

2021 sep

$$1a) f(x) = \frac{x-3}{x^2-3x+2} = \frac{x-3}{(x-2)(x-1)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} = \frac{2}{(x-1)} + \frac{-1}{(x-2)}$$

$$Ax - 2A + Bx - BA = x - 3$$

$$\begin{cases} A+B=1 \\ 2A+B=3 \end{cases} \rightarrow \begin{cases} A=2 \\ B=-1 \end{cases}$$

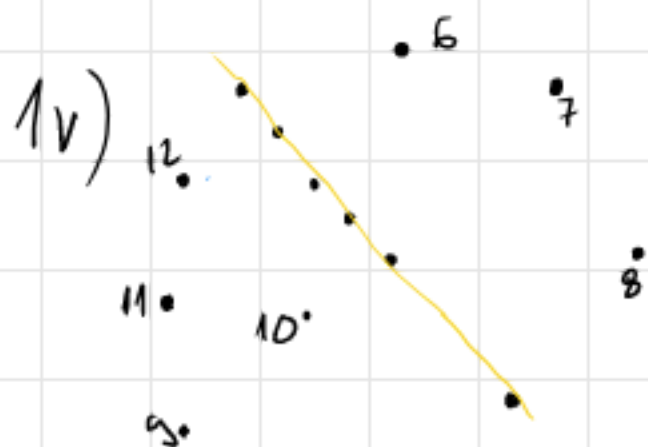
$$= -2(1+(-x))^{-1} + \frac{1}{2}(1+(-\frac{x}{2}))^{-1} = -2 \sum_{n=0}^{\infty} \binom{-1}{n} (-1)^n x^n + \frac{1}{2} \sum_{n=0}^{\infty} \binom{-1}{n} (-\frac{x}{2})^n$$

$$= -2 \sum_{n=0}^{\infty} x^n + \frac{1}{2} \sum_{n=0}^{\infty} (\frac{x}{2})^n = \sum_{n=0}^{\infty} x^n (-2 + (\frac{1}{2})^{n+1}) \Rightarrow \text{uz } x^{50} \text{ je } -2 + \frac{1}{2^{51}}$$

1b) 25 vitezova, kraj A. 3 viteza ^{od kojih nijedan par} ne sedi jedan pored drugo

$$\binom{25}{3} - 23 - 24 \cdot (23-2) = \binom{25}{3} - 23 - 24 \cdot 21$$

$$1 + \sum_{k=(13-1-6)}^{13-1} k = 1 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 1 + 18 \cdot 3 + 9 = 1 + 54 + 9 = 64$$



2.

$$f_n = 2f_{n-1} - f_{n-2} - 6 \quad f_1 = f_0 = 0$$

$$t^2 - 2t + 1 = 0 \rightarrow (t-1)^2 = 0 \quad t_1 = t_2 = 1$$

$$-6 \text{ je } p_d \Rightarrow 1 = 1 \text{ jeste resenje } p_d = n^2 \cdot A$$

$$A \cdot n^2 - 2A(n^2 - 2n + 1) + A(n^2 - 4n + 4) = -6$$

$$A(n^2 - 2n^2 + 4n - 2 + n^2 - 4n + 4) = A \cdot 2 = -6 \Rightarrow A = -3$$

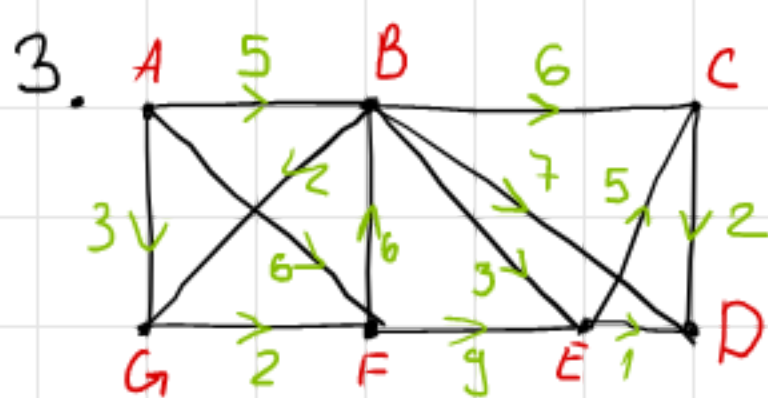
$$a_n = c_1 + n c_2 - 3 \cdot n^2$$

$$c_1 = 0$$

$$0 = c_1 + c_2 - 3$$

$$c_1 = 0, c_2 = 3$$

$$a_n = 3n - 3n^2 = 3n(1-n)$$

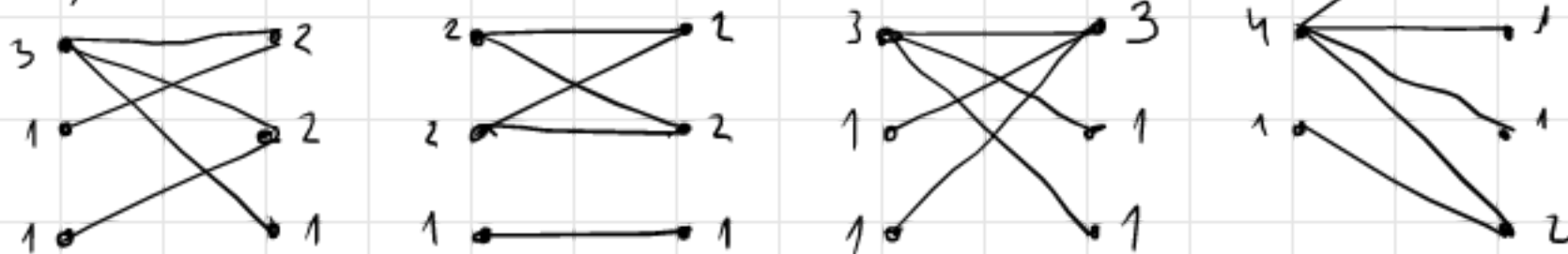


A	6	∞	∞	∞	∞	∞
B	5	5	5	5	5	5
C	∞	∞	11	11	11	11
D	∞	∞	12	12	9	9
E	∞	∞	8	8	8	8
F	6	5	5	5	5	5
G	3	3	3	3	3	3

$\Rightarrow 9$
 $A \rightarrow B \rightarrow E \rightarrow D$

b) D_a ABCDEFGA
 D_a' ABCDEFG

4a)



4b) 7 čvorova, 12 grana

Min broj grana potreban $= (7-1) = 6 < 12 \checkmark$

Max broj grana $\frac{7 \cdot 6}{2} = 21 > 12 \checkmark$

PPS Imamo dve komponente povezanosti

K_5 i 2 čvora $\Rightarrow \frac{5 \cdot 4}{2} = 10$ i 2 čvora povezana 1 granom

$\Rightarrow 11$ grana $< 12 \quad \hookrightarrow$ Mora biti jedna komponenta

4v) n čvorova, $\frac{1}{2} \cdot (n-1)(n-2) + 2$ Svaki čvor je stepena dva

Izdvojimo 1 čvor i pokušajmo da spojimo $(n-1)$ čvor u kompletan graf

Imaće $\frac{(n-1)(n-2)}{2}$ grana. Iskoristili smo maksimalan broj grana i ostaje

nam $\frac{1}{2} \cdot (n-1)(n-2) + 2 - \frac{(n-1)(n-2)}{2} = 2$ grane i 1 čvor. Sa tim tim

te grane moraju povezivati taj čvor sa neka dva iz K_{n-1} .

