1}{a_n}$

$$\sqrt{} a_{n+1}=(1+a_n)^{1/2}(1-a_n)^{-1/2}$$

$$=a_{n+1}\left(1+\tfrac{1}{2}a_n-\tfrac{1}{8}a_n^2\right)\left(1+\tfrac{1}{2}a_n+\tfrac{3}{8}a_n^2\right)$$

$$=1+\tfrac{1}{2}a_n+\tfrac{3}{8}a_n^2+\tfrac{1}{2}a_n+\cancel{\tfrac{1}{4}a_n^2}+\cancel{\tfrac{3}{16}a_n^3}$$

$$-\cancel{\tfrac{1}{8}a_n^2}-\cancel{\tfrac{1}{16}a_n^3}-\tfrac{3}{64}a_n^4$$

$$=1+a_n+\tfrac{1}{2}a_n^2-1=a_n+\tfrac{1}{2}a_n^2$$

$$=\frac{1}{a_n(1+\frac{1}{2}a_n)}-\frac{1}{a_n}=\frac{1-1-\frac{1}{2}\cancel{a_n}}{\cancel{a_n}(1+\underbrace{\tfrac{1}{2}a_n}_{0})}=-1/2$$

$$\Rightarrow \lim_{n\to+\infty} n\cdot a_n=-2$$

c) $a_1 = 1/2$ $\qquad \frac{1+a_n}{1-a_n} \geq 0$

a_n raste

$a_n \in [-1, 1)$ $\qquad a_2 = \sqrt{3} - 1 \approx 1.75 - 1 = 0.75 = 3/4$

$a_3 = \sqrt{\frac{7/4}{1/4}} - 1 > 1$ $\quad$ ↯

2. $tg\, x$ do x^3 $\qquad tg\, x = \frac{\sin x}{\cos x}$

$f'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$f''(x) = \frac{-2\cos x \sin x \cdot (-1)}{\cos^4 x}$

$f^{(3)}(x) = \frac{2(-\sin^2 x + \cos^2 x) - 2\sin 2x \cdot 4 \cdot \cos^3 x \cdot (-\sin x)}{\cos^8 x}$

$\Rightarrow tg\, x = 0 + x \cdot 1 + 0 + \frac{x^3}{3} = x + \frac{x^3}{3} + o(x^3)$

b) $\lim_{x \to 0} \frac{tg(tg(x)) - \sin(\sin x)}{tg(x) - \sin x} = \frac{x + \frac{x^3}{3} + \frac{(x+\frac{x^3}{3})^3}{3} - x + \frac{x^3}{3} + \frac{(x-\frac{x^3}{6})^3}{6}}{x + \frac{x^3}{3} - x + \frac{x^3}{6}}$

$= \frac{\cancel{x} + \frac{x^3}{3} + \frac{x^3}{3} - \cancel{x} + \frac{x^3}{6} + \frac{x^3}{6}}{x^3 \cdot 1/2} = \frac{x^3}{\frac{1}{2}x^3} = \boxed{2}$

3. $f(x) = arctg\left(\frac{x+1}{2x-3}\right) - \frac{x}{2}$

I $D_f: \mathbb{R} \setminus \{3/2\}$ $\quad$ II Teško nam je da nađemo

III Ni parna, ni neparna, ni periodična

IV $\lim_{x \to 3/2^+} arctg\left(\frac{x+1}{2x-3}\right) - x/2 = \begin{bmatrix} t = x - 3/2 \\ t \to 0^+ \\ x = 3/2 + t \end{bmatrix} = \lim_{t \to 0^+} \underbrace{arctg\left(\frac{t + 5/2}{2t}\right)}_{\pi/2} - \frac{(3/2 + t)}{2}$ $\quad$ $+\infty$

$\pi/2 - 3/4$

$\lim_{x \to 3/2^-} \ldots = -\pi/2 - 3/4$

$a = \lim_{x \to \pm\infty} \left(arctg\left(\frac{x+1}{2x-3}\right) \cdot 1/x\right) - 1/2 = -1/2 + \frac{1}{x\left(1 + \frac{x+1}{2x-3}\right)} \cdot \frac{\overbrace{2x-3-2x-2}^{-5}}{(2x-3)^2} - \frac{1}{x^2} \cdot arctg\left(\frac{x+1}{2x-3}\right)$

$= -1/2 + \lim_{x \to +\infty} \frac{-5}{x(3x-2)(2x-3)} - \frac{arctan\left(\frac{x+1}{2x-3}\right)}{x^2} = -1/2$

$b = \lim_{x \to \pm\infty} \left(arctan\left(\frac{x+1}{2x-3}\right) - \cancel{\frac{x}{2}} + \cancel{\frac{x}{2}}\right) = \lim_{x \to \pm\infty} arctan\left(\frac{x(1 + 1/x)}{x(2 - 3/x)}\right) = arctan(1/2)$

$\approx \pi/9$

$y = -\frac{x}{2} + \overline{\pi/9}$

V $\left(\arctan \frac{x+1}{2x-3} - \frac{x}{2}\right)' = \frac{1}{1+\left(\frac{x+1}{2x-3}\right)^2} \cdot \frac{2x-3-2x-2}{(2x-3)^2} - 1/2$

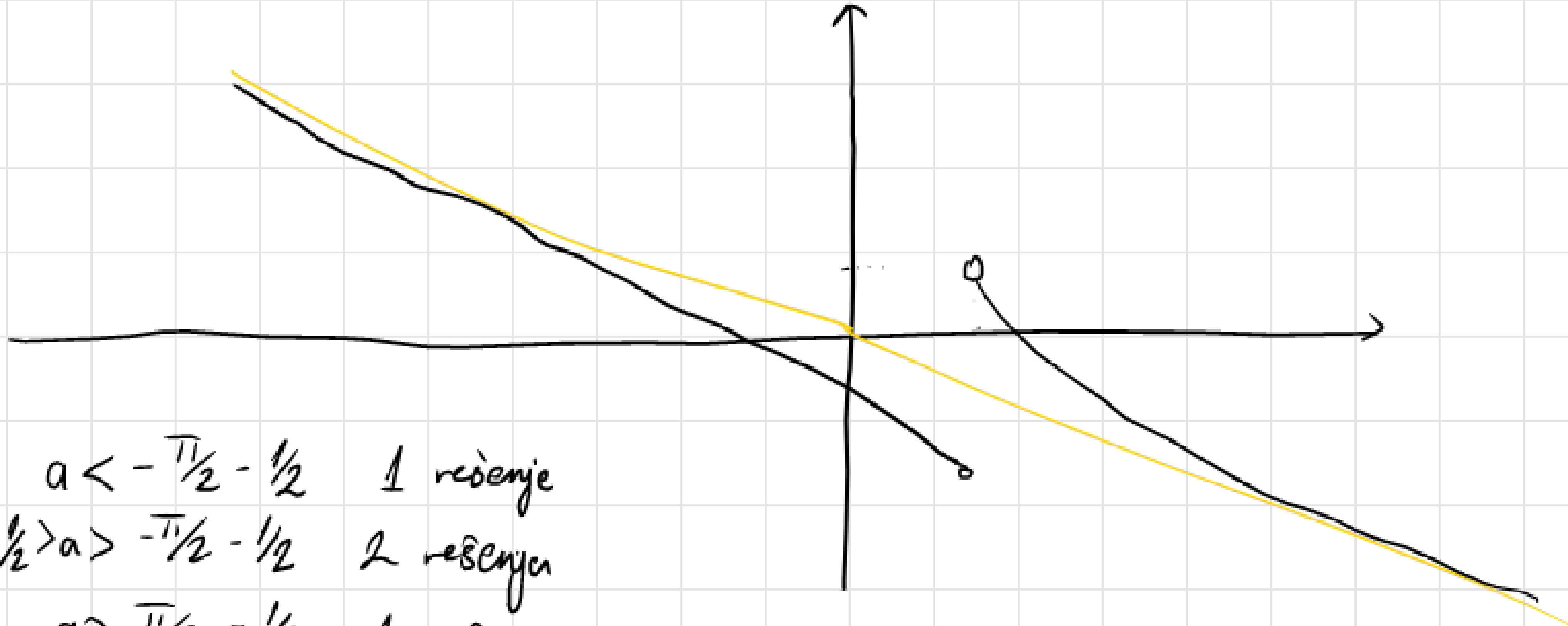
$= \frac{-5}{(2x-3)^2+(x+1)^2} - 1/2 = -\frac{\cancel{5}}{\cancel{5}x^2 - \frac{10x}{2} + \frac{10}{2}} - \frac{1}{2}$

$= \frac{-2-x^2+2x-2}{2(x^2-2x+2)} = \frac{-x^2+2x-4}{2(x^2-2x+2)} = \frac{x^2-2x+4}{-2\underbrace{(x^2-2x+2)}_{D=4-8<0 \Rightarrow poz}}$ $\to D = 4-16<0$ uvek poz

$\Rightarrow F$ opada na D_f.

VI $-1/2\left(\frac{(2x-2)\cdot(x^2-2x+2)-(2x-2)(x^2-2x+4)}{(x^2-2x+2)^2}\right) = -\frac{(x-1)(x^2-2x+2-x^2+2x-4)}{(x^2-2x+2)^2}$

$= \frac{2(x-1)}{(x^2-2x+2)^2}$ $\quad$ 1 $\quad$ + $\quad$ b) $F'(3/2) = -\frac{13}{10}$



c) $a < -\pi/2 - 1/2$ 1 rešenje

$\pi/2 - 1/2 > a > -\pi/2 - 1/2$ 2 rešenja

$a > \pi/2 - 1/2$ 1 rešenje

4. $f:(0,+\infty) \to \mathbb{R}$ dif.

$f(1)=7$ 7

$f(4)=6$ 12

$f(9)=5$ 15

$f(16)=3$ 12

Dokazati da $\exists c$ za koje važi $\frac{f(c)}{2\sqrt{c}} = -\sqrt{c}\cdot f'(c)$

$\frac{f(c)}{2\sqrt{c}} + \sqrt{c} f'(c) = 0 \iff (f(c)\cdot\sqrt{c})' = 0$

Hoćemo $\frac{f(a)-f(b)}{a-b} = f'(c) = 0$ tj $f(a)=f(b)$

$\frac{f(16)-f(4)}{16-4} = 0 = f'(c)$ pa postoji!

2021 sep 0 2. kol

2. $\lim_{x\to 0^+} (5(x+1)^{1/5} - 4)^{1/x} = e^{\wedge}\lim_{x\to 0^+} \frac{1}{x}\cdot\ln\left(\frac{5((x+1)^{1/5}-1)}{x}\cdot x + 1\right) = \frac{1}{x}\ln(x+1) = 1$

$\alpha = e$

$\lim_{x\to 0^-} \beta - (x^2+2)\sin(x^3+3) = \beta - 2\sin 3$; guess $\Rightarrow \beta = e + 2\sin 3$

2021 sep1 1. tok

1. $a_{n+1} = 4a_n - 3a_n^2 \quad a_1 < 0$

$$\left\lceil \begin{array}{l} L = 4L - 3L^2 \\ 3L^2 - 3L = 0 \\ 3L(1-L) = 0 \end{array} \right. \quad \begin{array}{l} L_1 = 0 \\ L_2 = 1 \end{array}$$

1) B: $a_1 < 0$ IH: $a_n < 0 \Rightarrow a_{n+1} < 0$?

$$a_{n+1} = \underbrace{4a_n}_{<0} \underbrace{- 3a_n^2}_{<0} \Big\} < 0 \Rightarrow a_n < 0 \ \forall n$$

2) $a_{n+1} - a_n = 4a_n - 3a_n^2 - a_n = 3\underbrace{a_n}_{<0}\underbrace{(1-a_n)}_{>0} < 0$

$\Rightarrow$ Niz opada

$\Rightarrow$ 0 i 1 ne mogu biti pa niz divergira na $-\infty$

b) $\lim\limits_{n\to\infty} \frac{n}{\sqrt{1-a_n}}$

Stolc $\lim\limits_{n\to\infty} \frac{1}{\sqrt{1-a_{n+1}} - \sqrt{1-a_n}} = \frac{\sqrt{1-a_{n+1}} + \sqrt{1-a_n}}{a_n - a_{n+1}} = \frac{\sqrt{1-4a_n+3a_n^2} + \sqrt{1-a_n}}{3a_n^2 - 3a_n}$

$$= \frac{a_n\sqrt{3 - \underbrace{\frac{4}{a_n}}_{\to 0} + \underbrace{\frac{1}{a_n^2}}_{\to 0}} + a_n\sqrt{\underbrace{\frac{1}{a_n^2}}_{\to 0} - \underbrace{\frac{1}{a_n}}_{\to 0}}}{3\cancel{a_n}(a_n - 1)} = \frac{\sqrt{3}}{a_n - 1} = 0$$

2. a) $\ln(\cos x)$ do x^4 $\quad (a^2+b^2+c^2+2ab+2ac+2cb)(a+b+c)$

$$\ln\Big(1 \underbrace{- \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + o(x^6)}_{t}\Big)$$

$$= -\frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} - \frac{1}{2}\left(-\frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}\right)^2 + \frac{1}{3}\left(-\frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}\right)^3 =$$

$$= -\frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} - \frac{x^4}{8} + \frac{x^6}{48} - \frac{x^6}{24}$$

$$= -\frac{x^2}{2} - \frac{x^4}{12} - \frac{x^6}{45} + o(x^6)$$

b) $f(x) = \begin{cases} \frac{\ln\cos x + \sqrt{1+x^2} - 1}{x^4}, & x \in (-\pi/2, 0) \cup (0, \pi/2) \\ b, & x = 0 \end{cases}$

$$-\frac{1/4 \cdot -3/2}{2} = \frac{3}{16} \qquad \frac{1/2 \cdot (1/2 - 1)(1/2 - 2)}{2 \cdot 3}$$

$$\lim_{x\to 0^\pm} \frac{\ln\cos x + (1+x^2)^{1/2} - 1}{x^4} = \frac{\cancel{-\frac{x^2}{2}} - \frac{x^4}{12} - \frac{x^6}{45} + \cancel{\frac{1}{2}x^2} + \binom{1/2}{2}x^4 + \binom{1/3}{3}x^6}{x^4}$$

$$= \frac{-\frac{\cancel{x^4}}{12} - \underbrace{\frac{x^6}{45}}_{\to 0} - \frac{\cancel{x^4}}{8} + \underbrace{\frac{3x^6}{16}}_{\to 0}}{\cancel{x^4}} = -\frac{1}{4}\left(\frac{1}{3} + \frac{1}{2}\right) = -\frac{5}{24} = b$$

$$\lim_{h\to 0} \frac{\ln\cos h + \sqrt{1+h^2} - 1 + \frac{5x^4}{24}}{h^5} = \frac{-\frac{5\cancel{x^4}}{24} + \frac{5\cancel{x^4}}{24} + x^6\left(\frac{3}{16} - \frac{1}{45}\right)}{\cancel{x^5}} = 0$$

Jeste dif. na D_f.

3. $f(x) = \ln \frac{|2x-1|-1}{2x-1} - 2x$

$|2x-1| \begin{cases} x \ge 1/2, & 2x-1 \quad \Rightarrow 2(x-1) \\ x < 1/2, & 1-2x \quad \Rightarrow -2x \end{cases}$

	$-\infty$ … 0	0 … $1/2$	$1/2$ … 1	1 … $+\infty$
$2(x-1)$			$-$	$+$
$-2x$	$+$	$-$		
$\lvert 2x-1\rvert-1$	$+$	$-$	$-$	$+$
$2x-1$	$-$	$-$	$+$	$+$
$\ln \dots$	$-$	$+$	$-$	$+$

D_f

I $D_f: (0, 1/2) \cup (1, +\infty)$

II Nema presek sa y-osom. Nule ćemo naknadno

III Nije ni parna, ni neparna. Nije periodična

IV. V.A

$$\lim_{x\to 0^+} \ln\frac{|2x-1|-1}{2x-1} - 2x = \lim_{x\to 0^+} \ln\frac{-2x}{2x-1} - 2x \overset{LOP}{=} \lim_{x\to 0^+} \frac{\cancel{2x-1}}{-2x}\cdot\frac{\cancel{-4x}+2+\cancel{2x}-2}{(2x-1)^{\cancel{2}}} - 2$$

$$= \lim_{x\to 0^+} \frac{-1}{x\,|2x-1|} - 2 = -\infty$$

$$\lim_{x\to 1/2^-} \ln\frac{2(x-1)}{2x-1} - 2x = \lim_{x\to 1/2^-} \ln\frac{2x-1-1}{2x-1} - 2x = \left[\begin{matrix} t = x - 1/2 \\ t \to 0^- \end{matrix}\right]$$

$$= \lim_{t\to 0^-} \ln\frac{2t-1}{2t} - 2t + 1 \overset{LOP}{=} \lim_{t\to 0^-} \frac{\cancel{2t}}{2t-1}\cdot\frac{4t-4t+\cancel{2}}{\cancel{4t^2}\,2t} - 2$$

$$= \lim_{t\to 0^-} \frac{1}{t(2t-1)} - 2 = +\infty$$

$$\lim_{x\to 1^+} \ln\frac{2(x-1)}{2x-1} - 2x = \left[\begin{matrix} t = x-1 \\ t \to 0^+ \end{matrix}\right] \lim_{t\to 0^+} \ln\frac{2t}{2t+1} - 2t - 2 = -\infty$$

$$a_1 = \lim_{x\to+\infty} \frac{\ln\frac{|2x-1|-1}{2x-1} - 2x}{x} = \lim_{x\to+\infty} \frac{\ln\frac{2(x-1)}{2x-1} - 2x}{x}$$

$$= \lim_{x\to+\infty} \frac{\frac{\ln\left(1-\frac{1}{2x-1}\right)\cdot\left(-\frac{1}{2x-1}\right)}{-\frac{1}{2x-1}} - 2x}{x} = \lim_{x\to+\infty} -\frac{1}{x(2x-1)} - 2 = -2$$

$$a_2 = \lim_{x\to-\infty} \frac{\ln\frac{-2x}{2x-1} - 2x}{x} = \left[\begin{matrix} t = -x \\ t \to +\infty \end{matrix}\right] = \lim_{t\to+\infty} \frac{\ln\frac{-2t}{2t+1} + 2t}{-t}$$

$$\overset{LOP}{=} \lim_{t\to+\infty} \frac{\cancel{2t+1}}{2t}\cdot\frac{\cancel{-4t}-2+\cancel{4t}}{(2t+1)^{\cancel{2}}} - 2 = \frac{-1}{t(2t+1)} - 2 = -2$$

$$b_1 = \lim_{x\to+\infty} \ln\frac{2(x-1)}{2x-1} \overset{LOP}{=} \frac{\cancel{2x-1}}{\cancel{2}(x-1)}\cdot\frac{\cancel{4x}-2\cancel{-4x}+4}{(2x-1)^{\cancel{2}}} = \frac{1}{(x-1)(2x-1)} = 0$$

$$b_2 = \lim_{x\to-\infty} \ln\frac{-2x}{2x-1} \overset{LOP}{=} \frac{\cancel{2x-1}}{-\cancel{2x}}\cdot\frac{\cancel{-4x}+2\cancel{+4x}+2}{2x-1} = 0$$

