2022 Jun 2

1)a $\sum_{k=0}^{n}(-1)^k \frac{k}{k+1}\binom{n}{k} = \sum_{k=0}^{n}(-1)^k\left(1+\frac{-1}{k+1}\right)\binom{n}{k} = \sum_{k=0}^{n}(-1)^k\binom{n}{k}$

$- \sum_{k=0}^{n}(-1)^k\left(\frac{1}{k+1}\right)\cdot\frac{n!}{k!(n-k)!}\cdot\frac{n+1}{n+1} = 0 - \sum_{k=0}^{n}(-1)^k\binom{n+1}{k+1}\frac{1}{n+1}$

$= -\frac{1}{n+1}(0+1) = -\frac{1}{n+1}$

1b) $n+1$ od $\{1,2,\dots,2n\}$ $\Rightarrow$ Postoje dva uzajamno prosta

Brojeve podelimo u parove $\{1,2\}, \{3,4\} \dots \{2n-1, 2n\}$. Brojevi u paru su uzajamno prosti. Parova postoji $n \Rightarrow$ U najgorem slučaju će biti odabran po jedan iz para, ali $n+1$-vi el mora da pripada jednom od već odabranih parova pa će postojati dva uzajamno prosta

1v) $S:8$ $O:9$ $K:10$ 3 učenika

min 3, 5 min 3, 6 min 3, 7

$x_1+x_2+x_3=5$ $\binom{5+2}{5}$ $y_1+y_2+y_3=6$ $\binom{6+2}{6}$ $z_1+z_2+z_3=7$ $\binom{7+2}{7}$ $\Big\}$ $\binom{7}{5}\binom{8}{6}\binom{9}{7}$

2. $a_{n+3} = a_{n+2} + 16\cdot a_{n+1} + 20a_n + 49\cdot 5^n$ $a_0=2$, $a_1=2$, $a_2=47$

$a_{n+3} - a_{n+2} - 16a_{n+1} - 20a_n = 49\cdot 5^n \Rightarrow a_n = h_n + p_n$

$t^3 - t^2 - 16t - 20 = 0$ $(t+2)(t^2-3t-10) = (t+2)(t-5)(t+2)$

$t_1=-2$, $t_2=5$, $t_3=-2$ $k\cdot b^n = 49\cdot 5^n \to 5=5$ jeste rešenje

$p_n = n\cdot 5^n\cdot A$

$(n+3)5^3\cdot \cancel{5^n}\cdot A - A(n+2)25\cdot\cancel{5^n} - 16A(n+1)5\cdot\cancel{5^n} - 20\cdot A\,n\cdot\cancel{5^n} = 49\cdot\cancel{5^n}$

$A(125n + 375 - 25n - 50 - 80n - 80 - 20n) = 49$

$A\cdot 245 = 49 \Rightarrow A = 49/245 \Rightarrow p_n = 49\cdot n\cdot 5^n\cdot\frac{1}{245}$

