Symmetric multiple chessboard complexes and some theorems of Tverberg type

Siniša Vrećica, University of Belgrade
joint work with Duško Jojić and Rade Živaljević
Chessboard complex $\Delta_{m,n}$

- **vertices of $\Delta_{m,n}$**: squares in a chessboard which has $n$ rows and $m$ columns,
Chessboard complex $\Delta_{m,n}$

- **vertices of $\Delta_{m,n}$**: squares in a chessboard which has $n$ rows and $m$ columns,
- **simplices of $\Delta_{m,n}$**: non-taking rooks placements, i.e. at most one vertex from each row and each column,
Chessboard complex $\Delta_{m,n}$

- **vertices of $\Delta_{m,n}$**: squares in a chessboard which has $n$ rows and $m$ columns,
- **simplices of $\Delta_{m,n}$**: non-taking rooks placements, i.e. at most one vertex from each row and each column,
- The first examples: $\Delta_{3,2}$ is a hexagon, $\Delta_{4,3}$ is a torus.
The first examples

\[ \Delta_{3,2} \text{ is a hexagon} \]
The first examples

<table>
<thead>
<tr>
<th></th>
<th>(1, 1)</th>
<th>(2, 1)</th>
<th>(3, 1)</th>
<th>(4, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 2)</td>
<td>(2, 2)</td>
<td>(3, 2)</td>
<td>(4, 2)</td>
<td></td>
</tr>
<tr>
<td>(1, 3)</td>
<td>(2, 3)</td>
<td>(3, 3)</td>
<td>(4, 3)</td>
<td></td>
</tr>
</tbody>
</table>

Figure: $\Delta_{4,3}$ is a torus
Chessboard complexes appear as...

coset complex of certain subgroups in the symmetric group
Chessboard complexes appear as...

coset complex of certain subgroups in the symmetric group

\[ \Delta_{m,n} = \Delta(\mathcal{S}_n, \mathcal{H}_m) \]
Chessboard complexes appear as...

coset complex of certain subgroups in the symmetric group

\[ \Delta_{m,n} = \Delta(S_n, \mathcal{H}_m) \]

matching complex in a complete bipartite graph
Chessboard complexes appear as...

coset complex of certain subgroups in the symmetric group

\[ \Delta_{m,n} = \Delta(S_n, \mathcal{H}_m) \]

matching complex in a complete bipartite graph

\[ \Delta_{m,n} = M(K_{m,n}) \]
Chessboard complexes appear as...

coset complex of certain subgroups in the symmetric group

\[ \Delta_{m,n} = \Delta(\mathcal{S}_n, \mathcal{H}_m) \]

matching complex in a complete bipartite graph

\[ \Delta_{m,n} = M(K_{m,n}) \]

n-fold 2-deleted join of vertices of the \((m - 1)\)-simplex
Chessboard complexes appear as...

coset complex of certain subgroups in the symmetric group

\[ \Delta_{m,n} = \Delta(\mathcal{S}_n, \mathcal{H}_m) \]

matching complex in a complete bipartite graph

\[ \Delta_{m,n} = M(K_{m,n}) \]

n-fold 2-deleted join of vertices of the \((m - 1)\)-simplex

\[ \Delta_{m,n} = \left( (\sigma^{m-1})^{(0)} \right)_{\Delta(2)}^* \Delta(2) = \left( [1]_{\Delta(2)}^m \right)_{\Delta(2)}^* \]
Properties of chessboard complexes are important!

- \( \Delta_{m,n} \) is \((n - 2)\)-connected for \( m \geq 2n - 1 \).
Properties of chessboard complexes are important!

- $\Delta_{m,n}$ is $(n - 2)$-connected for $m \geq 2n - 1$.
- For a prime-power $r = p^{\alpha}$ there is no $(\mathbb{Z}_p)^{\alpha}$-map $(\Delta_{r,2r-1})^{(d+1)} \to (\mathbb{R}^d)^{*r}_\Delta$. 
Properties of chessboard complexes are important!

