2020. Jun 1 grupa 1

1a) $M \times$ maska, N stud $\quad M>N$

↳ Svaki student treba da dobije po 1 masku, ostane $M-N$ maski

$p_1+p_2+\dots+p_N = M-N \qquad \binom{M-N+N-1}{M-N} = \binom{M-1}{M-N}$

1b) $\left(\sqrt[4]{a^2x}+\sqrt[5]{\frac{1}{ax^2}}\right)^{13} \qquad x^{1/4\cdot k_1}+x^{-2/5\, k_2}=x^0$

$\binom{13}{8}\cdot a^4\cdot 1/a = \boxed{a^3\cdot\binom{13}{8}} \qquad 5k_1-8k_2=0 \Rightarrow 5k_1=8k_2$

$k_1=8,\ k_2=5$

1v) Paja, Ana, Ogi $\quad$ 10 autica

$\underbrace{\text{min } 2 \quad \text{min } 3}_{5 \text{ odlučeno}} \quad \text{max } 5$

$x_1+x_2+x_3=10$

$p+2+a+3+o=10 \rightarrow p+a+o=5$

$\binom{5+3-1}{5}=\binom{7}{5}=21$

2.
$$\begin{cases} a_{n+1}=4b_{n+1}+3a_n \\ 4b_{n+1}=\frac{32}{3}b_n+\frac{1}{3}\cdot a_n \\ a_0=12,\ b_0=0 \end{cases}$$

$a_{n+1}=4b_{n+1}+3a_n \Rightarrow b_{n+1}=\frac{a_{n+1}-3a_n}{4}$

$4b_{n+1}=\frac{32}{3}b_n+\frac{1}{3}a_n \Rightarrow a_{n+1}-3a_n=\frac{32}{3}b_n+\frac{1}{3}a_n$

$a_{n+1}-\frac{10a_n}{3}=\frac{32}{3}b_n \;/\cdot\frac{3}{32}$

$b_n=\frac{3a_{n+1}-10a_n}{32}$

$b_{n+1}=\frac{3a_{n+2}-10a_{n+1}}{32}$

iz * $\frac{a_{n+1}-3a_n}{4}=\frac{3a_{n+2}-10a_{n+1}}{32} \;/\cdot 32$

$8a_{n+1}-24a_n=3a_{n+2}-10a_{n+1} \Rightarrow 3a_{n+2}-18a_{n+1}+24a_n=0 \;/:3$

$a_{n+2}-6a_{n+1}+8a_n=0 \Rightarrow t^2-6t+8=0 \quad (t-2)(t-4)=0 \quad t_1=2,\ t_2=4$

$\Rightarrow a_n=c_1\cdot2^n+c_2\cdot4^n$ Tražimo c_1 i c_2

iz * ćemo naći b_1 i a_1 $\Rightarrow 4b_1=\frac{32}{3}\cdot0+\frac{1}{3}\cdot12=4 \Rightarrow \boxed{b_1=1}$ $\boxed{a_1=4\cdot1+3\cdot12=40}$

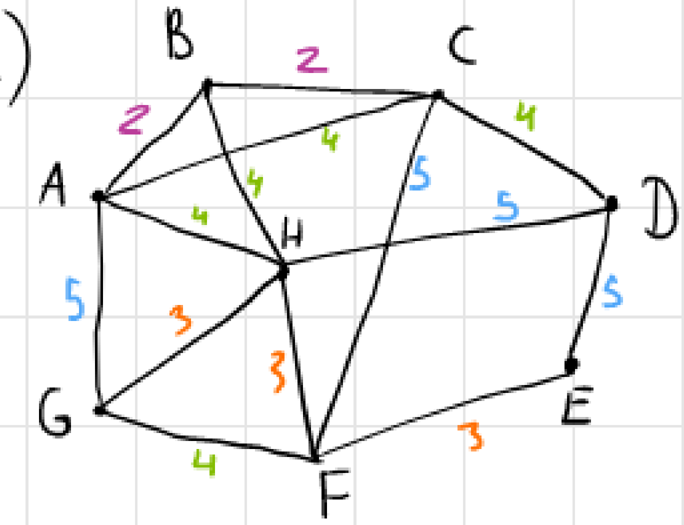
$12=c_1+c_2 \wedge 40=2c_1+4c_2 \;/:2$

$12=c_1+c_2 \wedge 20=\underbrace{c_1+c_2}_{=12}+c_2 \Rightarrow c_2=8,\ c_1=4$ tj. $\boxed{a_n=4\cdot2^n+8\cdot4^n}$