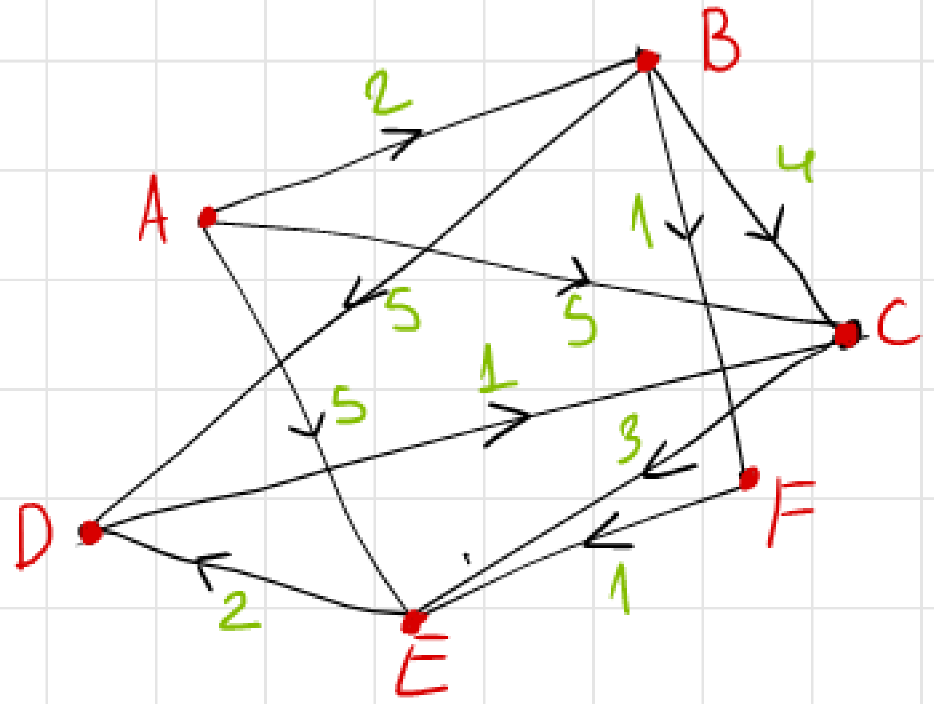
$a_n = c_1(-2)^n + c_2\cdot n\cdot(-2)^n + c_3\cdot 5^n + \frac{49}{245}\cdot n\cdot 5^n$

$2 = c_1 + c_3$ $2 = -2c_1 - 2c_2 + 5c_3 + \frac{49}{49}$ $47 = 4c_1 + 8c_2 + c_3\cdot 25 + 2\cdot 5$

$\left.\begin{array}{l} c_1+c_2=2 \\ -2c_1-2c_2+5c_3 = 1 \\ 4c_1+8c_2+25c_3=37 \end{array}\right\}$ $\left.\begin{array}{l} c_1+c_2=2 \\ c_3=1 \\ 4c_1+8c_2=12 /:4 \end{array}\right\}$ $\left.\begin{array}{l} c_3=1 \\ c_1=2-c_2 \\ 2-c_2+2c_2=3 \end{array}\right\}$ $\begin{array}{l} c_3=1 \\ c_2=1 \\ c_1=1 \end{array}$

$\Rightarrow a_n = (-2)^n + n(-2)^n + 5^n + n\cdot 5^{n-1}$

3 a)



k=0

$$D = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \\ \left[\begin{array}{cccccc} \infty & 2 & 5 & \infty & 5 & \infty \\ \infty & \infty & 4 & 5 & \infty & 1 \\ \infty & \infty & \infty & \infty & 3 & \infty \\ \infty & \infty & 1 & \infty & \infty & \infty \\ \infty & \infty & \infty & 2 & \infty & \infty \\ \infty & \infty & \infty & \infty & 1 & \infty \end{array}\right] \\ \begin{array}{cccccc} A & B & C & D & E & F \end{array} \end{array}$$

$$E = \begin{bmatrix} 0 & & 0 \\ 0 & & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & & & & 3 & 0 \\ 0 & & & & 0 & \\ 0 & & & & 3 & \\ 0 & & & 4 & 0 & 4 \\ 0 & & & 0 & 0 & 0 \;\; 0 \end{bmatrix}$$

k=1

$$D = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \\ \left[\begin{array}{cccccc} \infty & 2 & 5 & \infty & 5 & \infty \\ \infty & \infty & 4 & 5 & \infty & 1 \\ \infty & \infty & \infty & \infty & 3 & \infty \\ \infty & \infty & 1 & \infty & \infty & \infty \\ \infty & \infty & \infty & 2 & \infty & \infty \\ \infty & \infty & \infty & \infty & 1 & \infty \end{array}\right] \\ \begin{array}{cccccc} A & B & C & D & E & F \end{array} \end{array}$$