$y = -2x$

V Monotonost $\ln \frac{|2x-1|-1}{2x-1} - 2x$ $\quad (0, 1/2) \cup (1, +\infty)$

$x \in (0, 1/2)$ $\left(\ln \frac{-2x}{2x-1} - 2x\right)' = \frac{\cancel{2x-1}}{-2x} \cdot \frac{\cancel{-4x} + 2 + \cancel{4x}}{(2x-1)^{\cancel{2}}} - 2$

$= \frac{-1 - 4x^2 + 2x}{x(2x-1)} = \frac{\overbrace{-(4x^2-2x+1)}^{<0}}{\underbrace{x}_{>0}\underbrace{(2x-1)}_{<0}} \to D = 4-16 < 0$ uvek poz

$\Rightarrow$ Uvek raste na $(0, 1/2)$

$x \in (1, +\infty)$ $\left(\ln \frac{2(x-1)}{2x-1} - 2x\right)' = \frac{\cancel{2x-1}}{2(x-1)} \cdot \frac{4x-2-4x+4}{(2x-1)^{\cancel{2}}} - 2$

$= \frac{1 - 2(2x^2-3x+1)}{(x-1)(2x-1)} = \frac{-(4x^2-6x+1)}{(x-1)(2x-1)} \to x_{1/2} = \frac{6 \pm \sqrt{36-16}}{8} = \frac{6 \pm 2\sqrt{5}}{8} = \frac{3 \pm \sqrt{5}}{4}$

$= \frac{-4\left(x - \frac{3+\sqrt{5}}{4}\right)\left(x - \frac{3-\sqrt{5}}{4}\right)}{2\underbrace{(x-1)}_{>0}\underbrace{(x-1/2)}_{>0}}$

	$\frac{3-\sqrt{5}}{4}$	1	$\frac{3+\sqrt{5}}{4}$	$+\infty$
$x - \frac{3+\sqrt{5}}{4}$	$-$	$-$	$-$	$+$
$x - \frac{3-\sqrt{5}}{4}$	$-$	$+$	$+$	$+$
			$\nearrow$	$\searrow$

$x \in \left(1, \frac{3+\sqrt{5}}{4}\right)$ $f \nearrow$

$x \in \left(\frac{3+\sqrt{5}}{4}, +\infty\right)$ $f \searrow$

IV $x \in (0, 1/2)$ $-\left(\frac{(8x-2)(2x^2-x)-(4x-1)(4x^2-2x+1)}{x^2(2x-1)^2}\right) =$

$= \frac{-1}{x^2(2x-1)^2} \cdot \left(\cancel{16x^3} - \cancel{8x^2} - \cancel{4x^2} + \cancel{2x} - \cancel{16x^3} + \cancel{8x^2} - 4x + \cancel{4x^2} - \cancel{2x} + 1\right)$

$= \frac{-1}{x^2(2x-1)^2}$ $\cap$ uvek $\quad (2x^2-3x$

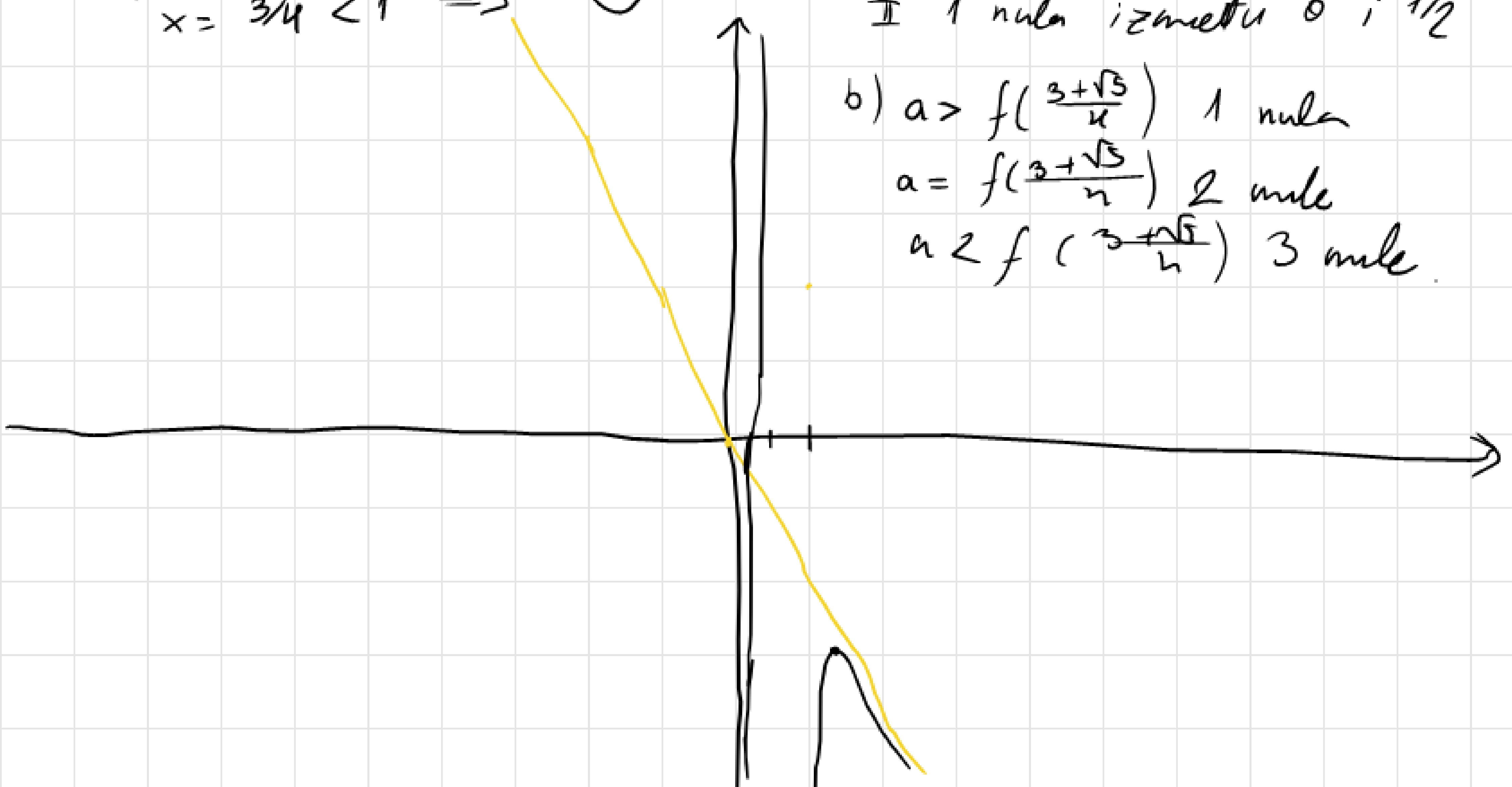
$x \in (1, +\infty)$ $-\left(\frac{(8x-6)(x-1)(2x-1)-(4x-3)(4x^2-6x+1)}{(x-1)^2(2x-1)^2}\right)$

$= \frac{-1}{(x-1)^2(2x-1)^2} \cdot \left((4x-3)\left(\cancel{4x^2} - \cancel{6x} + 2 - \cancel{4x^2} + \cancel{6x} - 1\right)\right) = \frac{4x-3}{(x-1)^2(2x-1)^2}$

$4x = 3$
$x = 3/4 < 1 \Rightarrow$ $\cup$ uvek

II 1 nula između 0 i 1/2

b) $a > f\left(\frac{3+\sqrt{5}}{4}\right)$ 1 nula
$a = f\left(\frac{3+\sqrt{5}}{4}\right)$ 2 nule
$a < f\left(\frac{3+\sqrt{5}}{4}\right)$ 3 nule



4. a) $x^3 = 6\,\mathrm{arctg}\,x + 1$ tačno 3 rešenja

$F(x) = x^3 - 6\,\mathrm{arctg}\,x - 1 \qquad F'(x) = 3x^2 - 6\cdot\frac{1}{1+x^2} = \frac{3x^2+3x^4-6}{\underbrace{1+x^2}_{>0}}$

$F'(x) = 0 = 3(x^4+x^2-2) \qquad x_{1/2}^2 = \frac{-1\pm\sqrt{1+8}}{2} \Rightarrow x^2 = \frac{3-1}{2} = 1$

$x = \pm 1$

	$-\infty$	$-\sqrt{\frac{\sqrt{13}-1}{2}}$	$\sqrt{\frac{\sqrt{13}-1}{2}}$	$+\infty$
$F'(x)$	$+$	$-$	$+$	
$F(x)$	$\nearrow$	$\searrow$	$\nearrow$	

$\lim_{x\to\pm\infty} F(x) = \pm\infty$

$F(-1) = -1 + \frac{3\pi}{2} - 1 > 0$

$F(1) = 1 - \frac{3\pi}{2} - 1 < 0$

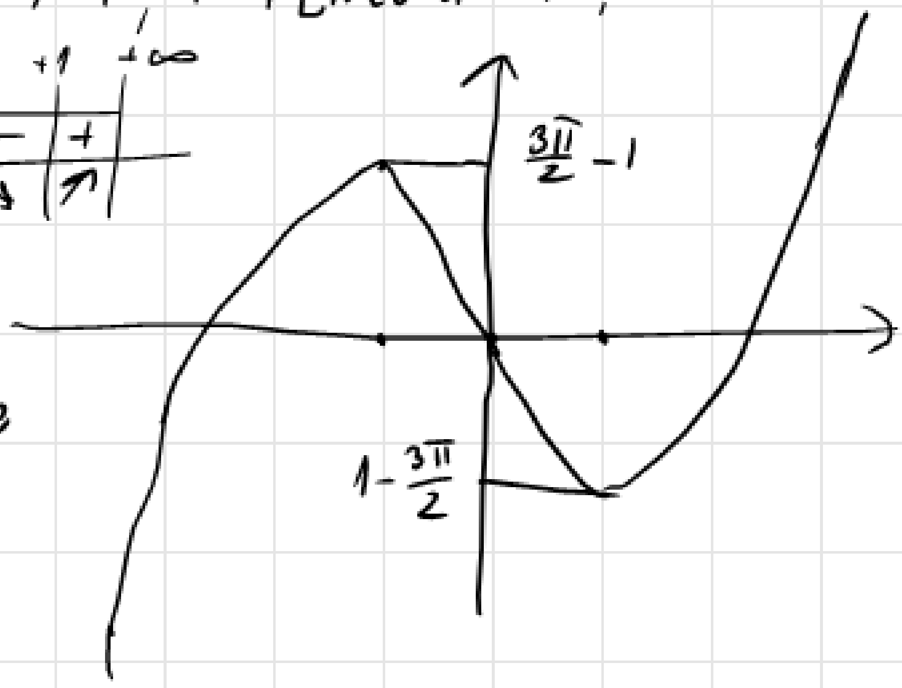
$\exists$ rešenje između $-\infty$ i -1, jedno između -1 i 1, i između 1 i $+\infty$

b) $F(x) = x^3 - 6\,\mathrm{arctg}\,x$

	$-\infty$	-1	$+1$	$+\infty$
$F'(x)$	$+$	$-$	$+$	
$F(x)$	$\nearrow$	$\searrow$	$\nearrow$	

$F(1) = 1 - \frac{3\pi}{2}$

$F(-1) = -1 + \frac{3\pi}{2}$



$\lambda \in \left(-\infty, 1-\frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}-1, +\infty\right)$ 1 rešenje

$\lambda \in \left\{\frac{3\pi}{2}-1, 1-\frac{3\pi}{2}\right\}$ 2 rešenja

$\lambda \in \left(1-\frac{3\pi}{2}, \frac{3\pi}{2}-1\right)$ 3 rešenja

2021 sept 1 drugi tok

2 a) $\lim_{x\to 0} \frac{x+1-\sin x - \sqrt{1+2x^2}}{x^2} \overset{LOP}{=} \frac{1-\cos x - \frac{1}{2}\cdot\frac{4x}{\sqrt{1+2x^2}}}{2x}$

$\overset{LOP}{=} \frac{\sin x}{2} - \frac{\sqrt{1+2x^2} - \frac{1}{2}\frac{2x^2}{\sqrt{1+2x^2}}}{1+2x^2} = 0 - \frac{1-0}{1} = -1 = b$

$\lim_{h\to 0^-} \frac{h+1-\sin h - \sqrt{1+2h^2} + h^2}{h^3} \overset{LOP}{=} \lim_{h\to 0^-} \frac{2h+1-\cos h - 2\frac{h}{\sqrt{1+2h^2}}}{3h^2}$

$\overset{LOP}{=} \frac{2+\sin h - 2\,\frac{\sqrt{1+2h^2} - \frac{2h^2}{\sqrt{1+2h^2}}}{1+2h^2}}{6h} \overset{LOP}{=} \frac{\cos h}{6h} - \frac{1}{h}3\cdot\frac{1}{(1+2h^2)^{3/2}}$

$\overset{LOP}{=} \frac{\cos h}{6} - \frac{1}{3}\cdot\left(-\frac{3}{2}\right)\cdot 2h\cdot\sqrt{1+2h^2} = \boxed{1/6} \neq \underset{0}{\overset{\shortparallel}{(-1)'}}$ Nije diferencijabilna

2021 sep 2 1. ton

1. a) $\lim\limits_{n\to+\infty} \frac{1}{n^3+n^2} + \dots + \frac{1}{n^3+n\cdot n^2}$

$\frac{\cancel{n}}{n^2(n+n)} \leq a_n \leq \frac{\cancel{n}}{n^2(n+1)}$

$\frac{1}{2n^2} \to 0 \leq a_n \leq \frac{1}{n^2+n} \to 0$

$a_n = 0$?

b) $\lim\limits_{n\to+\infty} \frac{e^{\sin 1/n}-1}{\ln(1+3/n)\cdot 3/n} = \lim\limits_{n\to+\infty} \frac{n\cancel{(e^{\sin 1/n}-1)}\sin 1/n}{3\sin 1/n} = \frac{n\cdot\sin 1/n}{3} = \frac{\sin 1/n}{3\ 1/n} = 1/3$

c) $(-1)^n\cdot(a_n - b_n) + b_n\cos\frac{2n\pi}{3}$

$\frac{10\pi}{3} - \frac{6\pi}{3} = \frac{4\pi}{3}$

$n=0 \quad -1/3 + 1/3\cdot 1 = 0$

$n=1 \quad 1/3 - 1/3\cdot 1/2 = 1/6$

$n=2 \quad -1/3 - 1/3\cdot 1/2 = -1/2$

$n=3 \quad +1/3 + 1/3\cdot 1 = 2/3$

$n=4 \quad -1/3 - 1/6 = -1/2$

$n=5 \quad 1/3 - 1/6 = 1/6$

$n=6 \Rightarrow 0$

$0, -1/2, 1/6, 2/3$

2. $f(x) = \sin\frac{x}{2}\cdot|x^2 - \pi x - 2\pi^2|$

a) $|x^2 - 2\cdot x\cdot\frac{\pi}{2} + (\frac{\pi}{2})^2 - \frac{\pi^2}{4} - 2\pi^2|$

$= |(x-\frac{\pi}{2})^2 - (\frac{3\pi}{2})^2| = |(x-2\pi)(x+\pi)|$

	$-\infty$	$-\pi$	2π	$+\infty$
$x-2\pi$	$-$	$-$	$+$	
$x+\pi$	$-$	$+$	$+$	
	$+$	$-$	$+$	

I $x\in(-\infty,-\pi)\cup(2\pi,+\infty)$ $\quad \sin\frac{x}{2}\cdot(x-2\pi)(x+\pi)$

$\lim\limits_{h\to0^-} \frac{\sin\frac{h-\pi}{2}\cdot|(h-\pi)^2 - \pi(h-\pi) - 2\pi^2| - 0}{h} = \frac{\sin\frac{h-\pi}{2}(h^2 - 2h\pi + \cancel{\pi^2} - \pi h + \cancel{\pi^2} - \cancel{2\pi^2})}{h}$

$= \lim\limits_{h\to0^-} \sin\frac{h-\pi}{2}\cdot(h-3\pi) = -3\pi$ $\qquad h(h-4) - \pi(h-4)$

$\lim\limits_{h\to0^+} \frac{\sin\frac{h+2\pi}{2}(h^2 - 4\pi h + 4\pi - \pi h + \cancel{2\pi^2} - \cancel{2\pi^2}) - 0}{h} = \frac{\sin\frac{4\pi+h}{2}(h^2 - 5h\pi + 4\pi)}{h}$

$= \frac{\sin\frac{2\pi+h}{2}(h-\pi)(h-4)}{h} \overset{LOP}{=} \sin\pi\,(2h-5\pi) = 0$

II $x\in(-\pi, 2\pi)$ $\quad \sin\frac{x}{2}(2\pi^2 + \pi x - x^2)$

$\lim\limits_{h\to0^+} 1/h\cdot\sin\frac{h-\pi}{2}(2\pi^2 + \pi\cancel{h} - \pi^2 - \cancel{h^2} + 2\pi\cancel{h} - \pi^2) = 1\cdot(3\pi - h) = -3\pi$

$\lim\limits_{h\to0^-} 1/h\cdot\sin\frac{2\pi+h}{2}(\cancel{2\pi^2} + \pi\cancel{h} + \cancel{2\pi^2} - \cancel{h^2} - 4\pi\cancel{h} - \cancel{4\pi^2}) = 0$

Jeste diferencijabilna

b) $\lim\limits_{x\to+\infty} \sin\frac{x}{2}\cdot(x^2-\pi x-2\pi^2)$ Ne postoji.

