2021 Jun 1 Grupa 1

1a) 3 prirodna broja manja od 37 da nisu oblika $x_1 \equiv_3 1$ $x_2 \equiv_3 2$ $x_3 \equiv_3 0$

$[1-36]$ $\left[\frac{36}{3}\right] = 12$

x_1: 12, $r=1$ $\quad x_2$: 12, $r=2$ $\quad x_3$: 12, $r=0$ $\Rightarrow 12^3$

$12^3 + 6 \cdot \binom{12}{2} \cdot 12$

001 110 220
002 112 221

1b) $x_1+x_2+x_3+x_4 = 2021$ $\quad x_1'+x_2+x_3'+x_4 = 2021-4 = 2017 \Rightarrow \binom{2017+3}{2017} = \binom{2020}{2017}$

1v) 1-999 deljivi samo sa 11 ili 13

$S_{11}+S_{13}-S_{11}\cap S_{13} \Rightarrow \left[\frac{999}{11}\right] + \left[\frac{999}{13}\right] - \left[\frac{999}{143}\right] = 90+76-6 = 160$

2. $\begin{cases} a_{n+1} = 4b_{n+1} + 3a_n \\ 4b_{n+1} = \frac{32}{3} b_n + \frac{1}{3} \cdot a_n \\ a_0 = 12,\ b_0 = 0 \end{cases}$

$\Rightarrow b_{n+1} = \frac{a_{n+1} - 3a_n}{4}$

$\Rightarrow a_{n+1} - 3a_n = \frac{32}{3} b_n + \frac{1}{3} a_n$

$a_{n+1} - \frac{10a_n}{3} = \frac{32}{3} b_n \quad / \cdot \frac{3}{32}$

$b_n = \frac{3a_{n+1} - 10a_n}{32}$

$b_{n+1} = \frac{3a_{n+2} - 10a_{n+1}}{32}$ (II)

iz $\frac{a_{n+1} - 3a_n}{4} = \frac{3a_{n+2} - 10a_{n+1}}{32} \quad / \cdot 32$

$8a_{n+1} - 24a_n = 3a_{n+2} - 10a_{n+1} \Rightarrow 3a_{n+2} - 18a_{n+1} + 24a_n = 0 \quad / :3$

$a_{n+2} - 6a_{n+1} + 8a_n = 0 \Rightarrow t^2 - 6t + 8 = 0 \quad (t-2)(t-4) = 0 \quad t_1 = 2,\ t_2 = 4$

$\Rightarrow a_n = c_1 \cdot 2^n + c_2 \cdot 4^n$ Tražimo c_1 i c_2

iz * čemo naći b_1 i a_1 $\Rightarrow 4b_1 = \frac{32}{3} \cdot 0 + \frac{1}{3} \cdot 12 = 4 \Rightarrow \boxed{b_1 = 1}$ $\boxed{a_1 = 4 \cdot 1 + 3 \cdot 12 = 40}$

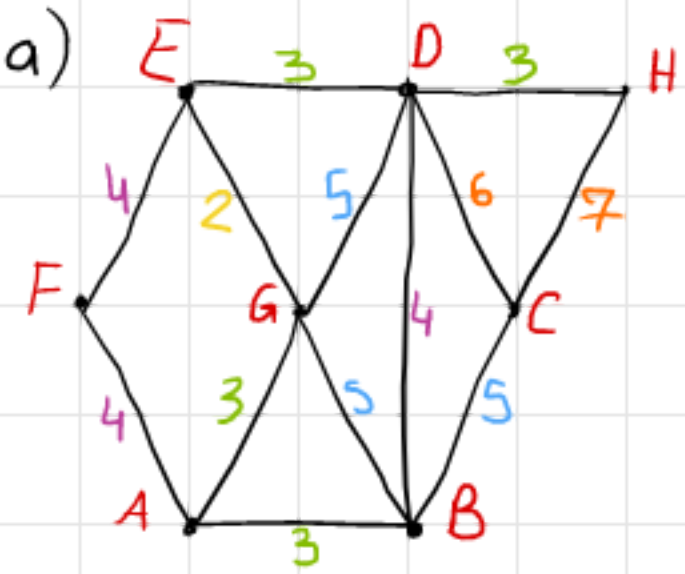
$12 = c_1 + c_2 \ \wedge \ 40 = 2c_1 + 4c_2 \quad / :2$