k=2

$$D = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \\ \left[\begin{array}{cccccc} \infty & 2 & 5 & 7 & 5 & 3 \\ \infty & \infty & 4 & 5 & \infty & 1 \\ \infty & \infty & \infty & \infty & 3 & \infty \\ \infty & \infty & 1 & \infty & \infty & \infty \\ \infty & \infty & \infty & 2 & \infty & \infty \\ \infty & \infty & \infty & \infty & 1 & \infty \end{array}\right] \\ \begin{array}{cccccc} A & B & C & D & E & F \end{array} \end{array}$$

k=3

$$D = \begin{array}{c} \\ \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} & & & & 5 & \end{array} \\ \left[\begin{array}{cccccc} \infty & 2 & 5 & 7 & 5 & 3 \\ \infty & \infty & 4 & 5 & \boxed{7} & 1 \\ \infty & \infty & \infty & \infty & 3 & \infty \\ \infty & \infty & 1 & \infty & \boxed{4} & \infty \\ \infty & \infty & \infty & 2 & \infty & \infty \\ \infty & \infty & \infty & \infty & 1 & \infty \end{array}\right] \\ \begin{array}{cccccc} A & B & C & D & E & F \end{array} \end{array}$$

k=4

$$D = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \\ \left[\begin{array}{cccccc} \infty & 2 & 5 & 7 & 5 & 3 \\ \infty & \infty & 4 & 5 & 7 & 1 \\ \infty & \infty & \infty & \infty & 3 & \infty \\ \infty & \infty & 1 & \infty & 4 & \infty \\ \infty & \infty & \boxed{3} & 2 & \boxed{5} & \infty \\ \infty & \infty & \infty & \infty & 1 & \infty \end{array}\right] \\ \begin{array}{cccccc} A & B & C & D & E & F \end{array} \end{array}$$

$$n=5 \qquad D=\begin{array}{c}A\\B\\C\\D\\E\\F\\ \end{array}\begin{bmatrix}\infty & 2 & 5 & 7 & 5 & 3\\ \infty & \infty & 4 & 5 & 7 & 1\\ \infty & \infty & \boxed{6} & \boxed{5} & 3 & \infty\\ \infty & \infty & 1 & \boxed{6} & 4 & \infty\\ \infty & \infty & 3 & 2 & 6 & \infty\\ \infty & \infty & \boxed{4} & \boxed{3} & 1 & \infty\end{bmatrix}$$
$$\qquad\quad A \quad B \quad C \quad D \quad E \quad F$$

$$E=\left[\begin{array}{ccc|cccc}0&0&0&2&0&2&\\0&0&0&0&0&3&0\\0&0&0&5&5&0&0\\0&0&0&0&5&&\\0&0&0&&5&3&0\\0&0&0&4&0&4&0\\0&0&0&5&5&0&0\end{array}\right]$$

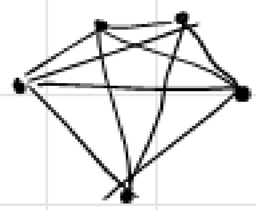
$$n=6 \qquad D=\begin{array}{c}A\\B\\C\\D\\E\\F\\ \end{array}\begin{bmatrix}\infty & 2 & 5 & \boxed{6} & \boxed{4} & 3\\ \infty & \infty & 4 & \boxed{4} & \boxed{2} & 1\\ \infty & \infty & 6 & 5 & 3 & \infty\\ \infty & \infty & 1 & 6 & 4 & \infty\\ \infty & \infty & 3 & 2 & 6 & \infty\\ \infty & \infty & 4 & 3 & 1 & \infty\end{bmatrix}$$
$$\qquad\quad A \quad B \quad C \quad D \quad E \quad F$$

$$E=\begin{array}{c|cccccc} & A & B & C & D & E & F\\ \hline A & 0&0&0&6&6&2\\ B&0&0&0&6&6&0\\ C&0&0&5&5&0&0\\ D&0&0&0&5&3&0\\ E&0&0&4&0&4&0\\ F&0&0&5&5&0&0\end{array}$$