• \( \Delta_{m,n} \) is \((n-2)\)-connected for \( m \geq 2n-1 \).
• For a prime-power \( r = p^\alpha \) there is no \((\mathbb{Z}_p)^\alpha\)-map \((\Delta_{r,2r-1})^{*(d+1)} \rightarrow (\mathbb{R}^d)^*\).
• This property was used in the proofs of some results in Discrete Geometry and Combinatorics.
**Multiple chessboard complex** \( \Delta_{m,n}^{k_1,\ldots,k_n;l_1,\ldots,l_m} 

- **vertices**: squares in a chessboard which has \( n \) rows and \( m \) columns,
Multiple chessboard complex $\Delta_{m,n}^{k_1,\ldots,k_n,l_1,\ldots,l_m}$

- **vertices**: squares in a chessboard which has $n$ rows and $m$ columns,
- **simplices**: having at most $k_i$ vertices from the $i$-th row and at most $l_j$ vertices from the $j$-th column for each $i, j,$
Multiple chessboard complex $\Delta_{m,n}^{k_1,\ldots,k_n;l_1,\ldots,l_m}$

- **vertices**: squares in a chessboard which has $n$ rows and $m$ columns,
- **simplices**: having at most $k_i$ vertices from the $i$-th row and at most $l_j$ vertices from the $j$-th column for each $i, j$,
The important special case

- If $k_1 = \cdots = k_n = k$ and $l_1 = \cdots = l_m = l$, we denote it by $\Delta^{k;l}_{m,n}$.
The important special case

- If $k_1 = \cdots = k_n = k$ and $l_1 = \cdots = l_m = l$, we denote it by $\Delta^{k;l}_{m,n}$.
- We are mainly interested in $\Delta^{k_1;\ldots;k_n;1}_{m,n}$ and $\Delta^{k;1}_{m,n}$, i.e. in the case $l_1 = \cdots = l_m = 1$. 

• The first examples: $\Delta^{2;1}_{1,3}, 2 \approx S^1 \times D^1, \Delta^{2;1}_{1,4}, 2 \approx S^2, \Delta^{2;1}_{1,5}, 2 \approx S^3$. 
The important special case

- If \( k_1 = \cdots = k_n = k \) and \( l_1 = \cdots = l_m = l \), we denote it by \( \Delta_{m,n}^{k;l} \).
- We are mainly interested in \( \Delta_{m,n}^{k_1,\ldots,k_n;1} \) and \( \Delta_{m,n}^{k;1} \), i.e. in the case \( l_1 = \cdots = l_m = 1 \).
- The first examples:

\[
\Delta_{3,2}^{2;1} \approx S^1 \times D^1, \quad \Delta_{4,2}^{2,1;1} \approx S^2, \quad \Delta_{5,2}^{2;1} \approx S^3.
\]
The new examples

Figure: $\Delta_{3,2}^{2,1}$ is a triangulation of cylinder
The new examples

Figure: $\Delta_{4,2}^{2,1;1} \cong S^2$
The new examples

\[ \Delta^{2,1}_{5,2} \approx S^3 \]
\[ \Delta^{2,1}_{5,2} \cong S^3 \]

- link of a vertex in \( \Delta^{2,1}_{5,2} \) is \( \Delta^{2,1;1}_{4,2} \cong S^2 \)
The new examples

$\Delta_{5,2}^{2,1} \cong S^3$

- link of a vertex in $\Delta_{5,2}^{2,1}$ is $\Delta_{4,2}^{2,1,1} \cong S^2$
- link of an edge in $\Delta_{5,2}^{2,1}$ is a circle
  (old chessboard complex $\Delta_{3,2}$ or $\Delta_{3,1}^{2,1}$)
The new examples

$\Delta_{5,2}^{2,1} \approx S^3$

- link of a vertex in $\Delta_{5,2}^{2,1}$ is $\Delta_{4,2}^{2,1;1} \cong S^2$
- link of an edge in $\Delta_{5,2}^{2,1}$ is a circle
  (old chessboard complex $\Delta_{3,2}$ or $\Delta_{3,1}^{2,1}$)
- link of a 2-simplex in $\Delta_{5,2}^{2,1}$ is a set of two points
The new examples

$\Delta_{5,2}^{2,1} \approx S^3$

- link of a vertex in $\Delta_{5,2}^{2,1}$ is $\Delta_{4,2}^{2,1;1} \cong S^2$
- link of an edge in $\Delta_{5,2}^{2,1}$ is a circle
  (old chessboard complex $\Delta_{3,2}^{3,1}$ or $\Delta_{3,1}^{2,1}$)
- link of a 2-simplex in $\Delta_{5,2}^{2,1}$ is a set of two points
- $\Delta_{5,2}^{2,1}$ is a 2-connected, simplicial 3-manifold.
The appearances