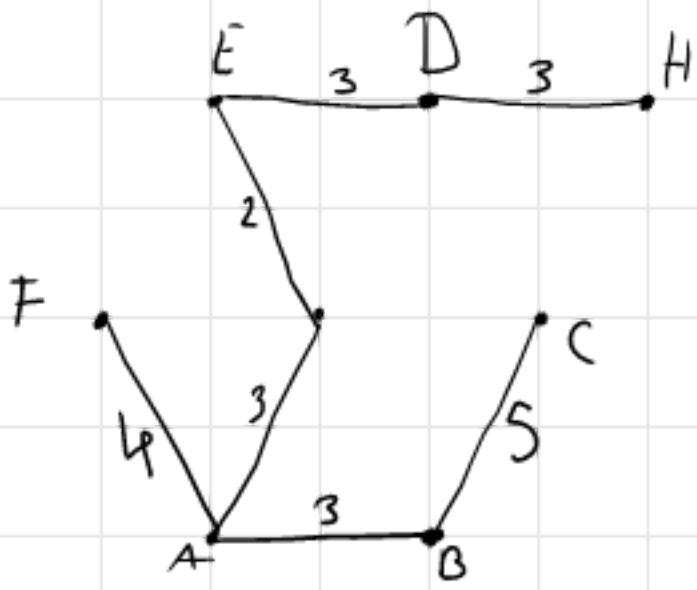
$12 = c_1 + c_2 \ \wedge \ 20 = c_1 + c_2 + c_2 \Rightarrow c_2 = 8,\ c_1 = 4$ tj. $\boxed{a_n = 4 \cdot 2^n + 8 \cdot 4^n}$

iz ✶ $b_n = \frac{3 \cdot 4 \cdot 2^{n+1} + 3 \cdot 8 \cdot 4^{n+1} - 10(4 \cdot 2^n + 8 \cdot 4^n)}{32}$

$b_n = \frac{{}^{3}\cancel{24} \cdot 2^n + {}^{3}\cancel{24} \cdot 4 \cdot 4^n - {}^{5}\cancel{40} \cdot 2^n - {}^{10}\cancel{80} \cdot 4^n}{4 \cdot \cancel{4 \cdot 2}} = \frac{-2^{n+1} + 2 \cdot 4^n}{4} = \boxed{2 \cdot 4^{n-1} - 2^{n-1} = b_n}$

3a)





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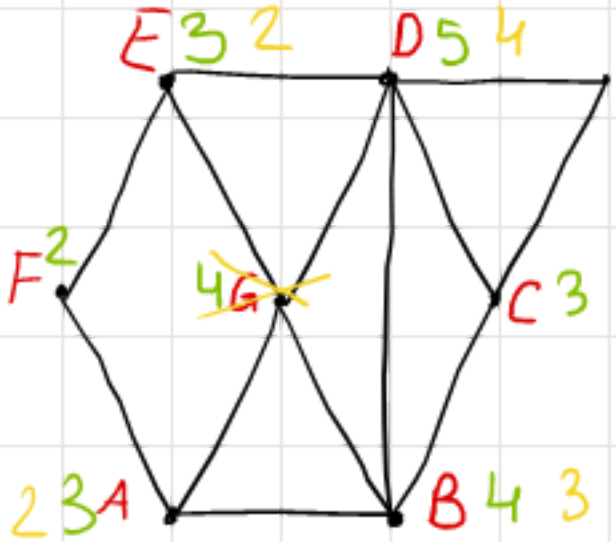