3. $f(x)=\sqrt[3]{\frac{x^2}{x+2}}$

a) $\lim\limits_{x\to+\infty}\frac{f(x)}{x^a}=1=\lim\limits_{x\to+\infty}\frac{\sqrt[3]{\frac{x^2}{x+2}}}{x^a}=\lim\limits_{x\to+\infty}\frac{\sqrt[3]{x\left(\frac{1}{1+\frac{2}{x}}\right)}}{x^a}\Rightarrow a=1/3$

b) $D_f: \mathbb{R}\setminus\{-2\}$ II $x>-2$ f-ja poz, $x<-2$ neg.

III $\boxed{x=0,\ y=0}$ IV $\lim\limits_{x\to-2^{\mp}}\sqrt[3]{\frac{x^2}{x+2}}=\sqrt[3]{\frac{4}{0^-}}=\mp\infty$

$a_1=\lim\limits_{x\to\pm\infty}\frac{\sqrt[3]{\frac{x^2}{x+2}}}{x}=\lim\limits_{x\to\pm\infty}\sqrt[3]{\frac{x^2}{x^3(x+2)}}=0$

V $\frac{1/3\cdot\frac{2x^2+4x-x^2}{(x+2)^2}}{\left(\sqrt[3]{\frac{x^2}{x+2}}\right)^2}$

(x^2+4x) 0 i -4

$b_1=\lim\limits_{x\to\pm\infty}\sqrt[3]{\frac{x^2}{x+2}}=\pm\infty$

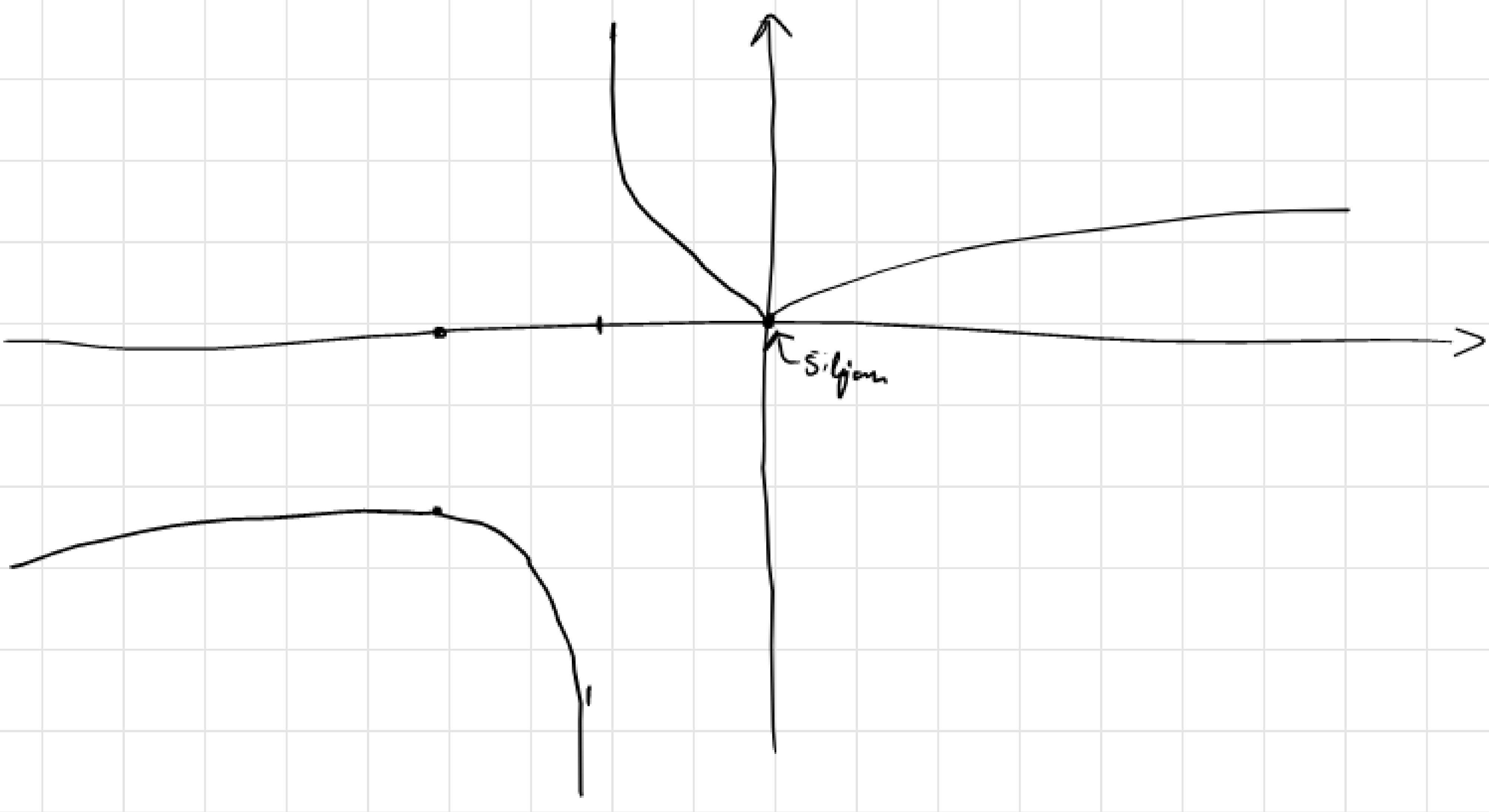
$\frac{16}{-2}=-8$

-4 + 0 +

$(-4,-2)$ $(0,0)$

VI $1/3\cdot\left(\frac{x(x+4)\cdot\sqrt[3]{(x+2)^2}}{(x+2)^2\cdot\sqrt[3]{x^4}}\right)'=\left(\frac{(x^2+4x)}{x^{4/3}\cdot(x+2)^{4/3}}\right)'$

$=\frac{(2x+4)\cdot x^{4/3}(x+2)^{4/3}-(x^2+4x)\left(4/3\cdot x^{1/3}\cdot(x+2)^{4/3}+4/3\cdot(x+2)^{1/3}\cdot x^{4/3}\right)}{x^{1/3}(x+2)^{1/3}\left((2x+4)(x^2+2x)-(x^2+4x)\cdot 4/3\,(2x+2)\right)}=\ldots=\frac{2(x^2+8x+4)}{3x^{4/3}(x+2)^{7/3}}$



4. $g: \mathbb{R} \to \mathbb{R}$ dva puta dif. poz drugi izvod svuda.

$$g(x+g'(x)) \geq g(x) \qquad g''(x) > 0 \Rightarrow g'(x) \text{ strogo rastuća}$$

$F(t) = g(x+t)$ x je fiks

I $\exists c \in (0, g'(x))$ $F'(c) = \frac{F'(g'(x)) - F(0)}{g'(x) - 0}$ $\qquad g'(x+c) = \frac{g(x+g'(x)) - g(x)}{g'(x)}$

$$0 < g'(x) \cdot g'(x+c) = g(x+g'(x)) - g(x)$$

$$\to g(x+g'(x)) - g(x) \geq 0 \Rightarrow g(x+g'(x)) \geq g(x)$$

II $\exists c \in (g'(x), 0)$

$$F'(c) = \frac{F'(0) - F'(g'(x))}{0 - g'(x)} = \frac{g(x+g'(x)) - g(x)}{g'(x)}$$

$$\underbrace{\overset{<0}{g'(x+c)}\, \overset{<0}{g'(x)}}_{>0} = g(x+g'(x)) - g(x) \Rightarrow g(x+g'(x)) - g(x) \geq 0$$

$$g(x+g'(x)) \geq g(x)$$

2022 jan 1

1. $x_1 = x > 5 \quad x_{n+1} = \frac{25 + x_n^2}{10} \quad \forall n \in \mathbb{N}$

a) $x_{n+1} - x_n = \frac{1}{10}(25 + x_n^2) - x_n = \frac{1}{10}(x_n^2 - 10x_n + 25)$

$= \frac{1}{10}(x_n - 5)^2 \geq 0 \Rightarrow$ niz raste

B: $x_1 = x > 5 \qquad x_n > 5 \Rightarrow x_{n+1} > 5$?

$x_{n+1} = \frac{25 + x_n^2}{10} \boxed{?} 5 \quad / \cdot 10$

$25 + x_n^2 \; \square \; 50$

$x_n^2 \; \square \; 25$

$x_n > 5 \Rightarrow x_n^2 > 25$ W

$L = \frac{25 + L^2}{10} \quad / \cdot 10$

$L^2 - 10L + 25 = 0 \Rightarrow (L-5)^2 = 0 \quad L = 5$

Niz divergira pa $\lim x_n \to +\infty$

b) $x_n^{(-1)^n} \cdot \left(\frac{2n+4}{2n+3}\right)^{3n} + \sin\frac{2n\pi}{3}$

$0 \quad +\infty$

$\left(1 + \frac{1}{2n+3}\right)^{2n+3} \to e^{\frac{3n}{2n+3}} = e^{3/2}$

$\left\{\pm\frac{\sqrt{3}}{2}, 0, +\infty\right\}$

$\sin\frac{2n\pi}{3}$

$0 \to 0 \qquad 3 \to 0$

$1 \to \frac{\sqrt{3}}{2} \qquad 4 \to \frac{\sqrt{3}}{2}$

$2 \to -\frac{\sqrt{3}}{2} \qquad 5 \to -\frac{\sqrt{3}}{2}$

2

$$f(x)=\begin{cases} \text{I} \quad \frac{1}{x^2}\left(e^{\sin x}-\ln(1-x)+\sqrt{1-4x}-2\cos x\right) & x<0 \\ \text{II} \quad \sqrt[3]{x^2-x}-\alpha, & x\in[0,1] \\ \text{III} \quad |x-1|\cdot\sin\frac{\pi}{x-1}, & x>1 \end{cases}$$

a) $t=-x,\ x\to 0^+$ $\lim_{x\to 0^+} \frac{1}{x^2}\left(e^{\sin(-x)}-\ln(1+x)+\sqrt{1+4x}-2\cos(-x)\right)$

$= \lim_{x\to 0^+} \frac{1}{x^2}\left(e^{-\sin x}-\ln(1+x)+\sqrt{1+4x}-2\cos x\right) \overset{LOP}{=} \frac{1}{2x}\left(e^{-\sin x}\cdot(-\cos x)-\frac{1}{1+x}+\frac{1}{2}\frac{\cdot 4}{\sqrt{1+4x}}+2\sin x\right)$

$\overset{LOP}{=} \frac{1}{2}\left(e^{-\sin x}(\cos^2 x+\sin x)+\frac{1}{(1+x)^2}+2\cdot(-\frac{1}{2})\cdot\frac{4}{\sqrt{1+4x}^3}+2\cos x\right)=\frac{1}{2}(1+1-4+2)=0$

$\lim_{x\to 0^+} \sqrt[3]{x^2-x}-\alpha=-\alpha$

$\lim_{x\to 1^-} \sqrt[3]{x^2-x}-\alpha=-\alpha$ $\alpha=0$

$t=x-1$
$t\to 0^+$

$\lim_{x\to 1^+} |x-1|\sin\frac{\pi}{x-1}=\lim_{t\to 0^+} t\cdot\sin\frac{\pi}{t}=0$

b) $\lim_{h\to 0^-} \frac{\frac{1}{h^2}\left(e^{\sin h}-\ln(1-h)+\sqrt{1-4h}-2\cos h\right)-0}{h} \overset{LOP}{=} -1\left(e^{-\sinh}\cdot(-\cos h)-\frac{1}{1+h}+\frac{1}{2}\cdot\frac{4}{\sqrt{1+4h}}+2\sin h\right)$

$=-1(-1-1+2)=0$

$\lim_{h\to 0^+} \frac{0-0}{h} \overset{LOP}{=} \frac{0}{1}=0$

$\lim_{h\to 0^+} \frac{|\cancel{h+1-1}|\sin\frac{\pi}{h+1-1}-0}{\cancel{h}}=\sin\frac{\pi}{h}$

Ne postoji

$\lim_{h\to 0^-} \frac{0}{h+1}=0$

Dif na $\mathbb{R}\setminus\{1\}$

3. $f(x)=\ln\left|\frac{x+3}{1-x}\right|$ I $D_f: \mathbb{R}\setminus\{1,-3\}$ II $\ln\left|\frac{3}{1}\right|=\ln 3$ presek sa y osom

$1=\frac{x+3}{1-x}$ /

$\frac{x+3-1+x}{1-x}=0$

$\frac{2(x+1)}{1-x}=0$ $x=-1$ nula

III Nije neparna/parna/periodična

IV $\lim_{x\to 1\pm} \ln\left|\frac{x+3}{1-x}\right|=+\infty$ $\lim_{x\to 3\pm} \ln\left|\frac{x+3}{1-x}\right|=-\infty$

$\lim_{x\to\pm\infty} \ln\left|\frac{x(1+3/x)}{x(-1+1/x)}\right|=\ln|1|=0$

V $\ln\left|\frac{3+x}{1-x}\right|$

	$-\infty$	-3	1	$+\infty$
$3+X$	$-$	$+$	$+$	
$1-X$	$+$	$+$	$-$	
	$-$	$+$	$-$	

I $x\in(-3,1)$ $=\left(\ln\frac{3+x}{1-x}\right)' = \frac{1-x}{3+x}\cdot\frac{1-x+3+x}{(1-x)^2} = \frac{4}{(3+x)(1-x)} > 0$ Raste

$(3+x)(1-x) = 3-2x-x^2$

II $x\in(-\infty,-3)\cup(1,+\infty)$

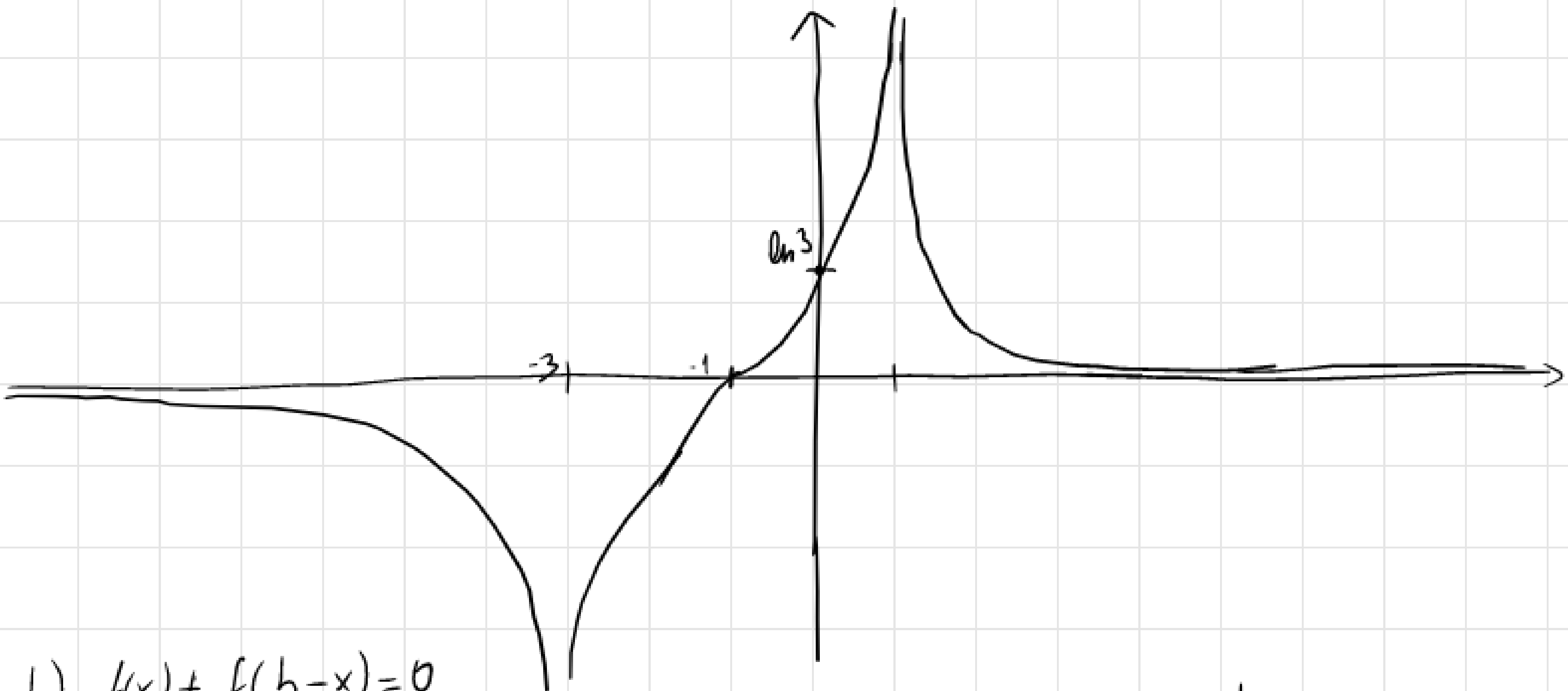
$\left(\ln\frac{3+x}{x-1}\right)' = \frac{\cancel{x-1}}{3+x}\cdot\frac{x-1-3-x}{(x-1)^{\cancel{2}}} = \frac{-4}{(3+y)(x-1)} < 0$ Opada