$A \to B \to F \to E \to D$

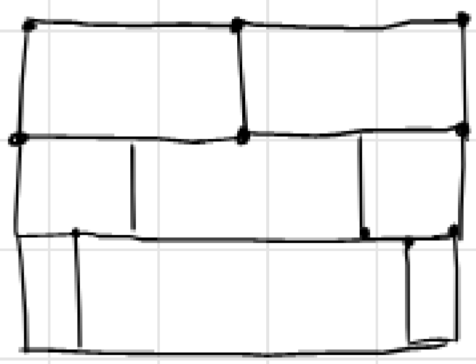
4. a) Da bi čvor koji je susedan sa listom bio stepena 3 on mora da ima 2 lista kao suseda, a kako listovi imaju samo jednog suseda to znači da svaki par listova ima zajedničkog suseda.

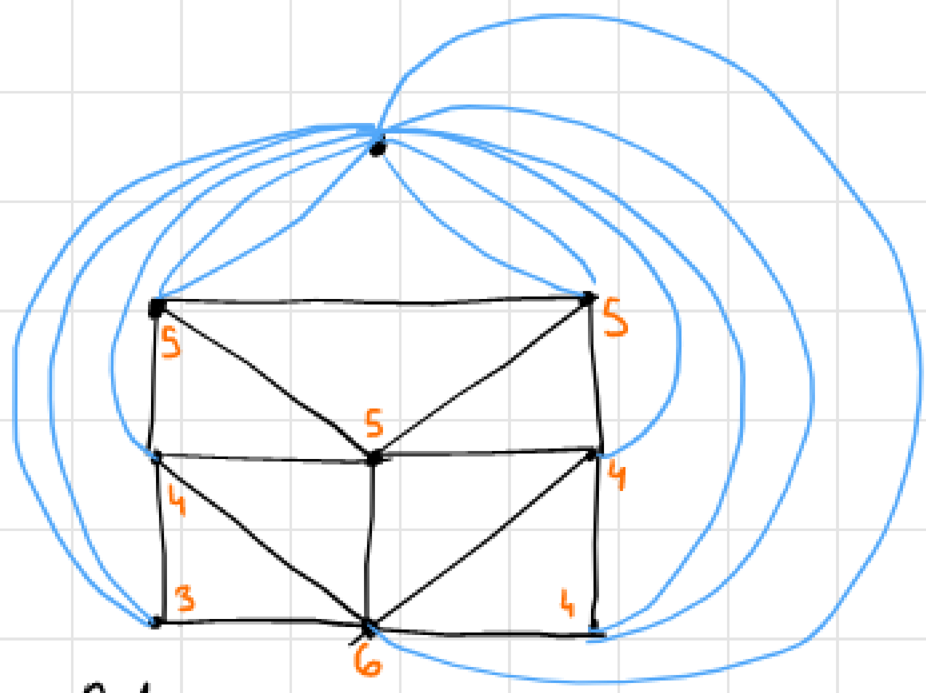
4b) 100 čvorova stepena 4



x 20

4v)





Sadrži 4 čvora neparnog stepena Prema Ojlerovoj teoremi ne može jer treba 0 ili 2

Druga grupa

1 a) $\sum_{k=0}^{n} (-1)^k \frac{k+2}{k+1}\binom{n}{k} = \sum_{k=0}^{n} (-1)^k \left(1+\frac{1}{k+1}\right)\binom{n}{k} = \sum_{k=0}^{n}(-1)^k\binom{n}{k} + \sum_{k=0}^{n}(-1)^k \frac{1}{k+1}\binom{n}{k}$

$= \sum_{k=0}^{n} (-1)^k \frac{n!}{k!(n-k)!}\cdot\frac{1}{k+1}\cdot\frac{n+1}{n+1} = \frac{1}{n+1}\sum_{k=0}^{n}(-1)^k\binom{n+1}{k+1} = \frac{1}{n+1}(0+1) = \frac{1}{n+1}$

1b) n+1 različitih elemenata skupa $\{1,2,\dots,2n\}$ Postoje dva uzajamno prosta

Formirajmo parove uzajamno prostih brojeva koji se razlikuju za 1 $(1,2), (3,4), \dots, (2n-1, 2n)$ => imamo ih n U najgorem slučaju od tih n+1, mi ćemo izabrati po jedan iz svakog para tj. njih n, ali n+1-vi će morati da bude iz jednog od već odabranih parova. Pa će postojati.