- $\Delta_{m,n}^{k;l}$ is a "matching" complex of $K_{m,n}$ where each red vertex is matched with at most $k$ blue vertices, and each blue vertex is matched with at most $l$ red vertices.
The appearances

- $\Delta_{m,n}^{k;l}$ is a ”matching” complex of $K_{m,n}$ where each red vertex is matched with at most $k$ blue vertices, and each blue vertex is matched with at most $l$ red vertices.
- $\Delta_{m,n}^{k;l}$ is $n$-fold $(l + 1)$-deleted join of the $(k - 1)$-skeleton of the $(m - 1)$-simplex or $n$-fold $(l + 1)$-deleted join of $m$-fold $(k + 1)$-deleted join of a point.
The appearances

- $\Delta^{k;l}_{m,n}$ is a "matching" complex of $K_{m,n}$ where each red vertex is matched with at most $k$ blue vertices, and each blue vertex is matched with at most $l$ red vertices.

- $\Delta^{k;l}_{m,n}$ is $n$-fold $(l + 1)$-deleted join of the $(k - 1)$-skeleton of the $(m - 1)$-simplex or $n$-fold $(l + 1)$-deleted join of $m$-fold $(k + 1)$-deleted join of a point.

- $\Delta^{k;l}_{m,n} = (\{(\sigma^{m-1})^{(k-1)}\})^*n_{\Delta(l+1)} = ([1]^m_{\Delta(k+1)})^*n_{\Delta(l+1)}$
The appearances

- \( \Delta_{m,n}^{k;l} \) is a "matching" complex of \( K_{m,n} \) where each red vertex is matched with at most \( k \) blue vertices, and each blue vertex is matched with at most \( l \) red vertices.
- \( \Delta_{m,n}^{k;l} \) is \( n \)-fold \((l+1)\)-deleted join of the \((k-1)\)-skeleton of the \((m-1)\)-simplex or \( n \)-fold \((l+1)\)-deleted join of \( m \)-fold \((k+1)\)-deleted join of a point.
- 
  \[
  \Delta_{m,n}^{k;l} = \left( \left( \sigma^{m-1}(k-1) \right)^* \right)^n \Delta(l+1) = \left[ 1 \right]^{m} \Delta(k+1) \Delta(l+1)
  \]

- Establishing the topological properties of these complexes was our main motivation.
Topological properties

Theorem:
Chessboard complex $\Delta^{k_1, \ldots, k_n; 1}$ is $\mu$-connected where
$\mu = \min\{m - n - 1; k_1 + \cdots + k_n - 2\}$. 
Topological properties

**Theorem:**
Chessboard complex $\Delta_{m,n}^{k_1,\ldots,k_n;1}$ is $\mu$-connected where $\mu = \min\{m - n - 1; k_1 + \cdots + k_n - 2\}$.

**Corollary:**
If $m \geq k_1 + \cdots + k_n + n - 1$, the complex $\Delta_{m,n}^{k_1,\ldots,k_n;1}$ is $(k_1 + \cdots + k_n - 2)$-connected. In particular, if $m \geq (k + 1)n - 1$, then $\Delta_{m,n}^{k;1}$ is $(kn - 2)$-connected.
Topological properties

Theorem:
Chessboard complex $\Delta_{m,n}^{k_1,\ldots,k_n;1}$ is $\mu$-connected where $\mu = \min\{m - n - 1; k_1 + \cdots + k_n - 2\}$.

Corrolary:
If $m \geq k_1 + \cdots + k_n + n - 1$, the complex $\Delta_{m,n}^{k_1,\ldots,k_n;1}$ is $(k_1 + \cdots + k_n - 2)$-connected. In particular, if $m \geq (k + 1)n - 1$, then $\Delta_{m,n}^{k;1}$ is $(kn - 2)$-connected.