>0

$(3+y)(x-1) = x^2+2x-3$

VI I $4\cdot(-1)\cdot\frac{-2-2x}{(3-2x-x^2)^2} = \frac{8}{(3-2x-x^2)^2}\cdot(1+x)$ $x\in(-3,-1)$ $\cap$, $x\in(-1,1)$ $\cup$

II $-4\cdot(-1)\cdot\frac{2x+2}{(x^2+2x-3)^2} = \frac{8}{(x^2+2x-3)^2}\cdot(x+1)$ $x\in(-\infty,-3)$ $\cap$, $x\in(1,+\infty)$ $\cup$



b) $f(x)+f(b-x)=0$

$\ln\left|\frac{3+x}{1-x}\right| + \ln\left|\frac{3+b-x}{1-b+x}\right| = \ln 1 \Rightarrow \left|\frac{3+x}{1-x}\right|\cdot\left|\frac{3+b-x}{1-b+x}\right| = 1$

	$-\infty$	-3	1	$+\infty$
$3+X$	$-$	$+$	$+$	
$1-X$	$+$	$+$	$-$	
	$-$	$+$	$-$	

$\left|\frac{9+3b+xb-x^2}{(1-x)(1-b+x)}\right| = 1$

$|y|=1 \Rightarrow$ I $y=1$

$9+3b+xb-x^2-(1-b+xb-x^2)=0$ $\vee$

$8+4b=0 \Rightarrow b=-2$

II $y=-1$

$10+2b+2xb-2x^2=0$

$b(2+2x)=2(x^2-5)$

$b=\frac{x^2-5}{1+x}$

$x\neq -1$

Za svako x iz domena postoje ova dva rešenja sem za $x=-1$ gde postoji jedno

4. $x^2-3 = a\cdot e^x$

$\frac{x^2-3}{e^x} = a \rightarrow F(x) = \frac{x^2-3}{e^x}$ F je neprekidna na $\mathbb{R}$ i

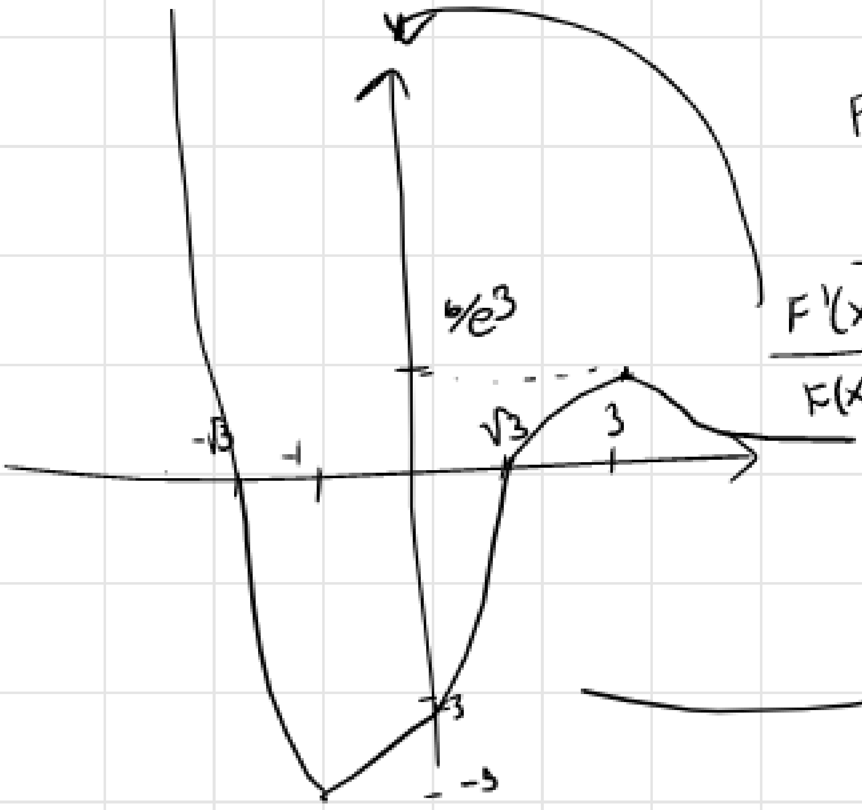
nema V.A.

$\frac{(x-\sqrt{3})(x+\sqrt{3})}{e^x} \rightarrow \pm\sqrt{3}$

$F'(x) = \frac{2x\cancel{e^x} - \cancel{e^x}(x^2-3)}{e^{2x}} = \frac{1}{e^x}(2x - x^2+3)$

$x_{1/2} = \frac{-2\pm\sqrt{4+12}}{-2}$, $\frac{-2+4}{-2} = -1$, $\frac{-2-4}{-2} = 3$

	$-\infty$	-1		3	$+\infty$
$F'(x)$	$-$		$+$		$-$
$F(x)$	$\searrow$		$\nearrow$		$\searrow$
		$-2e \simeq -5$		$\frac{6}{e^3}$	



$a = -2e$ 1 rešenje

$a \in (-2e, 0]$ 2 rešenja

$a \in (0, 6/e^3)$ 3 rešenja

$a = 6/e^3$ 2 rešenja

$a > 6/e^3$ 1 rešenje

2022 jun 1

1. $x_1 = 2022$; $x_{n+1} = \frac{x_n}{1+\sqrt{1+x_n^2}}$

Ako $\exists L$; $L = \frac{L}{1+\sqrt{1+L^2}}$ $\quad \cancel{1} + L\sqrt{1+L^2} = \cancel{1}$ $\quad L\sqrt{1+L^2} = 0 \Rightarrow L = 0$

$\frac{x_{n+1}}{x_n} = \frac{\frac{\cancel{x_n}}{1+\sqrt{1+x_n^2}}}{\frac{\cancel{x_n}}{1}} = \frac{1}{1+\sqrt{1+x_n^2}} < 1 \Rightarrow x_n > x_{n+1}$ tj. niz opada

B: $x_1 = 2022 > 0$ $\quad x_n > 0 \Rightarrow x_{n+1} > 0$?

$\frac{x_n > 0}{1+\sqrt{1+x_n^2} > 0} > 0$ ✓ $\Longrightarrow \lim\limits_{n\to+\infty} x_n = 0$

b) $\lim\limits_{n\to+\infty} n x_n \Rightarrow$ Tražimo $\frac{1}{n x_n} = \frac{\frac{1}{x_n}}{\frac{n}{1}}$ Štolc odmah nmg

$\lim\limits_{x\to+\infty} \frac{1+\sqrt{1+x_n^2}}{x_n} + \frac{1}{x_n} = \frac{2+\sqrt{1+x_n^2}}{x_n} = \frac{\frac{(1+x_n)^{\frac{1}{2}}-1\cdot x_n^2}{x_n^2}+1+2}{x_n} = \frac{x_n^2+6}{2x_n} = 0$?

2. a) $\operatorname{arctg} x =^{0} \frac{1}{1+x^2} =^{1} \frac{-2x}{(1+x^2)^2} =^{2} -2\,\frac{1+2x^2+x^4-2(1+x^2)\cdot 2x^2}{(1+x^2)^4}$ ^{3}

$0 + x + 0 - 2\cdot\frac{1}{3!}x^3 = x - \frac{x^3}{3} + o(x^3)$

b) $x<0$ $\quad 8\cos\left(\frac{x}{2}\right) - 8 + x^2 > 0 \Rightarrow F'(x) = -8\sin\left(\frac{x}{2}\right)\cdot\frac{1}{2} + 2x = 2\left(x - 2\sin\left(\frac{x}{2}\right)\right)$

$x - 2\sin\frac{x}{2} = 0$

$x = 2\sin\frac{x}{2}$

$x \in [-2, 2] \Rightarrow$ Samo 0

$x > 0 \Rightarrow \nearrow$ *

$x < 0 \Rightarrow \searrow$

$F(0) = 8 - 8 + 0 = 0$

F-ja nema V.A. i $D_f: \mathbb{R}$, neprekidna je iz * vidimo da na $(-\infty, 0]$ minimum dostiže u $(0,0)$ a kako $x<0$ f-ja $\searrow \Rightarrow$ sve vrednosti $x \in (-\infty, 0)$ su veće od 0

$\left[\lim\limits_{x\to-\infty} 8\cos\left(\frac{x}{2}\right) - 8 + x^2 = \lim\limits_{x\to+\infty} 8\cos\left(\frac{x}{2}\right) - 8 + x^2 = +\infty^{*}\right]$

c) $f(x) = \begin{cases} \dfrac{x\cdot\sin(\operatorname{arctg}(3x)) - 3x^2}{8\cos(\frac{x}{2}) - 8 + x^2}, & x<0 \\ A, & x=0 \\ 1 - B + \dfrac{x^{2x}-1}{(\ln(3x))^2} & x>0 \end{cases}$

$$\lim_{x\to 0^-} \frac{x\cdot\sin\left(3x - \frac{^{9}\cancel{27}x^3}{\cancel{3}}\right) - 3x^2}{8\left(1-\left(\frac{x}{2}\right)^2\cdot\frac{1}{2} + \left(\frac{x}{2}\right)^4\cdot\frac{1}{4!}\right) - 8 + x^2} = \frac{x\left(3x - 9x^3 - \frac{1}{6}(3x-9x^3)^3\right) - 3x^2}{\frac{1}{3}\cdot\frac{x^4}{16}}$$

$$= \frac{-18x^4 - \frac{9}{2}x^4}{\frac{x^4}{24}} = \frac{-27x^4}{\frac{x^4}{24}} = -648 \qquad = A = -648$$

$$\lim_{x\to 0+} \frac{e^{(2x)\cdot\ln x} - 1}{(\ln(3x))^2} = \frac{0-1}{+\infty} = 0 \implies 1 - B = -648 \implies B = 649$$

3. $f(x) = \operatorname{arctg}\left(\frac{x+1}{x-1}\right) - |x|$

I $x \neq 1 \Rightarrow D_f: \mathbb{R}\setminus\{1\}$ II $\operatorname{artg} -1$, $-\pi/4$ je presek sa y, nule kasnije

III Nije parna, neparna ni periodična

IV $\lim\limits_{x\to 1+} \operatorname{arctg}\left(\frac{x+1}{x-1}\right) - x = \operatorname{arctg}(\infty+) - 1 = \pi/2 - 1$

$-\infty - 1 = -\pi/2 - 1$

$$a = \lim_{x\to+\infty} \frac{\operatorname{arctg}\left(\frac{x+1}{x-1}\right) - x}{x} \overset{LOP}{=} \frac{1}{1+\left(\frac{x+1}{x-1}\right)^2}\cdot\frac{x-1-x-1}{(x-1)^2} - 1 =$$

$$\frac{1(x-1)^2}{x^2-2x+1+x^2+2x+1}\cdot\frac{-2}{(x-1)^2} - 1 = \underbrace{\frac{-2}{2(x^2+1)}}_{0} - 1 = -1$$

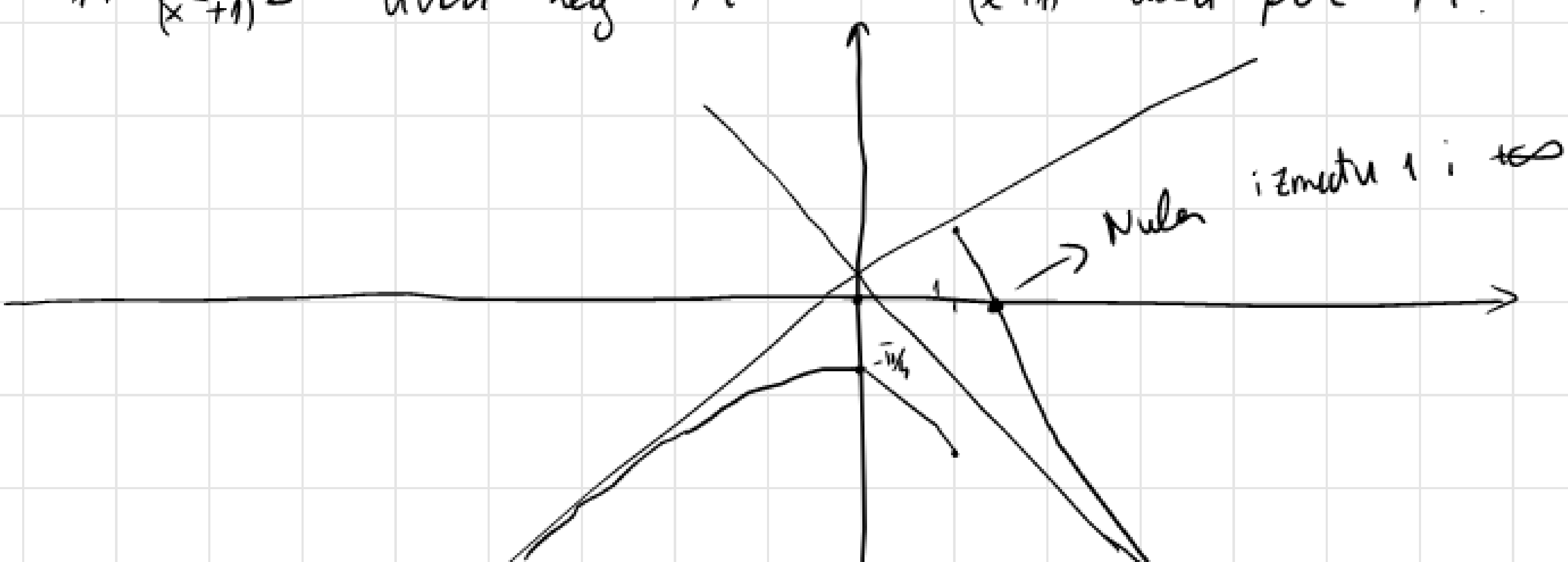
$$\lim_{x\to-\infty} \left. \begin{matrix} t = -x \\ t\to+\infty \end{matrix} \right] \overset{LOP}{=} \frac{1}{1+\left(\frac{1-x}{-x-1}\right)^2}\cdot\frac{x+1+1-x}{(-x-1)^2} + 1 = \underbrace{\frac{1}{(x^2+1)}}_{0} + 1 = 1$$

$$b = \lim_{x\to+\infty} \operatorname{arctg}\frac{x+1}{x-1} = \operatorname{arctg}\left(1 + \overset{\to 0}{\frac{2}{x-1}}\right) = \pi/4 \quad \text{isto za } \lim_{x\to-\infty}$$

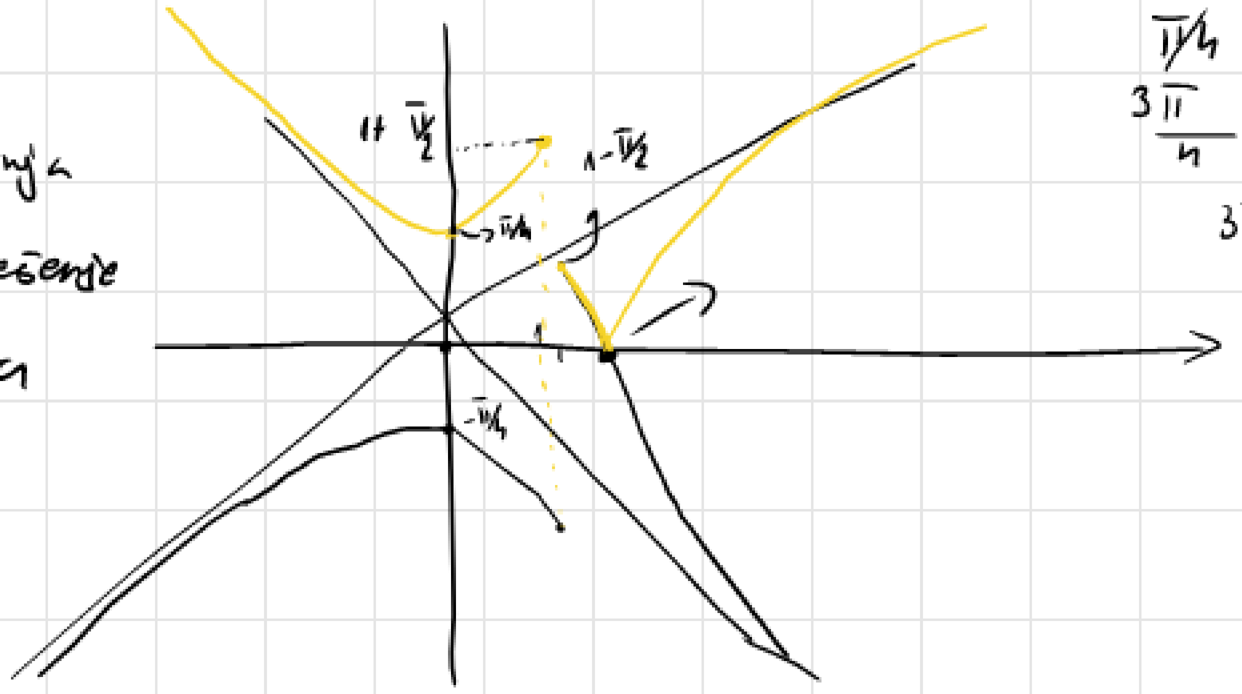
V $x>0$ $\quad -\frac{1}{(x^2+1)} - 1 =$ Uvek neg ↘

$x<0$ $\quad \frac{1}{x^2+1} + 1$ uvek poz ↗

VI $-1\cdot\frac{2x}{(x^2+1)^2}$ uvek neg ∩ $\quad -1\cdot\frac{2x}{(x^2+1)^2}$ uvek poz ∩.



