of a magnetic field on electron spins, for example); in this case, also, σ_ν ($\mathbf{n}$) might depend on the direction $\mathbf{n}$ of propagation of the radiation. If the scattering centers are randomly oriented, however, the quantity in square brackets in (2.103) vanishes in an isotropic medium, since then the differential scattering cross section depends only on the angle between $\mathbf{n}$ and $\mathbf{n}'$; moreover, $\sigma_\nu(\mathbf{n}) = \sigma_\nu$, independent of $\mathbf{n}$. We then have the important result that, *if the scattering centers are randomly oriented in an isotropic medium, the effects of induced emission and absorption in coherent scattering cancel out exactly in the equation of transfer and consequently do not have to be taken into account.* This exact cancelling is possible for scattering because there is both induced *absorption* and induced *emission* in the scattering process. We shall henceforth assume, unless we specifically state otherwise, that the scattering centers are randomly oriented in an isotropic medium, so that the effects of induced (coherent) scattering can be omitted from the equation of transfer (*i.e.,* we assume (2.68) and (2.69) to be valid); this assumption is permissible in the most cases of astrophysical interest. [1] We recall again that the assumption of coherence in the scattering is probably permissible with good accuracy as long as the electron gas is non-relativistic (*i.e.,* if $T \ll 5 \times 10^9 {}^\circ K$).

In the case of random orientations of the scattering centers and for coherent scattering it is convenient to define a *phase function* $p_\nu(\mathbf{n},\mathbf{n}')$ by the following relations:

$$d\sigma_\nu(\mathbf{n},\mathbf{n}') = \sigma_\nu p_\nu(\mathbf{n},\mathbf{n}')d\omega_n/4\pi = \sigma_\nu p_\nu(\mathbf{n}',\mathbf{n})d\omega_n/4\pi, \tag{1}$$

where σ_ν (total cross section) is given by (2.67). The second equality in (1) arises because p_ν ($\mathbf{n}$,$\mathbf{n}'$) depends only on the angle between $\mathbf{n}$ and $\mathbf{n}'$. From (2.67) it follows, also, that

$$\int_{4\pi} p_\nu(\mathbf{n},\mathbf{n}')\frac{d\omega_{n'}}{4\pi} = \int_{4\pi} p_v(\mathbf{n}',\mathbf{n})\frac{d\omega_n}{4\pi} = 1. \tag{2}$$

[1] It can be shown, by a simple application of Fermi statistics, in fact, that in the presence of a magnetic field H, the electron spins will be randomly oriented if $kT \gg 2(eh/2m_e c)H$ if the electron gas is partially degenerate or non-degenerate. The lower limit of T is much smaller than this if the electron gas is highly degenerate (*cf.* Chap. 24). Numerically, we have that random orientations of the electron spins will obtain if $T \gg 1.34\times10^{-4}\alpha H^\circ_{gauss}K$, where H is expressed in gauss and $\alpha = 1$ for non-degenerate cases and $\alpha \ll 1$ for highly degenerate cases. For $H = 10^8$ gauss (of the order of the strongest permissible field strengths in equilibrium stars with masses and radii of solar order (*cf.* Sect. 17.5)) and $\alpha = 1$, we have that $T \gg 13,400^\circ K$ for random orientations of spin. Since this condition is fulfilled throughout most of the mass of a magnetic star, it is probably safe to assume random orientations for the electron spins.

For *isotropic* coherent scattering we have
$p_\nu(\mathbf{n}, \mathbf{n}') = p_\nu(\mathbf{n}', \mathbf{n}) = const. = 1$, from (2).

For random orientations of the scattering centers we then have left in the scattering terms only the direct emission and absorption, and the equation of transfer becomes for coherent scattering and for bound-bound transitions

$$\begin{aligned}\mu_\nu^2 \frac{d}{\rho ds}\left(\frac{I_\nu}{\mu_\nu^2}\right) &= \frac{1}{\rho}\Bigg[h\nu_0 N_2 A_{21}\psi^*(\nu) + h\nu_0 N_2 B_{21} I_\nu \psi(\nu) - h\nu_0 N_1 B_{12} I_\nu \phi(\nu) \\ &\quad + N_e\sigma_\nu \int_{4\pi} I_\nu(\mathbf{n}') p_\nu(\mathbf{n}', \mathbf{n}) \frac{d\omega_{n'}}{4\pi} - N_e \sigma_\nu I_\nu\Bigg] \\ &= \frac{h\nu_0}{\rho} N_1 B_{12}\phi(\nu)\left\{\frac{N_2 A_{21}\psi^*(\nu)}{N_1 B_{12}\phi(\nu)} + \left[\frac{N_2 B_{21}\psi(\nu)}{N_1 B_{12}\phi(\nu)} - 1\right] I_\nu\right\} \\ &\quad + \frac{N_e\sigma_\nu}{\rho}\left\{\int_{4\pi} I_\nu(\mathbf{n}') p_\nu(\mathbf{n}', \mathbf{n}) \frac{d\omega_{n'}}{4\pi} - I_\nu\right\}.\end{aligned} \tag{3}$$

This equation may also be written in the form

$$\begin{aligned}\mu_\nu^2 \frac{d}{\rho ds}\left(\frac{I_\nu}{\mu_\nu^2}\right) &= \kappa_\nu^{(a)}\left[1 - \frac{N_2 B_{21}\psi(\nu)}{N_1 B_{12}\phi(\nu)}\right]\left\{\frac{A_{21}\psi^*(\nu)}{B_{21}\psi(\nu)}\left[\frac{N_1 B_{12}\phi(\nu)}{N_2 B_{21}\psi(\nu)} - 1\right]^{-1} - I_\nu\right\} \\ &\quad + \kappa_\nu^{(s)}\left\{\int_{4\pi} I_\nu(\mathbf{n}') p_\nu(\mathbf{n}', \mathbf{n}) \frac{d\omega_{n'}}{4\pi} - I_\nu\right\},\end{aligned} \tag{4}$$

where we have

$$\kappa_\nu^{(a)} \equiv \frac{h\nu_0}{\rho} N_1 B_{12}\phi(\nu) \text{ and } \kappa_\nu^{(s)} \equiv \frac{N_e \sigma_\nu}{\rho}, \tag{5}$$

representing the respective mass absorption coefficients for "true" absorption uncorrected for induced emission (*cf.* (2.84)) and for direct, coherent scattering (*cf.* (2.87)). We note that $N_2 B_{21}\psi(\nu)/N_1 B_{12}\phi(\nu)$ gives the ratio of the induced emission of the true absorption. Recall that effects of dispersion, if present, are contained in $\kappa_\nu^{(a)}$, $\psi^*(\nu)$, $\psi(\nu)$, $\phi(\nu)$, and σ_ν (*cf.* Sect. 3.6).

We wish to remind the reader once again that (3) is not a complete equation of transfer, as only bound-bound transitions have been explicitly included in the "true" emission and absorption terms (see the discussion at the beginning of Sect. 2.8). Two other terms, representing bound-free and free-free transitions, would appear on the right side of (3) in a complete equation of transfer.