Theorem:
If $m \geq k_1 + \cdots + k_n + n - 1$, the complex $\Delta_{m,n}^{k_1,\ldots,k_n;1}$ is shellable.
The applications

- A colored Tverberg type theorem with smaller number of colors, allowing $k$ vertices of the same color in each face.
The applications

- A colored Tverberg type theorem with smaller number of colors, allowing \( k \) vertices of the same color in each face.
- We could consider topological Tverberg theorem and require the dimensions of faces of a simplex whose images intersect to be prescribed.
The applications

• A colored Tverberg type theorem with smaller number of colors, allowing $k$ vertices of the same color in each face.
• We could consider topological Tverberg theorem and require the dimensions of faces of a simplex whose images intersect to be prescribed.
• Van Kampen-Flores theorem is an example of the result of this type.
The applications

- A colored Tverberg type theorem with smaller number of colors, allowing $k$ vertices of the same color in each face.
- We could consider topological Tverberg theorem and require the dimensions of faces of a simplex whose images intersect to be prescribed.
- Van Kampen-Flores theorem is an example of the result of this type.
- We have to allow more points, i.e. to consider the mapping from the higher dimensional simplex, to be able to prescribe the dimensions of faces.
**d-Tverberg prescribable r-tuple**

\textit{d-Tverberg prescribable r-tuple}


- \textbf{Definition:} For $d \geq 1$ and $r \geq 2$ an $r$-tuple $(d_1, d_2, \ldots, d_r)$ is $d$-Tverberg prescribable if there is $N$ such that for every continuous $f : \Delta_N \to \mathbb{R}^d$ there are disjoint faces $\sigma_1, \sigma_2, \ldots, \sigma_r$ such that $\text{dim} \sigma_i = d_i$ and $f(\sigma_1) \cap f(\sigma_2) \cap \cdots \cap f(\sigma_r) \neq \emptyset$. If we don't require the dimensions of faces to be prescribed, such faces exist for $N \geq (r - 1)(d + 1)$ by the Tverberg theorem.
**$d$-Tverberg prescribable $r$-tuple**


- **Definition:** For $d \geq 1$ and $r \geq 2$ an $r$-tuple $(d_1, d_2, \ldots, d_r)$ is $d$-Tverberg prescribable if there is $N$ such that for every continuous $f : \Delta_N \to \mathbb{R}^d$ there are disjoint faces $\sigma_1, \sigma_2, \ldots, \sigma_r$ such that $\dim \sigma_i = d_i$ and $f(\sigma_1) \cap f(\sigma_2) \cap \cdots \cap f(\sigma_r) \neq \emptyset$.

- If we don’t require the dimensions of faces to be prescribed, such faces exist for $N \geq (r - 1)(d + 1)$ by the Tverberg theorem.
By an affine general position mapping we see that a \( d \)-Tverberg prescribable \( r \)-tuple \((d_1, d_2, \ldots, d_r)\) has to satisfy \( d_1 + \cdots + d_r \geq (r - 1)d \).
**d-Tverberg prescribable r-tuple**

- By an affine general position mapping we see that a $d$-Tverberg prescribable $r$-tuple $(d_1, d_2, \ldots, d_r)$ has to satisfy $d_1 + \cdots + d_r \geq (r - 1)d$.
- By an affine map mapping the vertices to $N + 1$ different points on the moment curve, we obtain a cyclic polytope as the image, and this forces $d_i \geq \left\lfloor \frac{d}{2} \right\rfloor$ to hold for every $i$. 
**Definition:** For $d \geq 1$ and $r \geq 2$ an $r$-tuple $(d_1, d_2, \ldots, d_r)$ is $d$-admissible if $\left\lfloor \frac{d}{2} \right\rfloor \leq d_i \leq d$ for all $i$ and $d_1 + d_2 + \cdots + d_r \geq (r - 1)d$. 
**d-admissible r-tuple**

- **Definition:** For \( d \geq 1 \) and \( r \geq 2 \) an \( r \)-tuple \((d_1, d_2, \ldots, d_r)\) is \( d \)-admissible if \( \left\lfloor \frac{d}{2} \right\rfloor \leq d_i \leq d \) for all \( i \) and \( d_1 + d_2 + \cdots + d_r \geq (r-1)d \).

- By previous remarks, we see that a \( d \)-Tverberg prescribable \( r \)-tuple has to be \( d \)-admissible.
**Definition:** For $d \geq 1$ and $r \geq 2$ an $r$-tuple $(d_1, d_2, \ldots, d_r)$ is $d$-admissible if $\left\lfloor \frac{d}{2} \right\rfloor \leq d_i \leq d$ for all $i$ and $d_1 + d_2 + \cdots + d_r \geq (r - 1)d$.

