2018 1. k/k 1. grupa

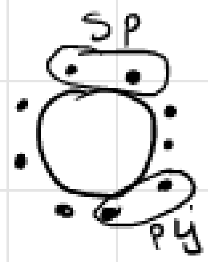
1. a) $x_1+x_2+x_3+x_4=25$

$x_1 \geq 0,\quad x_2 \geq 1,\quad x_3 \geq 4,\quad x_4 \geq -1$

$t_1=x_1,\ t_1\geq 0 \qquad x_2=t_2+1,\ t_2\geq 0 \qquad x_3=t_3+4 \qquad x_4=t_4-1$

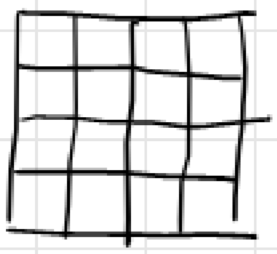
$t_1+t_2+1+t_3+4+t_4-1=25 \qquad t_1+t_2+t_3+t_4=21 \Rightarrow \binom{21+4-1}{21}=\binom{24}{21}$

b) Snežana + Princ, Pospanko + Ljutko, 5 patuljaka



Posmatramo S+P kao jednu osobu, P+Lj kao drugu

$\Rightarrow 6!\cdot 2\cdot 2 \quad \left(2 \begin{smallmatrix} \nearrow S+P \\ \searrow P+S \end{smallmatrix} \quad 2 \begin{smallmatrix} \nearrow P+Lj \\ \searrow Lj+P \end{smallmatrix}\right)$

v) 

plava, crvena, bela $=\{c,p,b\}$

bela: 1/0 crvena: 1/0 $\Rightarrow$

bela 0, crvena 1 $\to n^2$ opcija

bela 1, crvena 0 $\to n^2$ opcija

bela 0, crvena 0 $\to$ 1 opcija

bela 1, crvena 1 $\to n^2\cdot(n^2-1)=n^4-n^2$

$\Rightarrow n^4-n^2+2n^2+1=n^4+n^2+1$

2. $\sum_{k=0}^{n}(-1)^k\frac{k}{k+1}\binom{n}{k}=\frac{-1}{n+1}$

$\sum_{k=0}^{n}(-1)^k\left(1+\frac{-1}{k+1}\right)\binom{n}{k}=\sum_{k=0}^{n}(-1)^k\left(-\frac{1}{k+1}\right)\frac{n!}{k!\,(n-k)!}=\sum_{k=0}^{n}(-1)^k(-1)\binom{n+1}{k+1}\cdot\frac{1}{n+1}$

$=\sum_{k=0}^{n}(0+1)(-1)\cdot\frac{1}{n+1}=-\frac{1}{n+1}$

3. a) $(\underset{0}{0}, \underset{1}{2}, \underset{2}{0}, \underset{3}{4}, \underset{4}{0}, \underset{5}{6}, \underset{6}{0}, \underset{7}{8}, \dots) = 2x + 4x^3 + 6x^5 + 8x^7 + \dots = (1 + x^2 + x^4 + x^6 + \dots)'$

$= \lceil t = x^2 \rfloor \Rightarrow (1 + t + t^2 + t^3 + \dots)' = \left(\frac{1}{1-x^2}\right)' = \frac{1}{(1-x^2)^2}$

b) $(0, 1, 0, \underset{2\cdot3}{6}, 0, \underset{3\cdot5}{15}, 0, \underset{4\cdot7}{28} \dots) = x + 6x^3 + 15x^5 + 28 \cdot x^7 + \dots$

$= 1 \cdot x + 2 \cdot 3 \cdot x^3 + 3 \cdot 5 \cdot x^5 + 4 \cdot 7 \cdot x^7 + \dots =$

$= x\,(1 + 2 \cdot 3x^2 + 3 \cdot 5x^4 + 4 \cdot 7x^6 + \dots) = \lceil x^2 = t \rfloor = x(1 + 6 \cdot t + 15t^2 + 28t^3 + \dots)$

$= x(t + 3 \cdot t^2 + 5 \cdot t^3 + 7 \cdot t^4 + \dots)' = x(t + 2 \cdot t^2 + 3 \cdot t^3 + 4 \cdot t^4 + \dots$

$+ (t^2 + 2t^3 + 3t^4 + \dots\dots))' = x\,(t(1 + 2t + 3t^2 + 4t^3 \dots) + t^2(1 + 2t + 3t^2 + \dots))'$

$= x\left(t \cdot \frac{1}{(1-t)^2} + t^2 \cdot \frac{1}{(1-t)^2}\right)' = x\left(\frac{t+t^2}{(1-t)^2}\right)' = x\left(\frac{(1+2t)(1-t)^{\cancel{2}} - 2(\cancel{1-t}) \cdot (-1)}{(1-t)^{\cancel{4}}}\right)$

$= x\left(\frac{1 - t + 2t - 2t^2 + 2}{(1-t)^3}\right) = x\left(\frac{3 + x^2 - 2x^4}{(1-x^2)^3}\right)$

4. $a_{n+3} = a_{n+2} + 16 \cdot a_{n+1} + 20a_n + 49 \cdot 5^n \quad a_0 = 2,\ a_1 = 2,\ a_2 = 47$

$a_{n+3} - a_{n+2} - 16a_{n+1} - 20a_n = 49 \cdot 5^n \Rightarrow a_n = h_n + p_n$

$t^3 - t^2 - 16t - 20 = 0 \qquad (t+2)(t^2 - 3t - 10) = (t+2)(t-5)(t+2)$

$t_1 = -2,\ t_2 = 5,\ t_3 = -2 \qquad k \cdot b^n = 49 \cdot 5^n \rightarrow 5 = 5$ jeste rešenje

$p_n = n \cdot 5^n \cdot A$

$(n+3)\,5^3 \cdot \cancel{5^n} \cdot A - A(n+2)\,25 \cdot \cancel{5^n} - 16A(n+1)\,5 \cdot \cancel{5^n} - 20 \cdot A\,n \cdot \cancel{5^n} = 49 \cdot \cancel{5^n}$

$A(\cancel{125n} + 375 - \cancel{25n} - 50 - \cancel{80n} - 80 - \cancel{20n}) = 49$

$A \cdot 245 = 49 \Rightarrow A = 49/245 \Rightarrow p_n = 49 \cdot n \cdot 5^n \cdot \frac{1}{245}$

$a_n = c_1(-2)^n + c_2 \cdot n \cdot (-2)^n + c_3 \cdot 5^n + \frac{49}{245} \cdot n \cdot 5^n$

$2 = c_1 + c_3 \qquad 2 = -2c_1 - 2c_2 + 5c_3 + \frac{49}{49} \qquad 47 = 4c_1 + 8c_2 + c_3 \cdot 25 + 2 \cdot 5$

$\left.\begin{array}{l} c_1 + c_2 = 2 \\ -2c_1 - 2c_2 + 5c_3 = 1 \\ 4c_1 + 8c_2 + 25c_3 = 37 \end{array}\right\} \quad \left.\begin{array}{l} c_1 + c_2 = 2 \\ c_3 = 1 \\ 4c_1 + 8c_2 = 12 /:4 \end{array}\right\} \quad \left.\begin{array}{l} c_3 = 1 \\ c_1 = 2 - c_2 \\ 2 - c_2 + 2c_2 = 3 \end{array}\right\} \quad \begin{array}{l} c_3 = 1 \\ c_2 = 1 \\ c_1 = 1 \end{array}$

$\Rightarrow a_n = (-2)^n + n(-2)^n + 5^n + n \cdot 5^{n-1}$