1v) O: 10 G: 8 L: 7

$o_1+o_2+o_3+o_4=10$ $\qquad g_1+g_2+g_3+g_4=8$ $\qquad l_1+l_2+l_3+l_4=7$

$o_1'+1+o_2'+1+o_3'+1+o_4'+1=10$ $\qquad g_1'+g_2'+g_3'+g_4'=4$ $\qquad l_1+l_2+l_3+l_4=3$

$o_1'+o_2'+o_3'+o_4'=6 \Rightarrow \binom{6+4-1}{6} \cdot \binom{8-1}{4} \cdot \binom{3+4-1}{4}$

2.

$a_{n+3} = 6a_{n+2} - 12a_{n+1} + 8a_n + 19 - 5n$ $\qquad a_0=-3$, $a_1=7$, $a_2=34$

$t^3-6t^2+12t-8=0 \Rightarrow (t-2)^3 \qquad t_1=t_2=t_3=2$

$a_n = 2^n\cdot c_1 + c_2\cdot n\cdot 2^n + c_3 n^2 2^n + p_d \Rightarrow p_d = A+Bn$

$A+Bn+3B-6A-6Bn-\cancel{12B}+12A+12Bn+\cancel{12B}-8A-8Bn = 19-5n$

$-A+3B-Bn = 19-5n \Rightarrow B=5 \quad A=-4$ Možda $a_n = c_1 2^n + c_2 n 2^n + c_3 n^2 2^n - 4 + 5n$

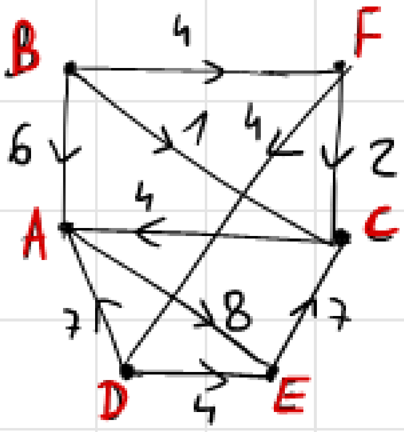
$a_0=-3=c_1-4 \Rightarrow c_1=1$

$a_1 = 7 = 2+c_2\cdot 2+2c_3+1 \Rightarrow 2=c_2+c_3$

$a_2 = 4+8c_2+16c_3+6=34 \qquad 3=c_2+2c_3 \Rightarrow 1=c_3$ $\quad c_2=1$

$a_n = 2^n+n2^n+n^2 2^n - 4+5n$

3.a)



$k=0$

$$D=\begin{array}{c}A\\B\\C\\D\\E\\F\end{array}\begin{bmatrix}\infty&\infty&\infty&\infty&8&\infty\\6&\infty&1&\infty&\infty&4\\4&\infty&\infty&\infty&\infty&\infty\\7&\infty&\infty&\infty&4&\infty\\\infty&\infty&7&\infty&\infty&\infty\\\infty&\infty&2&4&\infty&\infty\end{bmatrix}$$
$$\quad\ A\ \ B\ \ C\ \ D\ \ E\ \ F$$

$k=1$

$$D=\begin{array}{c}A\\B\\C\\D\\E\\F\end{array}\begin{bmatrix}\infty&\infty&\infty&\infty&8&\infty\\6&\infty&1&\infty&\boxed{14}&4\\4&\infty&\infty&\infty&\boxed{12}&\infty\\7&\infty&\infty&\infty&4&\infty\\\infty&\infty&7&\infty&\infty&\infty\\\infty&\infty&2&4&\infty&\infty\end{bmatrix}$$
$$\quad\ A\ \ B\ \ C\ \ D\ \ E\ \ F$$

$$E_1=\begin{bmatrix}0&0&0&0&0&0\\3&0&0&0&3&0\\0&0&0&0&1&0\\0&0&0&0&0&0\\3&0&0&0&3&0\\3&4&0&0&4&0\end{bmatrix}$$