By previous remarks, we see that a $d$-Tverberg prescribable $r$-tuple has to be $d$-admissible.

**Question ([BFZ]):** Is every $d$-admissible $r$-tuple $d$-Tverberg prescribable?
**d-admissible r-tuple**

- **Definition:** For $d \geq 1$ and $r \geq 2$ an $r$-tuple $(d_1, d_2, \ldots, d_r)$ is $d$-admissible if $\left\lfloor \frac{d}{2} \right\rfloor \leq d_i \leq d$ for all $i$ and $d_1 + d_2 + \cdots + d_r \geq (r - 1)d$.

- By previous remarks, we see that a $d$-Tverberg prescribable $r$-tuple has to be $d$-admissible.

- **Question ([BFZ]):** Is every $d$-admissible $r$-tuple $d$-Tverberg prescribable?

- **Theorem ([BFZ]):** Yes, if $d_1 = d_2 = \cdots = d_r$. 
**Definition:** For $d \geq 1$ and $r \geq 2$ an $r$-tuple $(d_1, d_2, \ldots, d_r)$ is $d$-admissible if $\left\lfloor \frac{d_i}{2} \right\rfloor \leq d_i \leq d$ for all $i$ and $d_1 + d_2 + \cdots + d_r \geq (r - 1)d$.

By previous remarks, we see that a $d$-Tverberg prescribable $r$-tuple has to be $d$-admissible.

**Question ([BFZ]):** Is every $d$-admissible $r$-tuple $d$-Tverberg prescribable?

**Theorem ([BFZ]):** Yes, if $d_1 = d_2 = \cdots = d_r$.

This follows also directly from the connectivity of multiple chessboard complexes.
A conjecture

• Conjecture ([BFZ]): Yes, if $d_1 = \cdots = d_s = d' + 1$ and $d_{s+1} = \cdots = d_r = d'$ for some $s$. 

• For an $r$-tuple $(d_1, d_2, \ldots, d_r)$, the configuration complex of all candidates is $K' = \Delta^{d_1+1}, \ldots, d_r+1; N+1$. 
A conjecture

• Conjecture ([BFZ]): Yes, if \( d_1 = \cdots = d_s = d' + 1 \) and \( d_{s+1} = \cdots = d_r = d' \) for some \( s \).

• Conjecture ([BFZ]): Let \( r \geq 2 \) be a prime power, \( d \geq 1 \), \( N \geq (r - 1)(d + 2) \), and \( r(k + 1) + s > N + 1 \) for integers \( k \geq 0 \) and \( 0 \leq s < r \). Then, for every continuous map \( f : \Delta_N \rightarrow \mathbb{R}^d \), there are \( r \) pairwise disjoint faces \( \sigma_1, \sigma_2, \ldots, \sigma_r \) of \( \Delta_N \) such that \( f(\sigma_1) \cap f(\sigma_2) \cap \cdots \cap f(\sigma_r) \neq \emptyset \), with \( \dim \sigma_i \leq k + 1 \) for \( 1 \leq i \leq s \) and \( \dim \sigma_i \leq k \) for \( s < i \leq r \).
A conjecture

• Conjecture ([BFZ]): Yes, if \( d_1 = \cdots = d_s = d' + 1 \) and \( d_{s+1} = \cdots = d_r = d' \) for some \( s \).

• Conjecture ([BFZ]): Let \( r \geq 2 \) be a prime power, \( d \geq 1 \), \( N \geq (r-1)(d+2) \), and \( r(k+1) + s > N + 1 \) for integers \( k \geq 0 \) and \( 0 \leq s < r \). Then, for every continuous map \( f : \Delta_N \to \mathbb{R}^d \), there are \( r \) pairwise disjoint faces \( \sigma_1, \sigma_2, \ldots, \sigma_r \) of \( \Delta_N \) such that \( f(\sigma_1) \cap f(\sigma_2) \cap \cdots \cap f(\sigma_r) \neq \emptyset \), with \( \dim \sigma_i \leq k + 1 \) for \( 1 \leq i \leq s \) and \( \dim \sigma_i \leq k \) for \( s < i \leq r \).