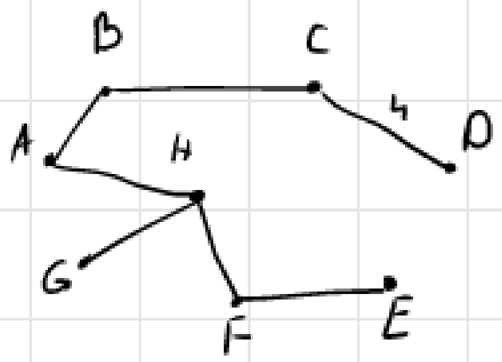
iz * $b_n=\frac{3\cdot4\cdot2^{n+1}+3\cdot8\cdot4^{n+1}-10(4\cdot2^n+8\cdot4^n)}{32}$

$b_n=\frac{\cancel{24}^{3}\cdot2^n+\cancel{24}^{3}\cdot4\cdot4^n-\cancel{40}^{5}\cdot2^n-\cancel{80}^{10}\cdot4^n}{4\cdot\cancel{4\cdot2}}=\frac{-2^{n+1}+2\cdot4^n}{4}=\boxed{2\cdot4^{n-1}-2^{n-1}=b_n}$

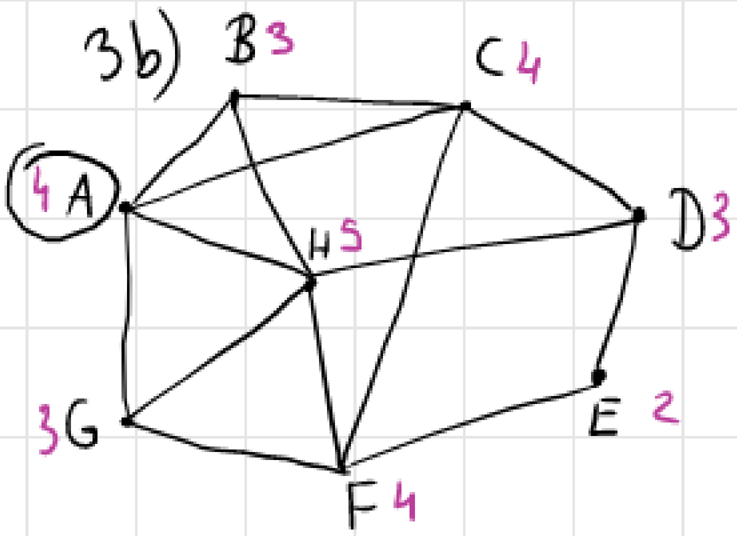
3a)



3v) BAGHFEDC,ba



3b)



4a) $G=(V,E)$ bipartitan sa n čvorova max $\frac{x^2}{4}$ grana

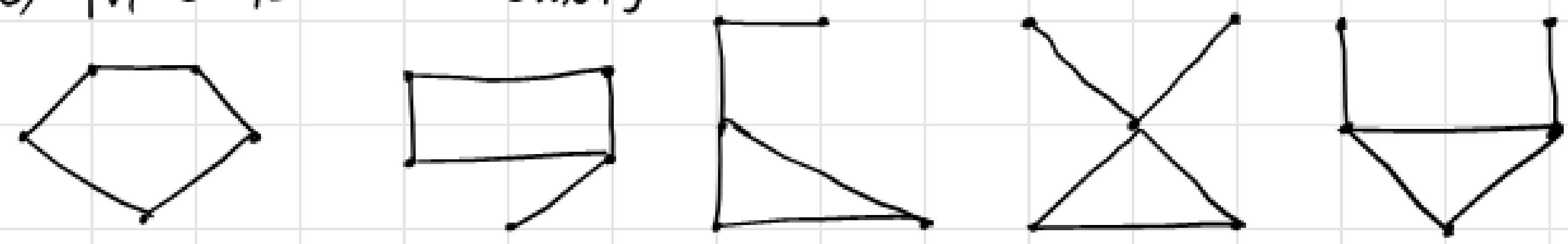
$|V|=n=l+d$ gde je l broj čvorova sa 'leve' strane tj. iz jednog skupa, a d sa 'desne' strane tj. iz drugog skupa.