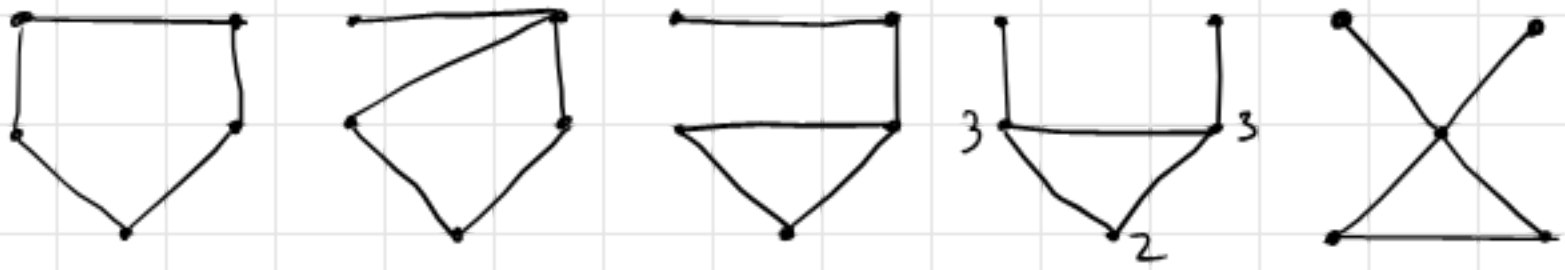
3b) Uklanjamo čvora G

3v) A B C H D G E F A

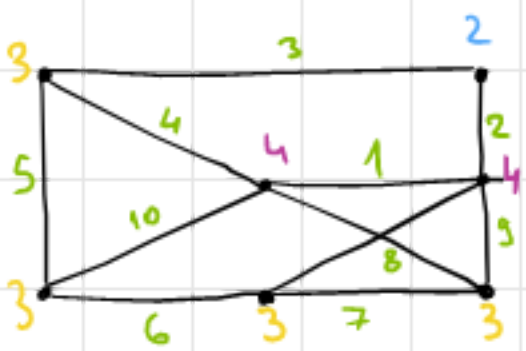
4a) $dg(v) = 3 \vee 4$ za svako v 2 razapinjuća stabla = $4(n-1)$ stepen

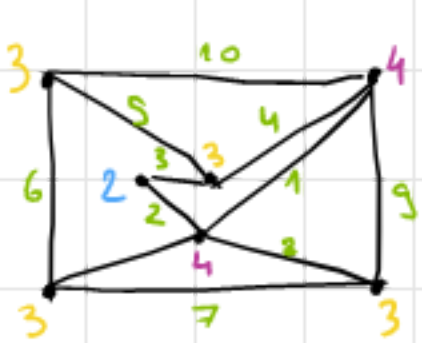
Neka su svi čvorovi stepena 4 $\Rightarrow D(G) = 4 \cdot n$ Oduzećemo $(4-3)x$ gde je x broj čvorova stepena 3 $\Rightarrow 4n - x$ Izjednačimo $\Rightarrow 4n - x = 4n - 4 \Rightarrow \boxed{x = 4}$

4b) 5 čvorova, 5 grana, 1 ciklus



4v)





Jesu

2021. Jun 1 Grupa 2

1a) $\left[\frac{46}{3}\right] = 15$ $\underbrace{r_1}_{16}$ $\underbrace{r_2}_{15}$ $\underbrace{r_3}_{15}$ $16 \cdot 15^2 + \binom{16}{2} \cdot 15 \cdot 2 + 2 \cdot \binom{15}{2} \cdot (15+16)$

1b) Isto kao gore

1v) 17, 19 -> $\left[\frac{999}{17}\right] + \left[\frac{999}{19}\right] - \left[\frac{999}{17 \cdot 19}\right] = 58 + 52 - 3 = 107$

2. $\begin{cases} a_{n+1} = 2b_{n+1} + 6a_n \\ \frac{4}{3} b_{n+1} = -\frac{1}{3} a_{n+1} + 2b_n \\ a_0 = 2, \; b_0 = -\frac{3}{2} \end{cases}$

$\Rightarrow$ * $b_{n+1} = \frac{a_{n+1} - 6a_n}{2}$

$b_n = \frac{1}{2}\left(\frac{4}{3} \cdot b_{n+1} + \frac{1}{3} a_{n+1}\right)$

$b_n = \frac{1}{6}(2a_{n+1} - 12a_n + a_{n+1})$

* $b_n = \frac{1}{2}(a_{n+1} - 4a_n) \overset{n=n+1}{\Rightarrow}$ * $b_{n+1} = \frac{1}{2}(a_{n+2} - 4a_{n+1})$

iz * $a_{n+1} - 6a_n = a_{n+2} - 4a_{n+1} \Rightarrow a_{n+2} - 5a_{n+1} + 6a_n = 0 \Rightarrow t^2 - 5t + 6 = 0$

$t_1 = 2 \quad t_3 = 3 \Rightarrow a_n = c_1 \cdot 2^n + c_2 \cdot 3^n$

U drugu jednačinu ćemo ubaciti $2b_{n+1}$ iz prve $\Rightarrow \frac{2}{3}(a_{n+1} - 6a_n) = -\frac{1}{3} a_{n+1} + 2b_n$