$b > 1+\pi/2$ 2 rešenja
$b = 1+\pi/2$ 3 rešenja
$b \in (\pi/4, 1+\pi/2)$ 3 rešenja
$b = \pi/4$ 2 rešenje
$b \in (\pi/4, 1-\pi/2)$ 1 rešenje
$b \in (0, 1-\pi/2)$ 2 rešenja
$b = 0$ 1 rešenje
$b < 0$ 0 rešenja



$\pi/4 \boxed{?} 1-\pi/2$
$\frac{3\pi}{4} \boxed{?} 1$ $/ \cdot 4$
$3\pi \boxed{?}$

4. $f: \mathbb{R} \to (0, +\infty)$ dif. $f(1) = 1$

$g(x) = \ln(f(x))$ $\qquad g'(x) = \frac{f'(x)}{f(x)}$

$g(1) = 0$ $\qquad \exists c \in (1, 2022)$ tkd. $\frac{f'(c)}{f(c)} = \frac{\ln(f(2022)) - 0}{2021}$

$g(2022) = \ln(f(2022))$ $\qquad 2021 \frac{f'(c)}{f(c)} = \ln(f(2022))$ sve na e

$e^{2021 \frac{f'(c)}{f(c)}} = f(2022)$

2022 jun 2

1. $x_0 > 0$ $\qquad x_{n+1} = \frac{x_n^2}{\sqrt{e^{x_n^2} - 1}}$

B: $x_0 > 0$ IH: $x_n > 0 \Rightarrow x_{n+1} > 0$?

$x_{n+1} = \frac{x_n^2}{\sqrt{e^{x_n^2} - 1}} > 0$ ✓

Ako $\exists L = \frac{L^2}{\sqrt{e^{L^2} - 1}}$

$e^{L^2} - 1 = L^2$

$(e^L - 1)(e^L + 1) = L^2$

$\boxed{L = 0}$

$x_n \left(\frac{x_n}{\sqrt{e^{x_n^2} - 1}} - 1 \right) =$

$x_n \,\square\, \sqrt{e^{x^2} - 1}$ $/^2 \Rightarrow x_n^2 - e^{x^2} + 1$

$f(0) = 0$ $\quad f'(x) = 2x - e^{x_n^2} \cdot 2x = 2x(1 - e^{x^2})$ opada

$\Rightarrow x_n < \sqrt{e^{x_n^2} - 1}$ na $(0, +\infty)$ $\Rightarrow$ NIZ opada

$\Rightarrow \lim_{n \to +\infty} x_n = 0$

b) $x_n \sim \frac{c}{\sqrt{n}} \Rightarrow \lim \frac{x_n \cdot \sqrt{n}}{c} = 1 \Rightarrow x_n\sqrt{n} = c$ $\qquad$ Tražimo $\frac{\frac{1}{x_n}}{\sqrt{n}} = k$

$\Rightarrow \lim (\sqrt{n+1} + \sqrt{n}) \cdot \frac{x_n - x_{n+1}}{x_{n+1} x_n} = (\sqrt{n+1} + \sqrt{n}) \cdot \frac{x_n \left(1 - \frac{x_n}{\sqrt{e^{x_n^2} - 1}}\right)}{\frac{x_n^3}{\sqrt{e^{x_n^2} - 1}}} =$

$= (\sqrt{n+1} + \sqrt{n}) \left(\frac{1}{x_n^2} \left(\sqrt{e^{x_n^2} - 1} - x_n \right) \right)$

$= (\sqrt{n+1} + \sqrt{n}) \frac{1}{x_n^2} \left(\sqrt{x_n^2 + \frac{x_n^4}{2}} - x_n \right) = \frac{1}{x_n} \left(\sqrt{1 + \frac{x_n^2}{2}} - 1 \right) (\sqrt{n+1} + \sqrt{n})$

$\frac{1}{x_n} \left(1 + \frac{x_n^2}{4} - 1 \right) (\sqrt{n+1} + \sqrt{n}) = \frac{x_n (\sqrt{n+1} + \sqrt{n})}{4} = \frac{2c}{4} = \frac{c}{2} \Rightarrow$

$k = \frac{c}{2}$ $/ \cdot 2c$
$2 = c^2$
$c = \pm\sqrt{2}$
$c > 0$
$\Rightarrow \boxed{c = +\sqrt{2}}$

2. a) $\arcsin x = \frac{1}{\sqrt{1-x^2}} = \frac{x}{\sqrt{1-x^2}^3} = \frac{\sqrt{1-x^2}^3 - \frac{3}{2}x\cdot(-2x)\cdot\sqrt{1-x^2}}{\sqrt{1-x^2}^5}$

$0 + x + \frac{x^2\cdot 0}{2} + \frac{1\, x^3}{6} = x + \frac{x^3}{6} + o(x^3)$

b) $f(x) = \begin{cases} \frac{1}{x^3}\left(e^{\arcsin x} - (1+x)^{\frac{2x+2}{x+2}}\right), & x\in(-1,0) \\ \alpha & x=0 \\ |x-\frac{1}{3}|\cos(3\pi x) & x>0 \end{cases}$

$\frac{1}{x+2} = \frac{A}{x+1} + \frac{B}{x}$
$Ax + Bx + B$
$B = 1,\ A = -1$

I $e^{\arcsin x} = e^{\wedge}\left(x + \frac{x^3}{6} + o(x^3)\right) = 1 + x + \frac{x^3}{6} + \frac{1}{2}\left(x + \frac{x^3}{6}\right)^2$

$= 1 + x + \frac{x^3}{6} + \frac{x^2}{2} + \frac{x^4}{6} + \frac{x^6}{72}$

II $(1+x)^{\frac{2(x+1)}{x+2}} = e^{\wedge}\, 2(x+1)\frac{1}{x+2}\ln(1+x) = e^{\wedge}(x+1)\left(1+\frac{x}{2}\right)^{-1}\ln(1+x)$

$= e^{\wedge}\left(x+1\right)\left(1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \frac{x^4}{4}\right)\left(x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24}\right)$

$e^{\wedge}\left(x+1\right)\left(x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^2}{2} + \frac{x^3}{4} + \frac{x^3}{4}\right)$

$e^{\wedge}\left(x - x^2 + \frac{2x^3}{3} + x^2 - x^3\right) = 1 + x - \frac{x^3}{3} + \frac{\left(x-\frac{x^3}{3}\right)^2}{2} + \frac{\left(x - \frac{x^3}{3}\right)^3}{6}$

$= 1 + x - \frac{x^3}{3} + \frac{x^2}{2} + \frac{x^3}{6} = 1 + x + \frac{x^2}{2} - \frac{x^3}{6}$

I i II
$\Rightarrow \lim_{x\to 0^-}$ od ovog $\frac{1}{x^3}\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} - 1 - x - \frac{x^2}{2} + \frac{x^3}{6}\right) = \frac{1}{x^3}\cdot\frac{x^3}{3} = \frac{1}{3}$

$\lim_{x\to 0^+} |x - \frac{1}{3}|\cos(3\pi x) = \frac{1}{3}\cos 0 = \frac{1}{3}$ $\alpha = \frac{1}{3}$

c) $|x - \frac{1}{3}|\cos(3\pi x)$

$\lim_{h\to 0^{\pm}} \frac{|h + \frac{1}{3} - \frac{1}{3}|\cos(3\pi h + \pi)}{h} = \frac{h\cos\pi}{h} = -1$ Nije

$\frac{-h\cos\pi}{h} = 1$

3. $f(x) = \frac{2(x-1)^2}{2x-1}\cdot e^{\frac{1}{x-1}}$ I $D_f: \mathbb{R}\setminus\{\frac{1}{2}, 1\}$

II $f(x) > 0 \quad x > \frac{1}{2}$
$f(x) < 0 \quad x < \frac{1}{2}$

III Nema nula, $-\frac{2}{e}$ je presek sa y-osom

IV Nije neparna, periodična ni parna

V $\lim_{x\to \frac{1}{2}^{\mp}} \frac{2(\frac{1}{2})^2}{0^{\mp}}\cdot e^{2} = \mp\infty$

$\left[\begin{array}{l} t = -(x-1) \\ t\to 0^+ \end{array}\right] = \lim_{t\to 0^+} \frac{2t^2}{1-2t}\cdot e^{\frac{-1}{t}} = 0$

$\left[\begin{array}{l} t = x-1 \\ t\to 0^+ \end{array}\right] \lim_{t\to 0^+} \frac{2t^2}{2t+1}\cdot e^{\frac{1}{t}} = +\infty$

$a = \frac{2x^2 - 4x + 2}{2x^2 - x}\cdot e^{\frac{1}{x-1}} = 1$

$b = \frac{2x^2-4x+2}{2x-1}\cdot e^{\frac{1}{x-1}} - x = \left(\frac{\left(\frac{1}{x-1}\right)e^{\frac{1}{x-1}} - 1}{\frac{1}{x-1}} + 1\right) = \frac{2x^3 - 4x^2 + 2x - x(2x^2 - 3x + 1)}{(x-1)(2x-1)} = \frac{-x^2 + x}{2x^2 - 3x + 1} = -\frac{1}{2}$

K.A. $y = x - \frac{1}{2}$

V $\frac{2\cdot(x-1)^2}{2x-1}\cdot e^{\frac{1}{x-1}} \Rightarrow 2\cdot \frac{e^{\frac{1}{x-1}}\left(2(x-1) - (x-1)^2\cdot\frac{1}{(x-1)^2}\right)(2x-1) - 2(x-1)^2\cdot e^{\frac{1}{x-1}}}{(2x-1)^2}$

$= 2\cdot\frac{e^{\frac{1}{x-1}}}{(2x-1)^2}\cdot\left((2x-3)(2x-1) - 2(x-1)^2\right) = 2\cdot\frac{e^{\frac{1}{x-1}}}{(2x-1)^2}\cdot(4x^2-8x+3-2x^2+4x-2)$

$= \underbrace{\cdots}_{>0}\cdot(2x^2-4x+1) \Rightarrow x = \frac{4\pm\sqrt{16-8}}{4} = 1\pm\frac{\sqrt2}{2}$ $\quad f\left(1-\frac{\sqrt2}{2}\right) = \frac{1}{1-\sqrt2}\cdot e^{-\sqrt2}$

$x\in\left(-\infty,1-\frac{\sqrt2}{2}\right)\cup\left(1+\frac{\sqrt2}{2},+\infty\right) \nearrow$ $\quad f\left(1+\frac{\sqrt2}{2}\right)\ \frac{1}{1+\sqrt2}\cdot e^{\sqrt2}$

$x\in\left(1-\frac{\sqrt2}{2},\frac12\right)\cup\left(\frac12,1\right)\cup\left(1,1+\frac{\sqrt2}{2}\right) \searrow$

VI $2\cdot\frac{e^{\frac{1}{x-1}}}{(2x-1)^2}\cdot(2x^2-4x+1) \Rightarrow 2\cdot\frac{e^{\frac{1}{x-1}}\left(\left(4x-4-\frac{1}{(x-1)^2}\cdot(2x^2-4x+1)\right)\right)(2x-1)^2 - 2\cdot2\cdot(2x-1)(2x^2-4x+1)}{(2x-1)^4}$

$= \frac{2e^{\frac{1}{x-1}}}{(2x-1)^4}\cdot\frac{\left(\frac{(4x-4)(x^2-2x+1)-2x^2-4x+1}{(x-1)^2}\right)\cdot(2x-1) - 4(2x^2-4x+1)}{(2x-1)^3}$

$= \frac{2e^{\frac{1}{x-1}}}{(2x-1)^4}\cdot \ldots = \frac{2(2x^2+2x+1)^{>0}}{\underbrace{(x-1)^2}_{>0}(2x-1)^3}\cdot \underbrace{e^{\frac{1}{x-1}}}_{>0}$ $\quad x<1/2\ \cap$ $\quad x>1/2\ \cup$

4. Dif na $\mathbb{R}$ $\quad a>0\ f(a)=0\ :\ c>0\ \ f(c)+2cf'(c)=0$

$F(x)=xf(x^2)\quad F'(x)=f(x^2)+2x^2f'(x^2)$

$\exists c_1\in(0,\sqrt a)$ tud. $\frac{\sqrt a\cdot f(a)-f(0)\cdot 0}{\sqrt a-0} = f(c_1^2)+2c_1^2\cdot f'(c_1^2) = 0 \Rightarrow c=c_1^2$ ■

2022 sep1

1. $x_1 = 5 \quad x_{n+1} = \ln(e^{x_n} - x_n)$

$L = \ln(e^L - L) \quad L = 0$

$x_{n+1} - x_n = \ln(e^{x_n} - x_n) - \ln e^{x_n} = \ln\left(1 - \frac{x_n}{e^{x_n}}\right) < 0$ Opada

B: $x_1 > 0 \quad IH: x_n > 0 \Rightarrow x_{n+1} > 0$? $\ln(e^{x_n} - x_n) > \ln 1$

$e^{x_n} - x_n > 1 - 0 > \ln 1 = 0$

$\Rightarrow \lim\limits_{n \to +\infty} x_n = 0$

b) $a_n = (-1)^n \sin\left(\frac{2n\pi}{3}\right) - x_n \cos(\ln(2022^n + 1)) \quad \in [-1, 1]$

$\cdot x_n = 0$

$1, -1$

$0 = 0, \quad 1 = \frac{\sqrt{3}}{2} \quad 2 \Rightarrow -\frac{\sqrt{3}}{2}$

$3 = 0, \quad 4 = \frac{\sqrt{3}}{2} \quad 5 \Rightarrow -\frac{\sqrt{3}}{2} \qquad \left\{0, \pm\frac{\sqrt{3}}{2}\right\}$

2.

$$f(x) = \begin{cases} \dfrac{1 - \cos(1 - \cos x)}{x^4}, & x < 0 \\ A, & x = 0 \\ \dfrac{(1 + \cos x)^{(1 - \cos x)} - 1}{(\ln(1 + x))^2} + B, & x > 0 \end{cases}$$

$$\lim_{x \to 0-} \frac{1 - \cos(1 - \cos x)}{x^4} = \frac{1 - \cos\left(1 - \left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right)\right)}{x^4} = \frac{1 - \cos\left(\frac{x^2}{2} - \frac{x^4}{24} + o(x^4)\right)}{x^4}$$

$$= \frac{1 - \left(1 - \frac{1}{2}\left(\frac{x^2}{2} - \frac{x^4}{24}\right)^2\right)}{x^4} = \frac{\frac{\cancel{x^4}}{4} - \frac{x^{6}}{24} + \frac{x^{8}}{24^2}}{2\cancel{x^4}} = 1/8$$

$$\lim_{x \to 0+} \frac{e^{(1 - \cos x)\ln(1 + \cos x)} - 1}{\frac{(\ln(1 + x))^2}{x^2} \cdot x^2} = \frac{(1 - \cos x)\ln(1 + \cos x)}{-\|-} \cdot \frac{\left(e^{(1 - \cos x)(\ln(1 + \cos x)} - 1\right)}{x^2}$$

$\frac{(\ln(1+x))^2}{x^2} \to 1$

$$= \frac{\left(\frac{x^2}{2}\right)\ln(1 + \cos x)}{x^2} = \frac{\ln 2}{2} \Bigg] \Rightarrow \frac{\ln 2}{2} + B = 1/8 \quad / \cdot 8$$

$4\ln 2 + 8B = 1 \Rightarrow B = \frac{1 - 4\ln 2}{8}$

$A = 1/8$

3. $f(x) = \left(x + \frac{1}{2x}\right) \cdot e^{\frac{1}{3x}}$ I $D_f: \mathbb{R} \setminus \{0\}$ II $\begin{matrix} x > 0 \\ x < 0 \end{matrix} \quad \begin{matrix} f(x) > 0 \\ f(x) < 0 \end{matrix}$ III Nema preseca sa y-osom ni nule.