Kada bismo povezali svaki iz l sa svakim iz desne imali bismo $l\cdot d$ grana. B.G.O. neka je $l\geq d$ tj. $d=l-x$

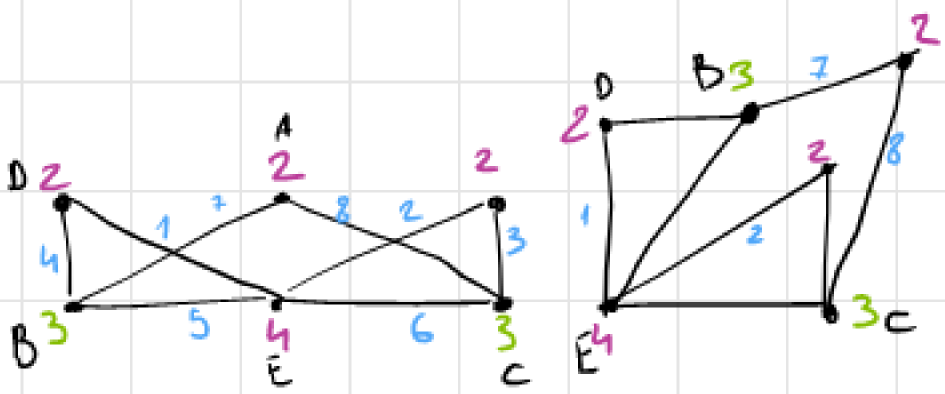
Pitamo se za koje x jednačina $(l-x)\cdot l$ dostiže max. $l^2-l\cdot x$ će biti najveće za $x\leq 0$ kako je x pozitivan broj $\Rightarrow x=0$ tj. $d=l$.

$n=2d \Rightarrow d=\frac{n}{2}$ a MAX grana je $\frac{n}{2}\cdot\frac{n}{2}=\frac{n^2}{4}$

4b) $|V|=5$ $|E|=5$ 1 ciklus



4v)



2020 Jun 1 Grupa 2

1a) M rukavica na N studenata, svako po dve

$x_1 + x_2 + \ldots + x_N = M$

$x_1' + 2 + x_2' + 2 + \ldots + x_N + 2 = M \Rightarrow x_1' + x_2' + \ldots + x_N' = M - 2N$

$\Rightarrow \binom{M-2N+N-1}{M-2N} = \binom{M-N-1}{M-2N}$

1b) isto kao ovaj gore

1v) Sofija, Nikolija, Maja $\quad x_1 + 2 + x_2 + 3 + x_3 = 10$

min 2, min 3, max 5 $\quad x_1 + x_2 + x_3 = 5 \Rightarrow \binom{7}{5} = 21$

2. $\begin{cases} a_{n+1} = 2b_{n+1} + 6a_n \\ \frac{4}{3} b_{n+1} = -\frac{1}{3} a_{n+1} + 2b_n \\ a_0 = 2,\ b_0 = -3/2 \end{cases}$

$\Rightarrow * \; b_{n+1} = \frac{a_{n+1} - 6a_n}{2}$

$\Rightarrow b_n = \frac{1}{2}\left(\frac{4}{3} \cdot b_{n+1} + \frac{1}{3} a_{n+1}\right)$

$b_n = \frac{1}{6}\left(2a_{n+1} - 12a_n + a_{n+1}\right)$

$* \; b_n = \frac{1}{2}\left(a_{n+1} - 4a_n\right) \overset{n=n+1}{\Longrightarrow} b_{n+1} \overset{*}{=} \frac{1}{2}\left(a_{n+2} - 4a_{n+1}\right)$

iz $*$

$a_{n+1} - 6a_n = a_{n+2} - 4a_{n+1} \Rightarrow a_{n+2} - 5a_{n+1} + 6a_n = 0 \Rightarrow t^2 - 5t + 6 = 0$