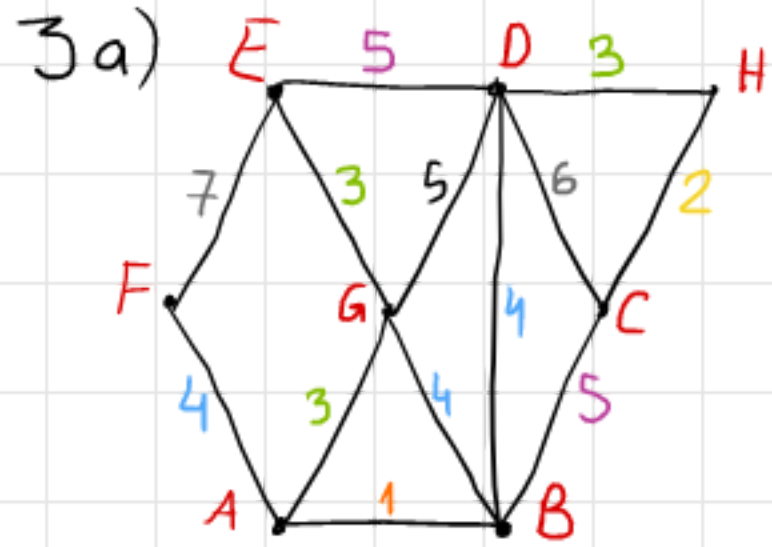
$\Rightarrow 3a_{n+1} = 12a_n + 2b_n \Rightarrow a_1 = \frac{12 \cdot 2 + 2 \cdot -\frac{3}{2}}{3} = \frac{24-3}{3} = 7$

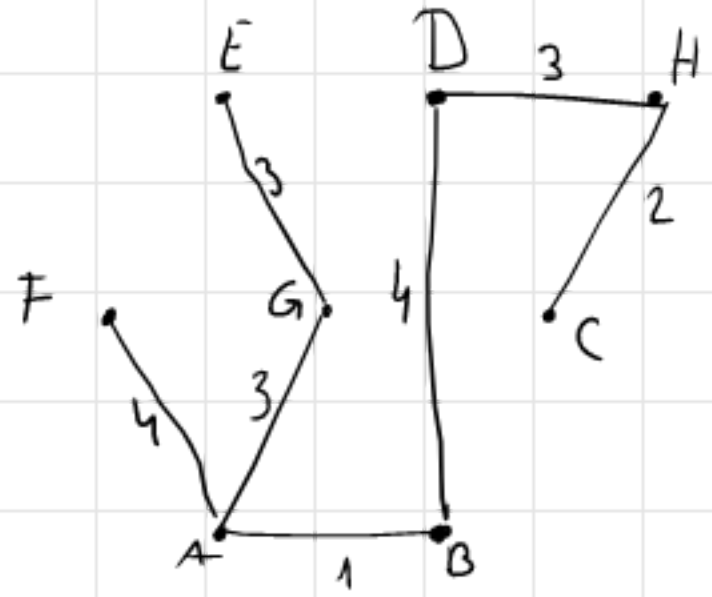
$2 = c_1 + c_2 \wedge 7 = 2c_1 + 3c_2 \Rightarrow 7 = 2 \cdot 2 + c_2 \Rightarrow c_2 = 3, \; c_1 = -1$

$a_n = 3^{n+1} - 2^n$

iz * $b_n = \frac{1}{2}(9 \cdot 3^n - 2 \cdot 2^n - 12 \cdot 3^n + 4 \cdot 2^n) = \frac{1}{2}(2^{n+1} - 3^{n+1})$

3a)





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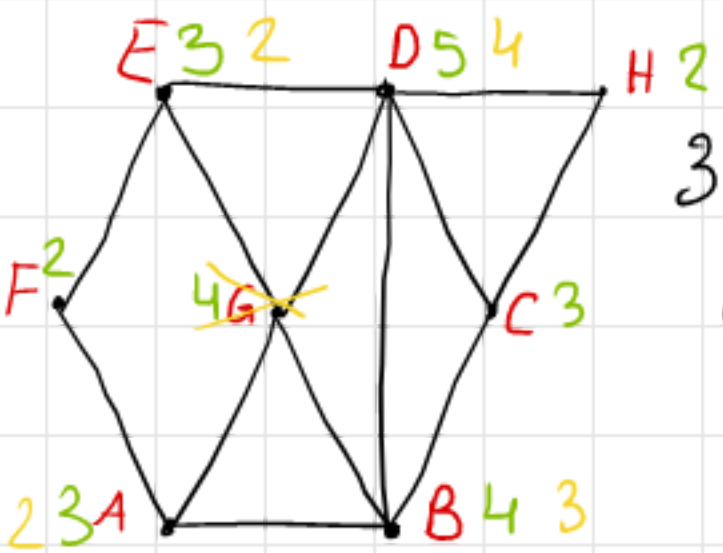

3b) Uhvatimo čvora G

3v) A B C H D G E F A

4. Isti kao gore