Nije ni par/ni nep/per IV $\lim\limits_{x \to 0\mp} \left(x + \frac{1}{2x}\right) \cdot e^{\frac{1}{3x}} = \begin{matrix} 0 \\ +\infty \end{matrix}$

$a = \lim\limits_{x \to \pm\infty} \left(1 + \frac{1}{2x^2}\right) e^{\frac{1}{3x}} = 1$

$b = \lim\limits_{x \to \pm\infty} x\left(1 + \frac{1}{2x^2}\right) \cdot e^{\frac{1}{3x}} - x = \left(x + \frac{1}{2x}\right)\left(1 + \frac{1}{3x} + \frac{1}{18x^2}\right) - x =$

$= x + \frac{1}{2x} + \frac{1}{3} + \frac{1}{6x^2} + \frac{1}{18x} + \frac{1}{36x^3} - x = 1/3$

V $\left(\left(x+\frac{1}{2x}\right)\cdot e^{\frac{1}{3x}}\right)' = e^{\frac{1}{3x}}\left(1-\frac{1}{2x^2}-\frac{1}{3x^2}\left(x+\frac{1}{2x}\right)\right)=$

$e^{\frac{1}{3x}}\left(1-\frac{1}{3x}-\frac{1}{2x^2}-\frac{1}{6x^3}\right)=\frac{e^{\frac{1}{3x}}}{6x^3}\left(6x^3-2x^2-3x-1\right)$

$\Rightarrow \frac{1}{x}(x-1)(6x^2+4x+1)$ $\quad D=16-24 \Rightarrow$ uvek poz

	$-\infty$	0	1 $+\infty$
x	$-$	$+$	$+$
$x-1$	$-$	$-$	$+$
	$\nearrow$	$\searrow$	$\nearrow$

$F(1)=\left(3/2\right)\cdot e^{1/3}$ minimum lokalni

VI $e^{\frac{1}{3x}}\left(1-\frac{1}{3x}-\frac{1}{2x^2}-\frac{1}{6x^3}\right) \Rightarrow e^{\frac{1}{3x}}\left(\left(1-\frac{1}{3x}-\frac{1}{2x^2}-\frac{1}{6x^3}\right)\cdot\frac{-1}{3x^2}+\frac{1}{3x^2}+\frac{1}{x^3}+\frac{1}{2x^4}\right)$

$\Rightarrow e^{\frac{1}{3x}}\left(\frac{1}{x^3}+\frac{1}{9x^3}+\frac{1}{6x^4}+\frac{1}{18x^5}+\frac{1}{2x^4}\right)=$

$e^{\frac{1}{3x}}\cdot\frac{1}{18x^5}\left(18x^2+2x^2+3x+9x+1\right)=\left(20x^2+12x+1\right)\cdot\frac{e^{\frac{1}{3x}}}{18x^5}$

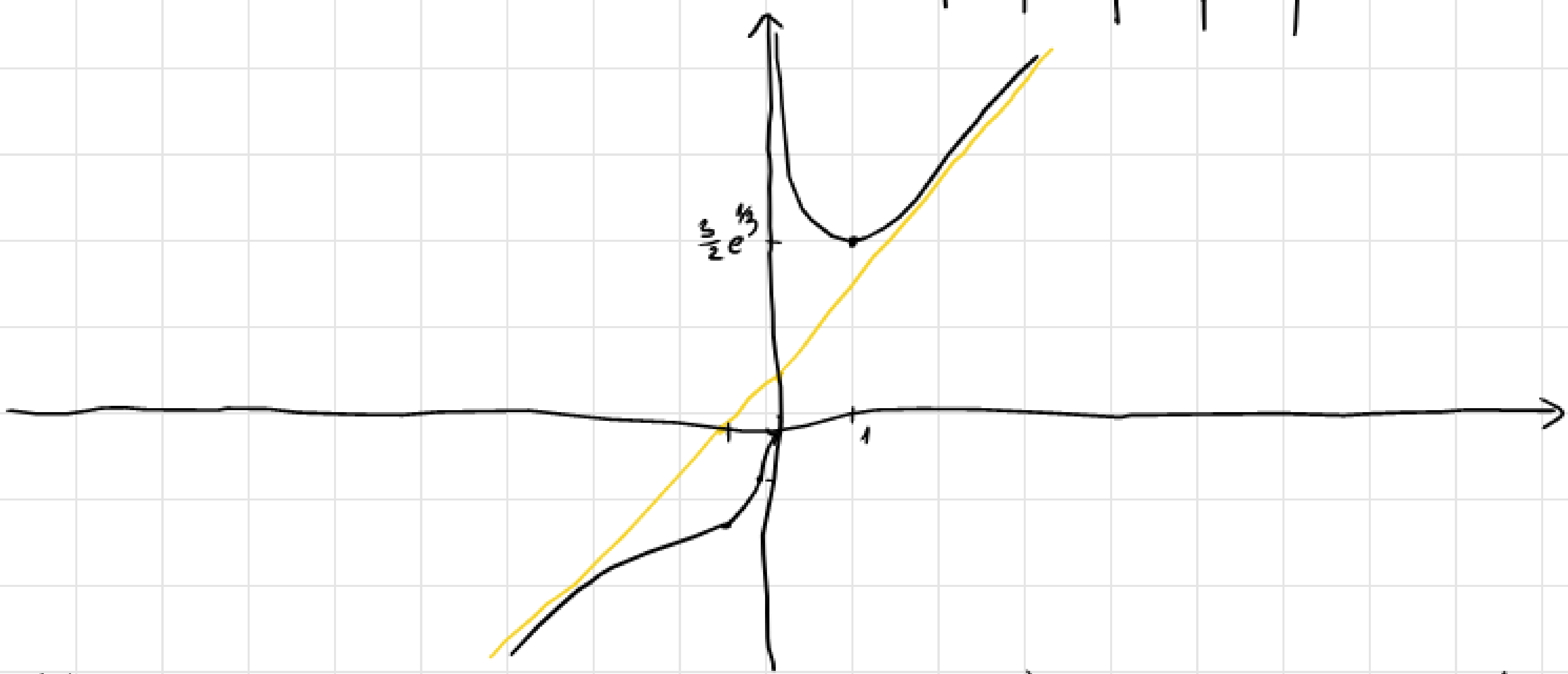
$=\left(20x^2+2\sqrt{5}x\cdot 2\cdot\frac{3}{\sqrt{5}}-\frac{4}{5}+\frac{9}{5}\right)$ $\left(2\sqrt{5}x+\frac{3}{\sqrt{5}}\right)^2-\left(\frac{2}{\sqrt{5}}\right)^2=\left(2\sqrt{5}x+\frac{1}{\sqrt{5}}\right)\left(2\sqrt{5}x+\sqrt{5}\right)$

$=\boxed{(2x+1)(10x+1)\cdot\frac{1}{x}}$

	$-\infty$	$-1/2$	$-1/10$	0 $+\infty$
x	$-$	$-$	$-$	$+$
$2x+1$	$-$	$+$	$+$	$+$
$10x+1$	$-$	$-$	$+$	$+$
$f(x)$	$\cap$	$\cup$	$\cap$	$\cup$

$f(-1/2)=(-3/2)e^{-2/3}$

$f(-1/10)=\left(-\frac{51}{10}\right)\cdot e^{-10/3}$



b) $b\in(-\infty,0]$ 0 rešenja $\quad b\in(0,\frac{3}{2}e^{1/3})$ 1 rešenje $\quad b=\frac{3}{2}e^{1/3}$ 2 rešenja ; $b>\frac{3}{2}e^{1/3}$ 3 rešenja

4. f nepr na $[0,1]$ i dif na $(0,1)$ $\quad f(0)=0$

$f(1)(f(1)-1)=f'(c)(2f(c)-1)$ $\qquad F(x)=f^2(x)-f(x)$

$F'(x)=2f(x)\cdot f'(x)-f'(x)$

$\Rightarrow \exists c\in(0,1)$ tkd. $\frac{F(1)-F(0)}{1}=f'(c)(2f(c)-1)=f(1)(f(1)-1)$ ∎

2022 sep 2

1. $a_1 = 1/5 \qquad a_{n+1} = \dfrac{5a_n}{2a_n^2+3}$

Ako $\exists$ $L = \dfrac{5L}{2L^2+3} \Rightarrow 2L^3 - 2L = 0 \ /:2 \Rightarrow L(L-1)(L+1)=0$

Ogr: I $a_1 = 1/5 > 0 \quad a_n > 0 \Rightarrow a_{n+1} > 0$?

$\dfrac{5a_n}{2a_n^2+3}$ zavisi od sgn $a_n \Rightarrow$ Jeste veće

II $a_1 = 1/5 < 1 \quad a_n < 1 \Rightarrow a_{n+1} < 1$?

$\dfrac{5a_n - 2a_n^2 - 3}{2a_n^2+3} < 0 \qquad 2a_n^2 - 5a_n + 3 > 0 \qquad a_{n_{1,2}} = \dfrac{5 \pm \sqrt{25-24}}{4} \nearrow 1, \searrow \frac{3}{2}$

$2(a_n - 1)(a_n - 3/2) > 0$

$\Rightarrow$ Jeste manje od 1

$a_{n+1} - a_n = a_n\left(\dfrac{5 - a_n^2 - 3}{a_n^2+3}\right) = a_n\left(\dfrac{2 - \overset{<1}{a_n^2}}{a_n^2+3}\right) > 0 \qquad \lim\limits_{n\to\infty} a_n = 1$

b) $\lim\limits_{n\to+\infty}\left(e^{-\frac{2a_n}{n}} - \dfrac{1}{\left(1+\ln\left(1+\frac{a_n}{n}\right)\right)^2}\right)\cdot n^2 =$

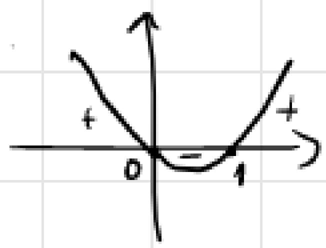
$= \left(1 - \dfrac{2a_n}{n} + \dfrac{2a_n^2}{n^2} - \left(1 + \dfrac{a_n}{n} - \dfrac{a_n^2}{2n^2}\right)^{-2}\right)\cdot n^2 = \left(\cancel{1} - \cancel{\dfrac{2a_n}{n}} + \dfrac{2a_n^2}{n^2} - \cancel{1} + \cancel{2\dfrac{a_n}{n}} - \dfrac{a_n^2}{n^2} - 3\cdot\dfrac{a_n^2}{n^2}\right)$

$= \left(-\dfrac{2a_n^2}{n^2}\right)\cdot n^2 = -2$

2. $f(x) = |x^2 - x| + \sqrt[3]{x^4}\cdot\sin\dfrac{\pi}{2x}$

a) $\lim\limits_{x\to 0^-} |x^2-x| + \sqrt[3]{x^4}\sin\dfrac{\pi}{2x} = 0 = \lim\limits_{x\to 0^+} \dots \Rightarrow a = 0$

b) $x(x-1)$



$f'(x) = \begin{cases} 2x - 1 + \frac{4}{3}\cdot\sqrt[3]{x}\cdot\sin\frac{\pi}{2x} + \sqrt[3]{x^4}\cdot\frac{\pi}{2}\cdot\frac{-1}{x^2}\cdot\cos\frac{\pi}{2x}, & x\in(-\infty,0)\cup(1,+\infty) \\ -2x + 1 + \frac{4}{3}\cdot\sqrt[3]{x}\cdot\sin\frac{\pi}{2x} + \text{—}\,||\,\text{—}, & x\in(0,1) \end{cases}$

$\lim\limits_{x\to 0^-} 2x - 1 + \frac{4}{3}\sqrt[3]{x}\cdot\sin\frac{\pi}{2x} + \underbrace{\sqrt[3]{x^4}\cdot\frac{-\pi}{2x^2}}_{\sqrt[3]{\frac{x^4}{x^6}} = \sqrt[3]{\frac{1}{x^2}}}\cdot\cos\frac{\pi}{2x} = 0 - 1 + 0 - \infty = -\infty = \lim\limits_{x\to 0^+}$

$\lim\limits_{x\to 1^-} -2x + 1 + \frac{4}{3}\cdot\sqrt[3]{x}\cdot\sin\frac{\pi}{2x} + \sqrt[3]{x^4}\cdot\frac{\pi(-1)}{2x^2}\cdot\cos\frac{\pi}{2x} = -2 + 1 + 4/3 = 1/3$

$1^+ \quad 2x - 1 + \dots = 2 - 1 + 4/3 = 7/3$

Nije dif u $\{0, 1\}$

3. $f(x) = \dfrac{x-2}{\ln^2(x-2)}$ I $x - 2 > 0$, $x > 2$ $\quad D_f: (2,3)\cup(3,+\infty)$ $\quad$ ($\ln(1) = 0$)

II Nema presek sa y osom, Nule i znak nisu važne

III Nije parna/neparna/periodična $\qquad$ IV $\lim\limits_{x\to 2^+} \dfrac{x-2}{\ln^2(x-2)} \overset{LOP}{=} \dfrac{x-2}{2\ln(x-2)} \overset{LOP}{=} \dfrac{x-2}{2} = 0$

$\lim\limits_{x\to 3\pm} \dfrac{1}{0} = +\infty$

$\lim\limits_{x\to+\infty} \dfrac{x-2}{\ln^2(x-2)} \overset{LOP}{=} \dfrac{1}{\frac{2\ln(x-2)}{(x-2)}} \overset{LOP}{=} \dfrac{x-2}{2} = +\infty$ Nema

V $\dfrac{\ln^2(x-2) - (x-2)\,2\ln(x-2)\cdot\frac{1}{x-2}}{\ln^4(x-2)} = \dfrac{\ln(x-2) - 2}{\ln^3(x-2)} = \dfrac{1}{\ln(x-2)}\cdot\dfrac{\ln(x-2) - 2}{\ln(x-2)}$ $\quad \Rightarrow 2 + e^2$, $\quad 3$

	2		3		$2+e^2$		$+\infty$
$\ln(x-2)-2$		$-$		$-$		$+$	
$\ln(x-2)$		$-$		$+$		$+$	
		$\nearrow$		$\searrow$		$\nearrow$	
					min		

$\dfrac{e^2}{\ln 2}$

VI $\left(\frac{\ln(x-2)-2}{\ln^3(x-2)}\right)' = \frac{\frac{1}{x-2}\cdot \ln^3(x-2) - (\ln(x-2)-2)\cdot 3\ln^2(x-2)\cdot \frac{1}{(x-2)}}{\ln^6(x-2)}$

$= \frac{1}{\ln^4(x-2)}\cdot\left(\frac{\ln(x-2)-3\ln(x-2)+6}{(x-2)}\right) = \frac{\ln\left(\frac{\cancel{(x-2)}}{(x-2)^{\cancel{3}\,2}}\right)+6}{x-2} = \frac{2\left(3-\ln(x-2)\right)}{x-2}$

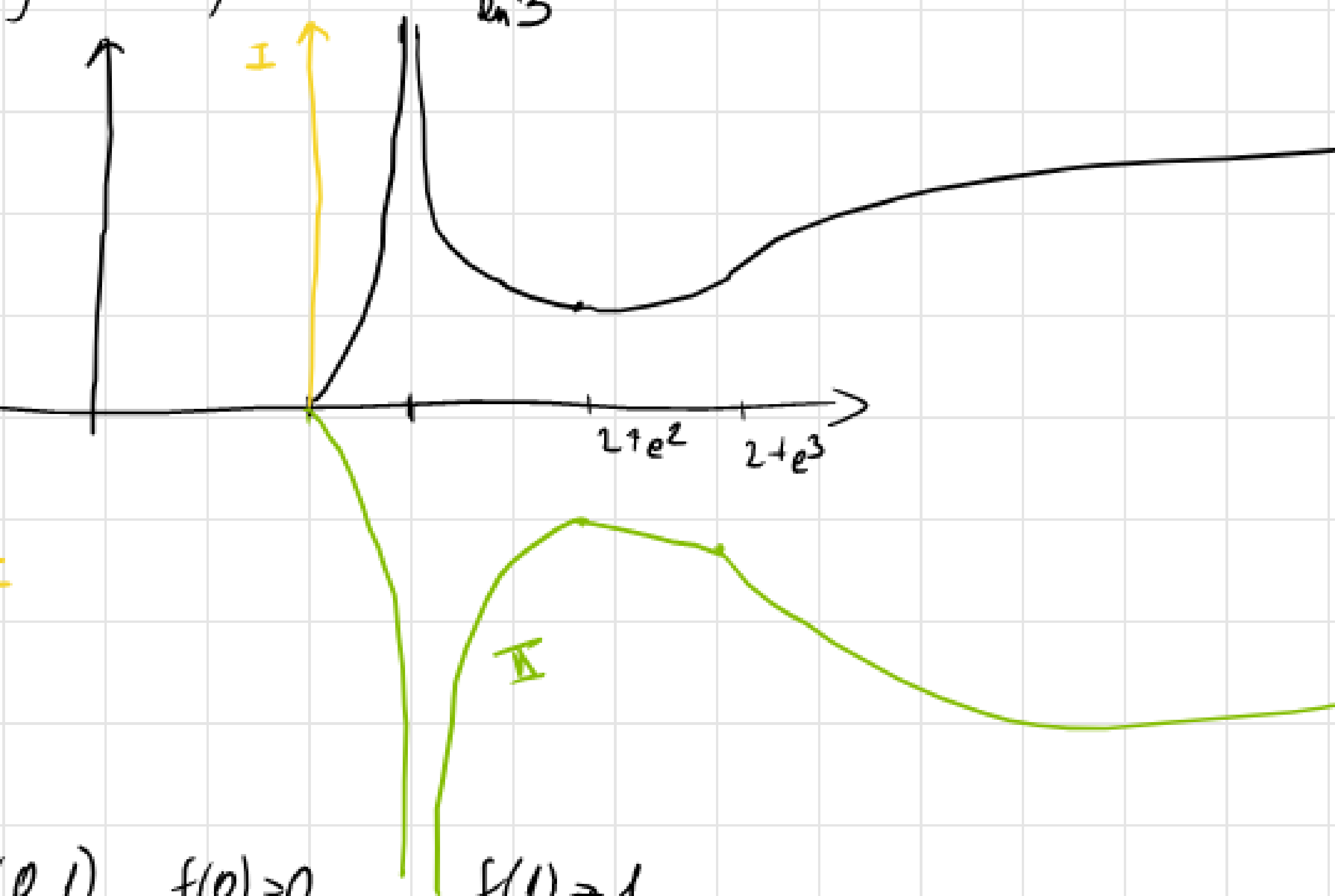
$\Rightarrow$

	2	3	$2+e^3$	$+\infty$
$3-\ln(x-2)$	+	+	−	
$x-2$	+	+	+	
$f(x)$	∪	∪	∩	

$\Rightarrow$ $x\in(2,3)\cup(2,2+e^3)$ ∪

$x\in(2+e^3,+\infty)$ ∩

$f(2+e^3) = \frac{e^3}{\ln 3}$



b) $g(x) = -f(x+2)$

Pomerimo ulevo za 2 $f(x)$ I

Potom obrnemo II

4. Nepr na $[0,1]$ dif na $(0,1)$ $f(0)=0$ $f(1)=1$

$f'(c) = \frac{\pi}{2}\cdot c\cdot(1+f^2(c))$ $\quad F(x)' = (1+f^2(x))\cdot x\cdot \frac{1}{2}$

$\frac{F(1)-F(0)}{1} = F'(c) = F(1)-F(0)$ $\quad \arctan(f\sqrt{x})' = \frac{f'(\sqrt{x})}{1+f(\sqrt{x})^2}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}} = \frac{f'(\sqrt{x})}{2\sqrt{x}(1+f(\sqrt{x})^2)}$

$\Rightarrow \frac{\pi}{4} - 0 = \frac{1}{2\sqrt{x}(1+f(\sqrt{x})^2)} = \frac{\pi}{2}\sqrt{x}\,(1+f(\sqrt{x})^2) = f'(\sqrt{x})$ $\quad \boxed{\sqrt{x}=c}$ ■

2023 jan 1

1. $x_0>0$ i $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{a}{x_n^2}\right)$ $\quad 3L = 2L + \frac{a}{L^2}$ /.

