2019 1 klk 1. GRUPA

1. a) 48 kuglica, biramo 35 ; 6 unapred odabrano

$$\binom{48-6}{35-6} = \binom{42}{29}$$

b) 8 gumica, 5 olovaka $\binom{9}{5} \cdot 5!$

2. $\sum_{k=0}^{n} \frac{1}{k^2+3k+2} \binom{n}{k} = \sum_{k=0}^{n} \frac{1}{(n+2)(n+1)} \cdot \frac{n!}{k!(n-k)!} \cdot \frac{(n+1)(n+2)}{(n+1)(n+2)}$

$= \sum_{k=0}^{n} \frac{1}{(n+1)(n+2)} \binom{n+2}{k+2} = \frac{1}{(n+1)(n+2)} \underbrace{\sum_{k=2}^{n} \binom{n+2}{k}}_{2^{n+2} - \binom{n+2}{0} - \binom{n+2}{1}} = \frac{1}{(n+1)(n+2)} \cdot (2^{n+2} - 1 - n - 2)$

$= \frac{2^{n+2} - 3 - n}{(n+1)(n+2)}$

2019. 1klk 2. grupa

1 a) 39 cedulja, bira se 25 7 unapred

$$\binom{39-7}{25-7} = \binom{32}{18}$$

1b) ×O×O×O×O×O×O×O×O×O× $\binom{10}{6} \cdot 6!$

2. $\sum_{k=0}^{n-1} \frac{1}{k^2+3k+2} \binom{n-1}{k} = \sum_{k=0}^{n-1} \binom{n+1}{k+2} \cdot \frac{1}{n(n+1)} = \frac{1}{(n+1) \cdot n} \underbrace{\sum_{k=2}^{n-1} \binom{n+1}{k}}_{2^{n+1} - 1 - n - 1}$

$= \frac{1}{n(n+1)} \cdot (2^{n+1} - n - 2)$

3. a) $(0, 1, \frac{1}{3}, 0, \frac{1}{27}, 0, \frac{1}{243}, \ldots) = x + \frac{x^2}{3} + \frac{x^4}{3^3} + \frac{x^6}{3^5} + \ldots =$

$x\left(1 + \frac{x}{3} + \frac{x^3}{3^3} + \frac{x^5}{3^5} + \ldots\right) = x \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^{2n+1} + x = x \cdot \frac{x}{3} \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^{2n} + x =$

$= x \cdot \frac{x}{3} \cdot \frac{1}{1 - \left(\frac{x}{3}\right)^2} + x = \frac{x^2}{3 - \frac{x^2}{3}} + x = \frac{3x^2}{9 - x^2} + x = \frac{-x^3 + 3x^2 + 9x}{9 - x^2}$

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b) $(0, 2, \frac{3}{3}, 0, \frac{5}{27}, 0, \frac{7}{243}, \ldots) = 2x + 3 \cdot \frac{1}{3} x^2 + 5 \cdot \left(\frac{1}{3}\right)^3 x^4 + 7 \cdot \left(\frac{1}{3}\right)^5 \cdot x^6$

$= \sum_{n=0}^{\infty} (2n+1) \cdot x^{2n} \cdot \left(\frac{1}{3}\right)^{2n-1} - 3 + 2x = 3 \cdot \sum_{n=0}^{\infty} \left(x^2 \cdot \frac{1}{9}\right)^n \cdot (2n+1) - 3 + 2x$

$= 3 \cdot \sum_{n=0}^{\infty} (x^2 \cdot 1/9)^n \cdot 2n + 3 \cdot \sum_{n=0}^{\infty} \left(x^2 \cdot \frac{1}{9}\right)^n - 3 + 2x = \frac{6t}{(1-t)^2} + \frac{3}{(1-t)} - 3 + 2x \qquad \frac{x^2}{9} = t$

$= \frac{6t + 3 - 3t}{(1-t)^2} - 3 + 2x = \frac{3(t+1)}{(1-t)^2} - 3 + 2x = \frac{1/3(x^2+9)}{(9-x^2)^2 \cdot 1/81} = \frac{27(x^2+9)}{(9-x^2)^2} - 3 + 2x$

NAPOMENA: Mislim da 3. nije tačan, ali me mrzi da prolazim kroz ovu agoniju treći/četvrti put

4. $a_{n+3} - 8a_{n+2} + 21a_{n+1} - 18a_n = 2^n$

$a_0 = 2, \ a_1 = 9, \ a_2 = 35$

$t^3 - 8t^2 + 21t - 18 = 0 \Rightarrow (t-2)(t^2 - 6t + 9) = (t-2)(t-3)^2$

$t_1 = 2, \ t_2 = t_3 = 3 \Rightarrow p = 2^n \quad b = 2 = t_1$ jeste $s = 1 \quad p_n = n \cdot 2^n \cdot A$

$A(n+3) \cdot 8 \cdot \cancel{2^n} - A8 \cdot 4 \cdot \cancel{2^n} \cdot (n+2) + A21 \cdot 2 \cdot \cancel{2^n} \cdot (n+1) - A18 \cdot n \cdot \cancel{2^n} = \cancel{2^n} \quad / : 2^n$

$A(\cancel{8n} + 24 - \cancel{32n} - 64 + \cancel{42n} + 42 - \cancel{18n}) = 1$

$A(66 - 64) = 1 \Rightarrow A = 1/2 \Rightarrow a_n = 2^n \cdot c_1 + c_2 \cdot 3^n + c_3 \cdot n \cdot 3^n + \frac{2^n \cdot n}{2}$

$$\left.\begin{aligned} 2 &= c_1 + c_2 \\ 9 &= 2c_1 + 3c_2 + 3c_3 + 1 \\ 35 &= 4c_1 + 9c_2 + 18c_3 + 4 \end{aligned}\right\} \quad \left.\begin{aligned} 2 &= c_1 + c_2 \\ 4 &= c_2 + 3c_3 \\ 31 &= 8 + 5c_2 + 18c_3 \end{aligned}\right\} \quad \left.\begin{aligned} 2 &= c_1 + c_2 \\ 4 &= c_2 + 3c_3 \\ 23 &= 5c_2 + 18c_3 \end{aligned}\right\} \quad \begin{aligned} 2 &= c_1 + c_2 \\ c_2 &= 4 - 3c_3 \\ 3 &= 3c_3 \end{aligned}$$

$c_3 = 1, \ c_2 = 1, \ c_1 = 1 \Rightarrow a_n = 2^n + 3^n + 3^n \cdot n + 2^{n-1} \cdot n$