$t_1 = 2 \quad t_2 = 3 \Rightarrow a_n = c_1 \cdot 2^n + c_2 \cdot 3^n$

U drugu jednačinu ćemo ubaciti $2b_{n+1}$ iz prve $\Rightarrow \frac{2}{3}\left(a_{n+1} - 6a_n\right) = -\frac{1}{3} a_{n+1} + 2b_n$

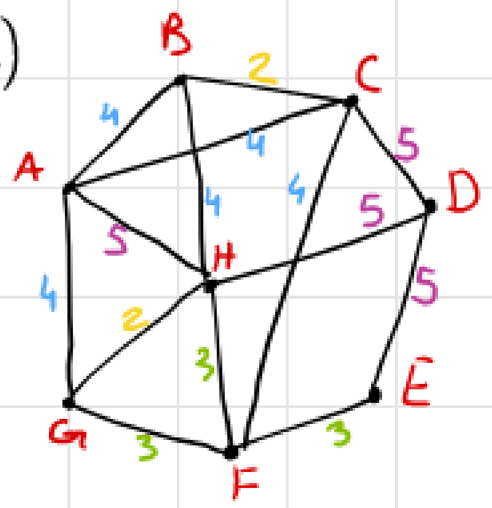
$\Rightarrow 3a_{n+1} = 12a_n + 2b_n \Rightarrow a_1 = \frac{12 \cdot 2 + 2 \cdot -3/2}{3} = \frac{24-3}{3} = 7$

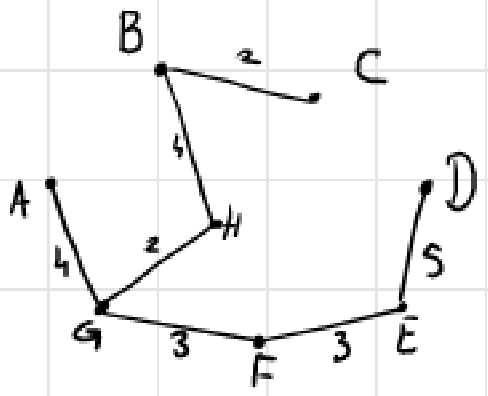
$2 = c_1 + c_2 \wedge 7 = 2c_1 + 3c_2 \Rightarrow 7 = 2 \cdot 2 + c_2 \Rightarrow c_2 = 3,\ c_1 = -1$

$a_n = 3^{n+1} - 2^n$

iz $*$ $\quad b_n = \frac{1}{2}\left(9 \cdot 3^n - 2 \cdot 2^n - 12 \cdot 3^n + 4 \cdot 2^n\right) = \frac{1}{2}\left(2^{n+1} - 3^{n+1}\right)$

3a)

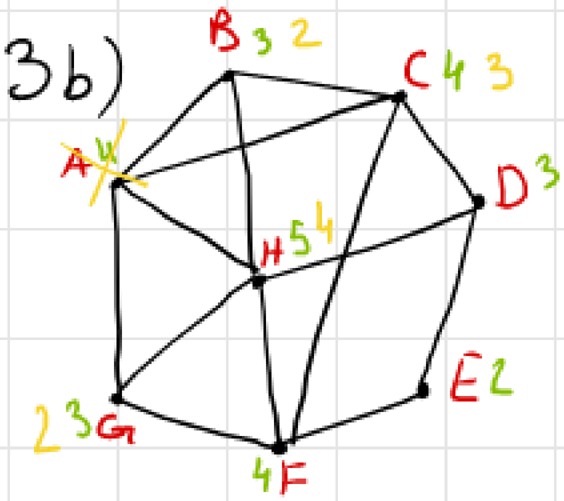




(Ima ih više)

$\boxed{23 \text{ je težina min}}$

3b)



Ako maknemo čvor A imaćemo 2 čvora neparnog stepena i možemo formirati Ojlerov put.

3v) AHGFEDCBA Da

4. Isti kao gore čini mi se