B: $x_0>0$ $\quad x_n>0 \Rightarrow x_{n+1}>0$? $\quad L = \frac{a}{L^2} \Rightarrow L = \sqrt[3]{a}$, $a\neq 0$

$2x_n + \frac{a}{x_n^2} > 0$ $/\cdot x_n^2$ $\quad 2x_n^3 + a > 0$ $\quad x_n^3 > -\frac{a}{2} \Rightarrow a\geq 0$

$x_{n+1} - x_n = \frac{2}{3}x_n - \frac{3}{3}x_n + \frac{a}{3x_n^2} = \frac{a}{3x_n^2} - \frac{x_n}{3} = \frac{a - x_n^3}{3x_n^2}$ >0 zavisi od a

2 slučaja

I $x_n > \sqrt[3]{a}$

$\Rightarrow$ Niz opada $\Rightarrow \lim_{n\to+\infty} x_n = 0$

II $x_n < \sqrt[3]{a}$ $\forall n\in\mathbb{N}$

$\Rightarrow$ Niz raste pa $\nexists$ $\lim_{n\to+\infty} x_n = \sqrt[3]{a}$

b) $\lim_{n\to+\infty}\frac{(n+1)x_{n+1}}{2n+1}$ $\to 0$, $\to \frac{\sqrt[3]{a}}{2}$

2. $$f(x)=\begin{cases} \text{I}\quad a^{\frac{\sqrt{\sin x^2 - x^3}}{x}}, & x<0 \\ \text{II}\quad b, & x=0 \\ \text{III}\quad \dfrac{1-\cos(\sin x)+\ln\sqrt{1+x^2}}{x^2}, & x>0 \end{cases}$$

III $$\frac{1}{x^2}\left(1-\cos(\sin x)+\ln(1+x^2)^{1/2}\right)=\frac{1}{x^2}\left(1-\cos\left(x-\frac{x^3}{6}\right)+\ln\left(1+\frac{1}{2}\cdot x^2\right)\right)=$$

$$=\frac{1}{x^2}\left(1-\left(1-\left(x-\frac{x^3}{6}\right)^2\cdot\frac{1}{2}\right)+\frac{x^2}{2}\right)=\frac{1}{x^2}\left(1-1+\frac{x^2}{2}+\frac{x^2}{2}\right)=1$$

I $$a^{\frac{\sqrt{\sin x^2-x^3}}{x}}=e^{\wedge}\left(\frac{1}{x}\cdot\sqrt{\frac{\sin x^2}{x^2}\cdot x^2-x^3}\right)\cdot\ln a=e^{\wedge}\frac{|x|\sqrt{1-x}}{x}\cdot\ln a=e^{-\ln a}$$

$$=\frac{1}{e^{\ln a}}=\frac{1}{a}$$

$$\frac{1}{a}=1 \Rightarrow a=1 \qquad b=1$$

3. $f(x)=(x+1)\,e^{\operatorname{arctg}\frac{1}{x}}$ I $D_f:\ \mathbb{R}\setminus\{0\}$ II $x>-1\ f(x)>0$; $x<-1\ f(x)<0$

$x=-1$ nula, nema presek sa y-osom III Nije parna, ni per., ni neparna

IV $$\lim_{x\to 0^{-}_{+}}(x+1)\,e^{\operatorname{arctg}\frac{1}{x}\to\pm\infty\to\mp\frac{\pi}{2}}=e^{-\pi/2}\ ,\ =e^{\pi/2}$$

$$\lim_{x\to\pm\infty}\frac{(x+1)\,e^{\operatorname{arctg}\frac{1}{x}\to 0}}{x}=e^0=1 \qquad b=(x+1)e^{\operatorname{arctg}\frac{1}{x}}-x=(x+1)\left(e^{\left(\frac{1}{x}-\frac{1}{3x^3}\right)}\right)-x$$

$$=(x+1)\left(1+\frac{1}{x}-\frac{1}{3x^3}\right)-x=x-x+1+1+\frac{1}{x}-\frac{1}{3x^3}-\frac{1}{3x^2}=2$$

V $$e^{\operatorname{arctg}\frac{1}{x}}\left(1+(x+1)\cdot\frac{-\frac{1}{x^2}}{\frac{1+x^2}{x^2}}\right)=\ldots\left(\frac{1+x^2-x-1}{1+x^2}\right)=\frac{x(x-1)}{1+x^2}\cdot e^{\operatorname{arctg}\frac{1}{x}}$$

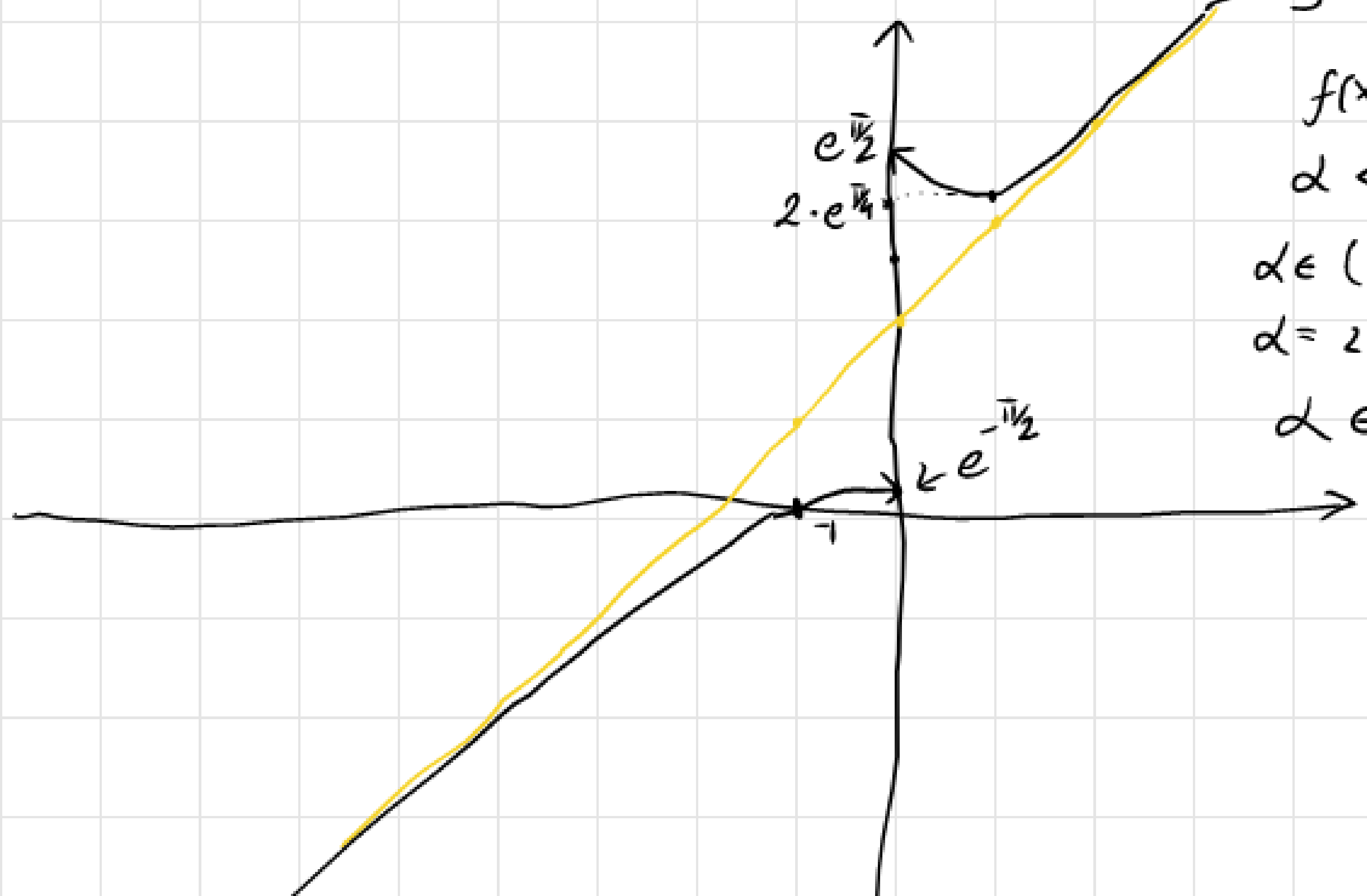
	$-\infty$	0	1	$+\infty$
x	$-$	$+$	$+$	
$x-1$	$-$	$-$	$+$	
	$\nearrow$	$\searrow$	$\nearrow$	

min $\boxed{2\cdot e^{\pi/4}}$

VI $$e^{\operatorname{arctg}\frac{1}{x}}\left(\frac{(2x-1)(1+x^2)-2x(x^2-x)}{(1+x^2)^2}+\frac{x(x-1)}{1+x^2}\cdot\frac{-\frac{1}{x^2}}{\frac{1+x^2}{x^2}}\right)$$

$$=e^{\operatorname{arctg}\frac{1}{x}}\left(\frac{\cancel{2x^3}+2x-\cancel{x^2}-1-\cancel{2x^3}+\cancel{2x^2}-\cancel{x^2}+x}{(1+x^2)^2}\right)=\frac{3x-1}{(1+x^2)^2}\cdot e^{\operatorname{arctg}\frac{1}{x}}$$

$x>1/3$:) | $x<1/3$:(



$f(x)=\alpha$

$\alpha<e^{-\pi/2}$ 1 rešenje

$\alpha\in(e^{-\pi/2},\,2\cdot e^{\pi/4})$ 0 rešenja

$\alpha=2\cdot e^{\pi/4}$ 1 rešenje

$\alpha\in(2\cdot e^{\pi/4},\,e^{\pi/2})$ 2 rešenje

$\alpha>e^{\pi/2}$ 1 rešenje

4. Nepr na $[a,b]$ dif na (a,b), $f(x) \neq 0 \ \forall x \in (a,b)$

$\exists ?\ c \in (a,b)\ \frac{f'(c)}{f(c)} = \frac{1}{a-c} + \frac{1}{b-c}$

$$F'(x) = f'(c)(a-c)(b-c) - f(c)(a+b-2c)$$
$$f'(c)(ab-ac-cb+c^2) - f(c)(a+b-2c)$$
$$= (f(c)\cdot(ab-ac-cb+c^2))'$$
$$F(x) = f(x)(ab-ax-bx+x^2)$$

$\exists c \in (a,b)$ tkd. $\frac{F(b)-F(a)}{b-a} = \frac{f(b)\cdot(ab-ab-b^2+b^2) - f(a)(ab-a^2-ab+a^2)}{b-a} = F'(c)$

$$\Rightarrow 0 = f'(c)(a-c)(b-c) - f(c)(a+b-2c)$$
$$f'(c)(a-c)(b-c) = f(c)(a+b-2c) \Rightarrow \frac{f'(c)}{f(c)} = \frac{1}{a-c} + \frac{1}{b-c}$$

2023 jun 1

1. $f(x) = \sqrt{\frac{x^3}{x+1}}$

	$-\infty$	-1	0	$+\infty$
x	$-$	$-$	$+$	
$x+1$	$-$	$+$	$+$	
	$\checkmark$	$\times$	$\checkmark$	

I D_f: $x \in (-\infty,-1) \cup [0,+\infty)$ II $f(x) > 0$ na domenu
$x=0, y=0$ nula i presek sa y-osom

III Ni parna, ni neparna. Nije periodična

IV $\lim_{x\to -1^-} \sqrt{\frac{x^3}{x+1}} = \sqrt{\frac{-1}{0^-}} = +\infty$ $\quad \lim_{x\to\pm\infty} \sqrt{\frac{\frac{x^3}{x+1}}{\frac{x^2}{1}}} = \sqrt{\frac{x^3\cdot 1}{x^3(1+\frac{1}{x^2})}} = \pm 1$

$\lim_{x\to+\infty} |x|\sqrt{\frac{x}{x+1}} - x = x\frac{\left(\left(1+\frac{-1}{x+1}\right)^{1/2} - 1\right)}{-\frac{1}{x+1}} \cdot \frac{-1}{x+1} = -\frac{x}{2(x+1)} = -1/2$

$\lim_{x\to-\infty} x\left(1-\sqrt{\frac{x}{x+1}}\right) = \ldots = 1/2$

V $\ 1/2 \cdot \sqrt{\frac{x+1}{x^3}} \cdot \frac{3x^2(x+1) - x^3}{(x+1)^2} = 1/2 \cdot \sqrt{\frac{x+1}{x^3}} \cdot \frac{1}{(x+1)^2} \cdot (x^2(2x+3))$

	$-3/2$	
	$-$	$+$
	$\searrow$	$\nearrow$

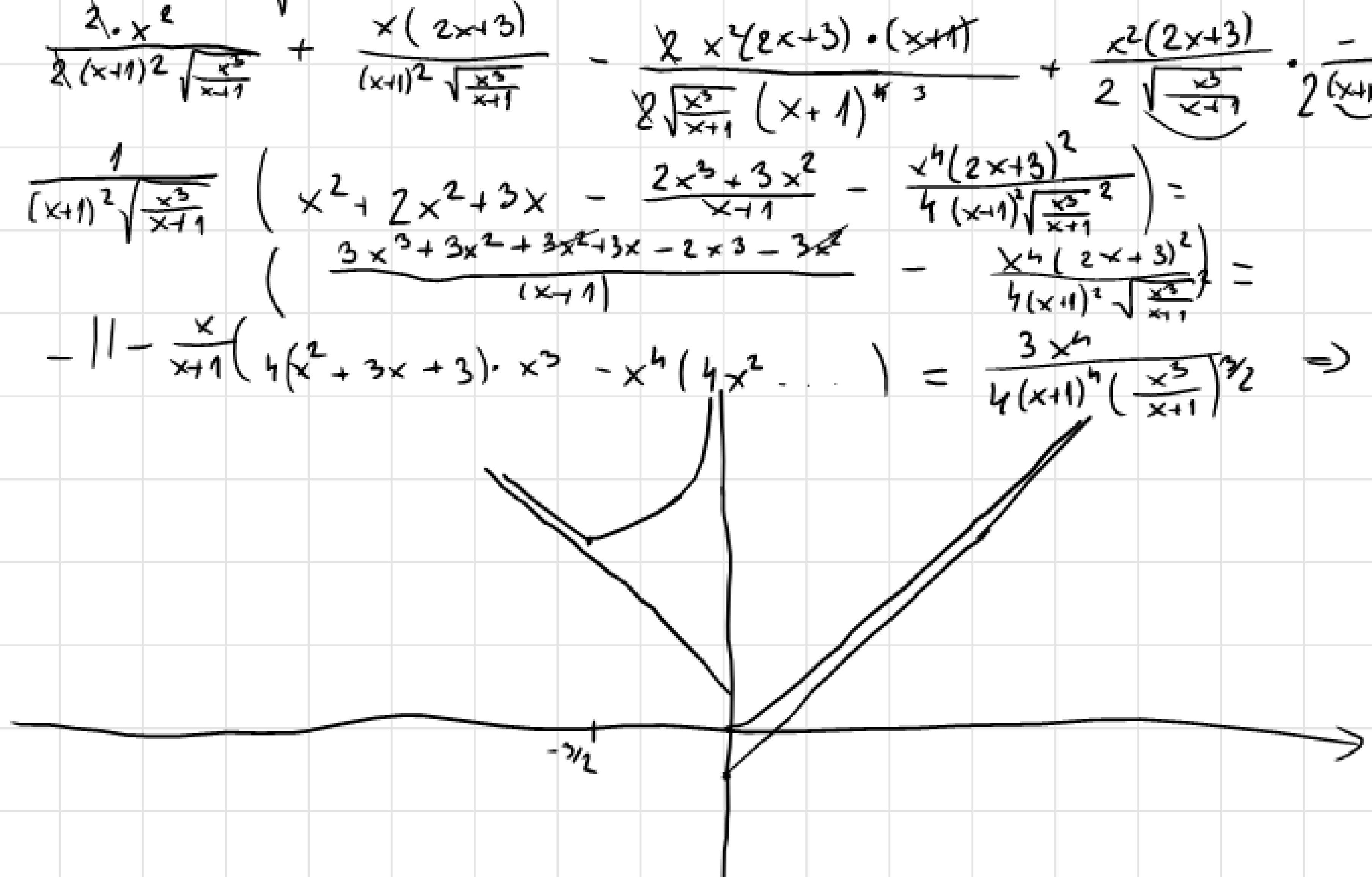
$f(-3/2) = \sqrt{\frac{-27}{8\cdot -1/2}} = \sqrt{\frac{27}{4}} = \frac{3\sqrt{3}}{2}$ lokalni minimum

VI $\frac{2\cdot x^2}{2(x+1)^2\sqrt{\frac{x^3}{x+1}}} + \frac{x(2x+3)}{(x+1)^2\sqrt{\frac{x^3}{x+1}}} - \frac{2x^2(2x+3)\cdot(x+1)}{2\sqrt{\frac{x^3}{x+1}}(x+1)^3} + \frac{x^2(2x+3)}{2\sqrt{\frac{x^3}{x+1}}} \cdot \frac{-(x^2(3x+3-x)}{2(x+1)^2(x+1)^2\cdot\sqrt{\frac{x^3}{x+1}}^3}$

$= \frac{1}{(x+1)^2\sqrt{\frac{x^3}{x+1}}}\left(x^2 + 2x^2 + 3x - \frac{2x^3+3x^2}{x+1} - \frac{x^4(2x+3)^2}{4(x+1)^2\sqrt{\frac{x^3}{x+1}}^2}\right) =$

$\left(\frac{3x^3+3x^2+3x^2+3x-2x^3-3x^2}{(x+1)} - \frac{x^4(2x+3)^2}{4(x+1)^2\sqrt{\frac{x^3}{x+1}}^2}\right) =$

$= -\|- \frac{x}{x+1}\left(4(x^2+3x+3)\cdot x^3 - x^4(4x^2 \ldots)\right) = \frac{3x^4}{4(x+1)^4\left(\frac{x^3}{x+1}\right)^{3/2}} \Rightarrow \cup$ na D_f.



