

2022 Jan 1

1. 1b) $\left(\sqrt[4]{a^2 x} + \sqrt[5]{\frac{1}{a x^2}}\right)^{13}$
 $\binom{13}{8} \cdot a^4 \cdot \frac{1}{a} = \boxed{a^3 \cdot \binom{13}{8}}$

$$x^{1/4 \cdot u_1} + x^{-2/5 \cdot u_2} = x^0$$

$$5u_1 - 8u_2 = 0 \Rightarrow 5u_1 = 8u_2$$

$$u_1 = 8, u_2 = 5$$

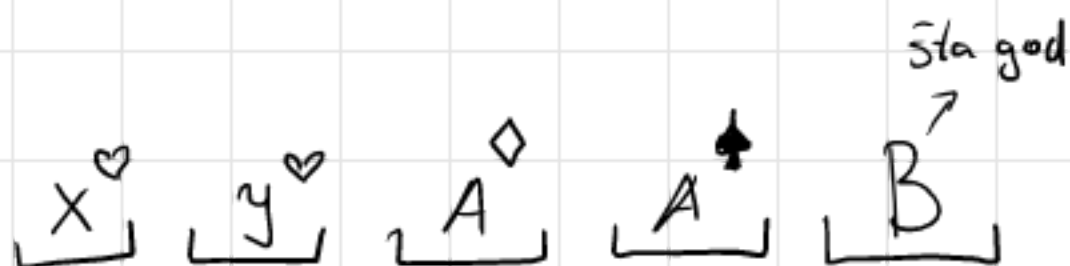
1b) 30 slova azbuke, 4 slova, 5 nishki, 2 isla slova

$$\binom{30}{2} \cdot 3 \cdot \frac{u!}{2!} = 5 \cdot 28 \cdot 28 \cdot 3 \cdot 4 \cdot 3 = \text{Br nishi}$$

(Br nishi) $\Rightarrow$ Reč

1v) 52 karte $\Rightarrow$ 13 po znaku

$$\binom{4}{2} \binom{13}{2} \cdot (3 \cdot 13 + 3 \cdot 11) \cdot 48$$



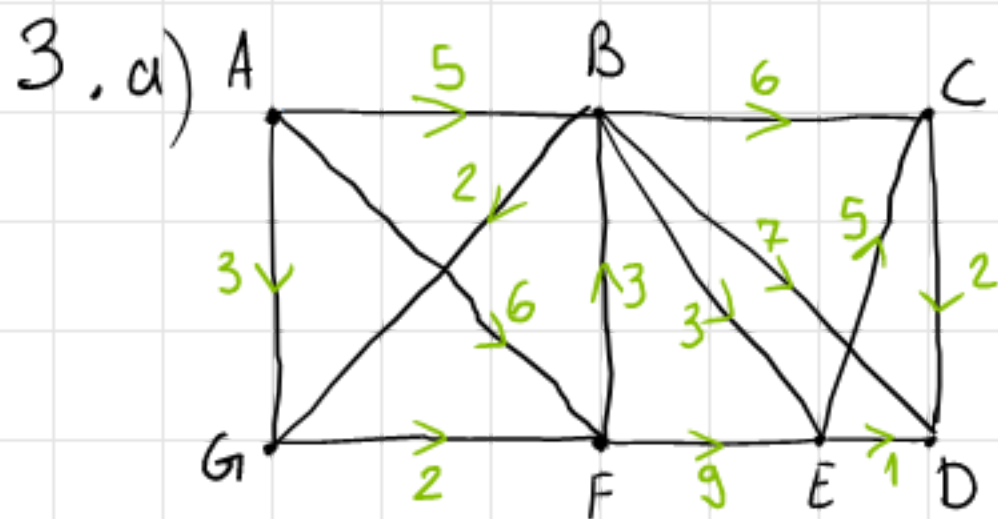
2. $a_{n+2} = \frac{2}{3} a_{n+1} + \frac{1}{3} a_n$ $a_0 = 0$ $a_1 = 1/3$

$$\frac{A(x) - \frac{x}{3}}{x^2} = \frac{2}{3} \frac{A(x)}{x} + \frac{1}{3} A(x)$$

$$A(x) \left(\frac{1}{x^2} - \frac{2}{3x} - \frac{1}{3} \right) = \frac{\frac{x}{3}}{\frac{x^2}{1}} \Rightarrow A(x) = \frac{3 - 2x - x^2}{3x^2} = \frac{1}{3x}$$

$$A(x) = \frac{-(3+x)(x-1)}{x^2} = 1 \quad A(x) = \frac{x}{(1-x)(3+x)} = \frac{1}{4(1-x)} - \frac{3}{4(3+x)}$$

$$\frac{1}{4} \sum_{n=0}^{\infty} x^n - \frac{1}{4} \left(1 + \frac{x}{3} \right) = \sum_{n=0}^{\infty} x^n \left(\frac{1}{4} \left(1 - (-1)^n \cdot \left(\frac{1}{3} \right)^n \right) \right)$$



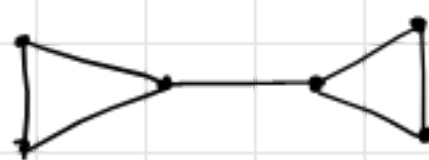
	A	B	C	D	E	F	G
A	∞	∞	∞	∞	∞	∞	∞
B	5	5	5	11	11	11	11
C	∞	∞	11	11	11	11	11
D	∞	∞	12	12	9	9	9
E	∞	∞	8	8	8	8	8
F	6	5	5	5	5	5	5
G	3	3	3	3	3	3	3

ostaje
...
sti $\Rightarrow$ 9
A $\rightarrow$ B $\rightarrow$ E $\rightarrow$ D

3. b) A B G F E C D $\Rightarrow$ put

Nije Hamiltonov ne postoji grana do A.

4 a) 6 čvorova, 2 ciklusa kao podgraf dužine 3



4b) $3k+2$ čvora, $2 \cdot k^2$ grana, 3 komponente povezanosti $k \in \mathbb{N}$

Minimalna komponenta povezanosti: •

$3k$ $2k^2$ Da li može da se napravi povezan graf? Ako ima $n-1$ ili više grana, onda može.

$$2k^2 - 3k + 1 \geq 0$$

$$k_{1/2} = \frac{3 \pm \sqrt{9-8}}{4} \rightarrow \frac{3+1}{4} = 1$$

$$\rightarrow \frac{3-1}{4} = \frac{1}{2}$$



Da li ima previše grana? $k_{3k} \Rightarrow$ broj grana = $\frac{3k(3k-1)}{2} = \frac{9k^2-3k}{2}$

$$2 \cdot k^2 - \frac{9k^2-3k}{2} \geq 0 \quad | \cdot 2 \quad -5k^2 + 3k \geq 0 \quad 3-5k < 0 \quad \forall k \in \mathbb{N}$$

$\Rightarrow$ Imaće manar 3 komponente povezanosti

4v) 4 povezan planarni graf sa bar 3 čvora, stepen $5_{\max}$.

$$3|V| - |E| \geq 6 \quad 2|E| = ?$$

$$3(|V|-2) \geq |E| \cdot 2$$

$$6(|V|-2) \geq 2|E| \quad \text{tj.} \quad 6(|V|-2) \geq 5 \quad |V| \text{ je min } 3$$

$$\Rightarrow 6(3-2) \geq 5 \quad \checkmark$$