$k=2$

$$D=\begin{array}{c}A\\B\\C\\D\\E\\F\end{array}\begin{bmatrix}\infty&\infty&\infty&\infty&8&\infty\\6&\infty&1&\infty&14&4\\4&\infty&\infty&\infty&12&\infty\\7&\infty&\infty&\infty&4&\infty\\\infty&\infty&7&\infty&\infty&\infty\\\infty&\infty&2&4&\infty&\infty\end{bmatrix}$$
$$\quad\ A\ \ B\ \ C\ \ D\ \ E\ \ F$$

$k=3$

$$D=\begin{array}{c}A\\B\\C\\D\\E\\F\end{array}\begin{bmatrix}\infty&\infty&\infty&\infty&8&\infty\\\boxed{5}&\infty&1&\infty&\boxed{13}&4\\4&\infty&\infty&\infty&12&\infty\\7&\infty&\infty&\infty&4&\infty\\\boxed{11}&\infty&7&\infty&\boxed{19}&\infty\\\boxed{6}&\infty&2&4&\boxed{14}&\infty\end{bmatrix}$$
$$\quad\ A\ \ B\ \ C\ \ D\ \ E\ \ F$$

$k=4$

$$D=\begin{array}{c}A\\B\\C\\D\\E\\F\end{array}\begin{bmatrix}\infty&\infty&\infty&\infty&8&\infty\\5&\infty&1&\infty&13&4\\4&\infty&\infty&\infty&12&\infty\\7&\infty&\infty&\infty&4&\infty\\11&\infty&7&\infty&19&\infty\\6&\boxed{11}&2&4&\boxed{8}&\infty\end{bmatrix}$$
$$\quad\ A\ \ B\ \ C\ \ D\ \ E\ \ F$$

$k=5$

$$\quad\ A\ \ B\ \ C\ \ D\ \ E\ \ F$$
$$D=\begin{array}{c}A\\B\\C\\D\\E\\F\end{array}\begin{bmatrix}\boxed{19}&\infty&\boxed{15}&\infty&8&\infty\\5&\infty&1&\infty&13&4\\4&\infty&\boxed{11}&\infty&12&\infty\\7&\infty&\boxed{11}&\infty&4&\infty\\11&\infty&7&\infty&19&\infty\\6&11&2&4&8&\infty\end{bmatrix}$$

$$E_1=\begin{array}{c}\\B\\\\D\\\\F\end{array}\begin{bmatrix}5&0&5&0&0&0\\3&6&0&6&6&0\\0&0&5&0&1&0\\0&0&5&0&0&0\\3&0&0&0&3&0\\3&4&0&0&4&0\end{bmatrix}$$
$$\qquad\qquad\quad D\ \ E\ \ F$$

$$D=\begin{array}{c}A\\B\\C\\D\\E\\F\end{array}\begin{bmatrix}19&\infty&15&\infty&8&\infty\\5&\boxed{15}&1&\boxed{8}&\boxed{12}&4\\4&\infty&19&\infty&12&\infty\\7&\infty&11&\infty&4&\infty\\11&\infty&7&\infty&19&\infty\\6&11&2&4&8&\infty\end{bmatrix}$$
$$\quad\ A\ \ B\ \ C\ \ D\ \ E\ \ F$$

$\underset{4}{(B,F)}$ +
(F,A) 6 = 10
(F,B) 11 = 15
(F,C) 2 = 6
(F,D) 4 = 8
(F,E) 8 = 12

B ⟶ E
B → F → E
B → F → D → E

4. isti za obe grupe