2. $g(x) = \begin{cases} \frac{\sin(\pi x) + \sqrt[3]{1+3x} - 1}{x} & , x \neq 0 \\ 1 & , x = 0 \end{cases}$ $\qquad \frac{1/3 \cdot -\frac{2}{3}}{2} = -\frac{1}{9}$

$$\lim_{x \to 0} \frac{\pi x - \frac{\pi^3 x^3}{6} + \cancel{1} + 1/3 \cdot 3x - x^2 - \cancel{1}}{x} = (\pi + 1) \implies \text{Nije neprekidna}$$

b) $$\lim_{x \to +\infty} \frac{\sin(\pi x) + \sqrt[3]{1+3x} - 1}{x} = \underbrace{\frac{\sin(\pi x)}{x}}_{\to} + \underbrace{\frac{x\sqrt[3]{\frac{3}{x^2} + \frac{1}{x^3}}}{x}}_{\to 0} - \underbrace{\frac{1}{x}}_{\to 0} = 0$$

c) $$\lim_{x \to 0} \frac{e^{-2x^2} + \sin^2 x - \cos^2 x}{\cos(\sin x) - \cos x} = \frac{f(x)}{g(x)}$$ (Pretpostavimo da ide do x^4)

$$f(x) = 1 - 2x^2 + \frac{4x^4}{2} + \left(x - \frac{x^3}{6}\right)\left(x - \frac{x^3}{6}\right) - \left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right)\left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right) =$$
$$= \cancel{1} - 2x^2 + \frac{4x^4}{2} + x^2 - 2\frac{x^4}{6} - \cancel{1} + \frac{x^2}{2} - \frac{x^4}{24} - \frac{x^4}{4} + \frac{x^2}{2} - \frac{x^4}{24}$$
$$= -x^2 + x^2 + x^4\left(2 - 1/3 - \frac{1}{24} - \frac{1}{24} - 1/4\right) = \frac{x^4}{12}(24 - 4 - 1 - 3) = \frac{16x^4}{12} = 4/3\, x^4$$

$$g(x) = \cos\left(x - \frac{x^3}{6}\right) - 1 + \frac{x^2}{2} - \frac{x^4}{24} = 1 - 1/2\left(x - \frac{x^3}{6}\right)^2 + \frac{1}{24}\left(x - \frac{x^3}{6}\right)^4 - 1 + \frac{x^2}{2} - \frac{x^4}{24}$$
$$= -\frac{x^2}{2} + \frac{x^4}{6} + \frac{x^4}{24} + \frac{x^2}{2} - \frac{x^4}{24} = \frac{x^4}{6}$$

$$\implies \lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{4 \cdot 6 x^4}{3x^4} = 8$$

3. a) $\lim_{n \to +\infty} \left(2 + \frac{1}{n}\right)^{\frac{n^2+1}{n}} = +\infty$ $\qquad$ b) $\lim_{n \to +\infty} \sqrt[n]{0.02023}\, \text{arctg}\, n = \pi/2$

c) $$\lim_{n \to +\infty} \sqrt[n]{a_n} = \frac{a_n}{a_{n-1}} = \frac{(1+n)(2+n)\cdot\ldots\cdot(n+n)\,(n-1)^{n-1}}{n^n \cdot (1+n-1)(2+n-1)\ldots(n-1+n-1)} = \frac{\cancel{(1+n)(2+n)}\ldots(n+n)\cdot n^{n-1}(1-1/n)^{n-1}}{n\cancel{(n+1)(n+2)}\ldots\cancel{(2n-2)}\cdot n^n} =$$
$$= \frac{(2n-1)(2\cancel{n})\cdot n^{n-1}(1-1/n)^{n-1}}{\cancel{n}\cdot n^n} = \frac{(4n-2)\cdot(n-1)^{n-1}}{n^n} = \frac{(4n-2)}{(n-1)} \cdot \frac{\cancel{n^n}(1-1/n)^n}{\cancel{n^n}} \to e^{-1} = \boxed{\frac{4}{e}}$$

2023 jun 2

$x_{1/2} = \frac{2 \pm \sqrt{4+12}}{2} \nearrow \frac{2+4}{2} = 3 \searrow -1$

1. $f(x) = \left(\frac{3}{x} + 2 - x\right) \cdot e^{-\frac{1}{x}}$ $\quad$ I $D_f: \mathbb{R} \setminus \{0\}$ $\quad$ II $\left(\frac{3 + 2x - x^2}{x}\right) \implies$ $f(x) > 0 \quad x \in (-\infty, -1) \cup (0, 3)$; $f(x) < 0 \quad x \in (-1, 0) \cup (3, +\infty)$

$x_1 = -1$, $x_2 = 3$ su nule, nema presека III Nije parna, ni neparna, ni periodična

IV $\lim_{x \to 0^-} \left(\frac{3}{x} + 2 - x\right) \cdot e^{-\frac{1}{x}} = -\infty$ $\qquad \lim_{x \to 0^+} \left(\frac{3}{x} + 2 - x\right) e^{-\frac{1}{x}} = 0$

$$\lim_{x \to \pm\infty} \left(\frac{3}{x^2} + \frac{2}{x} - 1\right) e^{-\frac{1}{x}} = -1$$

$$\lim_{x \to \pm\infty} \left(\frac{3}{x} + 2 - x\right) e^{-\frac{1}{x}} + x = \frac{3}{x} + 2 - x\left(e^{-\frac{1}{x}} + 1\right) \cdot \left(-\frac{1}{x}\right) = 3/x + 3 = 3$$

V $$\left(-\frac{3}{x^2} - 1\right) e^{-\frac{1}{x}} + \left(\frac{3}{x} + 2 - x\right) \cdot e^{-1/x} \cdot (-1) \cdot (-1) \cdot \frac{1}{x^2} = \frac{e^{-\frac{1}{x}}}{x^2}\left(-3 - x^2 - x + 2 + \frac{3}{x}\right)$$
$$= \frac{e^{-1/x}}{x^2}\left(-x^2 - x + \frac{3}{x} - 1\right) \implies \frac{1}{x}(-x^3 - x^2 - x + 3) = \frac{1}{x}(x-1)(-x^2 - 2x - 3) = \frac{-1}{x}(x-1)(x^2 + 2x + 3)$$

$$f(1) = \frac{4}{e} \approx 1.3$$

x		0		1		+∞
x	−		+		+	
x−1	−		−		+	
	↘		↗		↘	

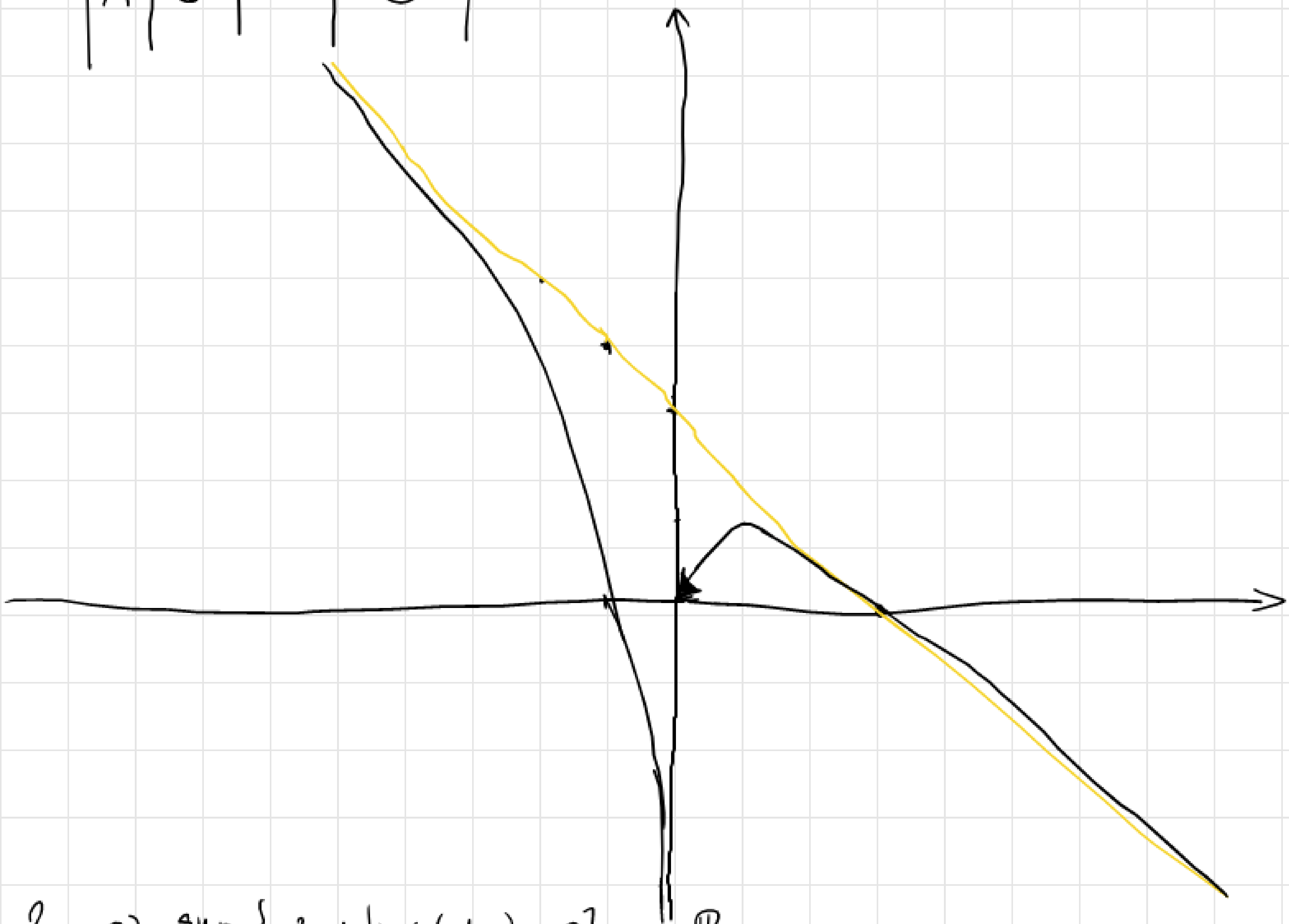
VI $-\dfrac{(3x^2+2x+1)e^{-\frac{1}{x}}}{x^3} - \dfrac{(x^3+x^2+x-3)e^{-1/x}}{x^5} - \dfrac{(x^3+x^2+x-3)e^{-1/x}(-3)}{x^4}$

$= \dfrac{e^{-1/x}}{x^5}\left(\cancel{3x^4}+\cancel{3x^3}+\underbrace{3x^2}-9x-\cancel{x^3}-\underbrace{x^2}-x+3-\cancel{3x^4}-\cancel{2x^3}-\underbrace{x^2}\right)$

$= \dfrac{e^{-1/x}}{x^5}(x^2-10x+3) \Rightarrow \dfrac{10\pm\sqrt{100-12}}{2} \to \dfrac{10\pm2\sqrt{22}}{2} \to 5\pm\sqrt{22}$

$5-\sqrt{22}$ $5+\sqrt{22}$

	$-\infty$ … 0	0 … $5-\sqrt{22}$	$5-\sqrt{22}$ … $5+\sqrt{22}$	$5+\sqrt{22}$ … $+\infty$
x	$-$	$+$	$+$	$+$
$x^2-10x+3$	$+$	$+$	$-$	$+$
	$\cap$	$\cup$	$\cap$	$\cup$



2. a) $\sup\{x^2+1 \mid x\in(-1,e)\cap\mathbb{Q}\}$ u $\mathbb{R}$

$\forall n\in\mathbb{N}$ $e^2+1\geq g(x)$? I $e^2+1\geq x^2+1 \overset{x=e}{\geq} e^2+1$ ✓

II $\varepsilon>0$? $(\exists n\in A)$ $n^2+\cancel{1} > e^2+\cancel{1}-\varepsilon$

$n^2>e^2-\varepsilon \Rightarrow$ biramo $n_0\in\mathbb{N}$ $n_0>2.8^2-\varepsilon$

[iz I i II e^2+1 je sup] $n_0^2\geq n_0>2.8^2-\varepsilon$

b) $\sup\left(\left\{ \frac{2+(-1)^n}{2n} \mid n\in\mathbb{N}\right\} \cup \left\{\frac{\pi}{6}\right\}\right)$ u $\mathbb{R}$

Pretpostavim: $3/4$ $\quad \frac{2+(-1)^n}{2n} \leq 3/4 \quad /\cdot 4n \Rightarrow 4+2(-1)^n \leq 3n$

$4-2 \leq 3n \qquad\qquad 4+2 \leq 3\cdot 2\cdot n$

$2 \leq 3n \qquad\qquad 6 \leq 6n$

$\forall n\in\mathbb{N}$ ✓ $\qquad\qquad 1 \leq k$ ✓

II $\varepsilon > 0 \quad \exists a\in A \quad a > 3/4 - \varepsilon \qquad \frac{2+(-1)^n}{2n}$ Biram veće $\Rightarrow \frac{3}{2n}$

$3/2n > \frac{3}{4} - \varepsilon \rightarrow \frac{3}{2n} > \frac{3-4\varepsilon}{4} /\cdot \frac{4n}{3-4\varepsilon} \Rightarrow \frac{3}{3-4\varepsilon} > n$

$n_0 = \left[\frac{3}{3-4n}\right]$

iz I i II $\sup = 3/4$

c) $\lim\limits_{x\to 0} \frac{tg(2x)}{\ln(2+x)} = \frac{tg\,0}{\ln 2} = 0$

d) $\lim\limits_{x\to 1} \frac{1}{\ln x} - \frac{x}{x-1} = \left[\begin{matrix} t = x-1 \\ t\to 0\end{matrix}\right] = \lim\limits_{t\to 0} \frac{1}{\underbrace{\frac{\ln(t+1)}{t}}_{\to 1}\cdot t} - \frac{t+1}{t} = \frac{-t-1+1}{t} = -1$

3. $\ln(x+2) = x^2$

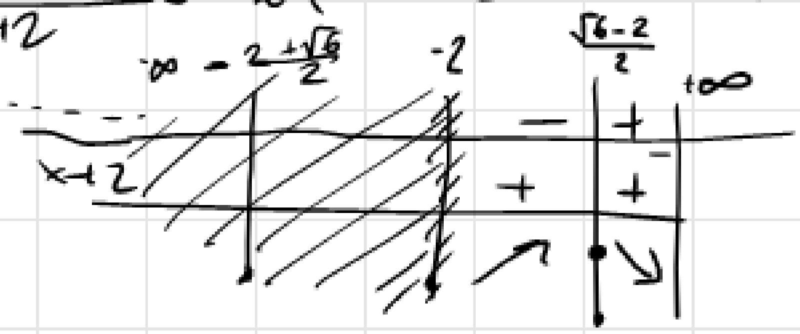
$F(x) = \ln(x+2) - x^2 \quad$ I $D_f: (-2, +\infty)$ Moramo da ispitamo V.A.

$\lim\limits_{x\to -2^+} \ln(x+2) - x^2 = -\infty$ Jeste V.A. $\quad \lim\limits_{x\to +\infty} \ln(x+2) - x^2 \overset{LOP}{=} \frac{1}{x+2} - 2x = -\infty$

Monotonost: $\frac{1}{x+2} - 2x = \frac{1-4x-2x^2}{x+2} = \frac{-(2x^2+4x-1)}{x+2} = -2\left(x+\frac{2+\sqrt{6}}{2}\right)\left(x+\frac{2-\sqrt{6}}{2}\right)$

$x_{1/2} = -\frac{2\pm\sqrt{6}}{2} \qquad F(x_1) = \ln(\sqrt{6}+2) - \ln 2 - \frac{(6+4-4\sqrt{6})}{4}$

$\ln(\sqrt{6}+2) - \ln 2 - \frac{5}{2} + \sqrt{6} > 0$



Presek sa y-osom $\Rightarrow x=0 \quad \ln 2 > 0$

a) $F(-1) = \ln 1 - 1 = -1 < 0 \qquad F(2) = 2\ln 2 - 4 < 0$

$\Rightarrow \exists\, a \in (-1, 0)$ tako da $F(a) = 0 \qquad \exists\, b\in(0,2)$ t.d. $F(b) = 0$

b) Ima dva rešenja po svemu ovom gore.