• For a \( r \)-tuple \( (d_1, d_2, \ldots, d_r) \), the configuration complex of all candidates is \( K' = \Delta_{N+1,r}^{d_1+1, \ldots, d_r+1,1} \).
Symmetric multiple chessboard complexes

- There is an action of a symmetric group $S_r$ on this complex only if $d_1 = d_2 = \cdots = d_r$. 
Symmetric multiple chessboard complexes

- There is an action of a symmetric group $S_r$ on this complex only if $d_1 = d_2 = \cdots = d_r$.
- We consider the symmetrized multiple chessboard complexes

$$\sum_{m,r}^k = S_r \cdot \Delta^{k_1,\ldots,k_r;1} = \bigcup_{\sigma \in S_r} \Delta^{k_{\sigma(1)},\ldots,k_{\sigma(r)};1}.$$
Symmetric multiple chessboard complexes

- There is an action of a symmetric group $S_r$ on this complex only if $d_1 = d_2 = \cdots = d_r$.
- We consider the symmetrized multiple chessboard complexes

$$
\sum_{m,r}^{k_1,\ldots,k_r;1} = S_r \cdot \Delta_{m,r}^{k_1,\ldots,k_r;1} = \bigcup_{\sigma \in S_r} \Delta_{m,r}^{k_{\sigma(1)},\ldots,k_{\sigma(r)};1}.
$$

- If $r = p^\alpha$ is a prime power, there is a fixed point free action of the group $(\mathbb{Z}/p)^\alpha$ on the complex $\sum_{m,r}^{k_1,\ldots,k_r;1}$. 
Topological properties

- **Theorem.** If $m \geq k_1 + \cdots + k_r + r - 1$, and $\max k_i - \min k_i = 1$, the complex $\Sigma^{k_1, \ldots, k_r, 1}_{m, r}$ is shellable.

- **Corollary.** Under the same conditions, this complex is $(k_1 + \cdots + k_r - 2)$-connected, and so there is no equivariant mapping from this complex to the appropriate representation sphere if $r$ is a prime power.

- An alternative proof of this consequence could be also provided using the established connectivity of $\Delta^{k_1, \ldots, k_r, 1}_{m, r}$, and the Sarkaria inequality.
Topological properties

• **Theorem.** If $m \geq k_1 + \cdots + k_r + r - 1$, and $\max k_i - \min k_i = 1$, the complex $\Sigma_{m,r}^{k_1,\ldots,k_r;1}$ is shellable.

• **Corollary.** Under the same conditions, this complex is $(k_1 + \cdots + k_r - 2)$-connected, and so there is no equivariant mapping from this complex to the appropriate representation sphere if $r$ is a prime power.
Topological properties

• **Theorem.** If $m \geq k_1 + \cdots + k_r + r - 1$, and \(\max k_i - \min k_i = 1\), the complex \(\Sigma_{m,r}^{k_1,\ldots,k_r;1}\) is shellable.

• **Corollary.** Under the same conditions, this complex is \((k_1 + \cdots + k_r - 2)\)-connected, and so there is no equivariant mapping from this complex to the appropriate representation sphere if \(r\) is a prime power.

• An alternative proof of this consequence could be also provided using the established connectivity of \(\Delta_{m,r}^{k_1,\ldots,k_r;1}\), and the Sarkaria inequality.
A conjecture is true

Using this result, we are able to confirm the stronger version of the conjecture, obtained by replacing the original condition $r(k+1) + s > N + 1$ by a weaker and more natural condition $rk + s \geq (r - 1)d$. 

Theorem:
Every $d$-admissible $r$-tuple $(d_1, \ldots, d_r)$ satisfying $\max d_i - \min d_i = 1$ is $d$-Tverberg prescribable.
A conjecture is true

- Using this result, we are able to confirm the stronger version of the conjecture, obtained by replacing the original condition $r(k + 1) + s > N + 1$ by a weaker and more natural condition $rk + s \geq (r - 1)d$.
- This weaker condition is actually contained in the assumption that given $r$-tuple is $d$-admissible.
A conjecture is true

• Using this result, we are able to confirm the stronger version of the conjecture, obtained by replacing the original condition $r(k + 1) + s > N + 1$ by a weaker and more natural condition $rk + s \geq (r - 1)d$.

• This weaker condition is actually contained in the assumption that given $r$-tuple is $d$-admissible.

• **Theorem:** Every $d$-admissible $r$-tuple $(d_1, ..., d_r)$ satisfying $\max d_i - \min d_i = 1$ is $d$-Tverberg prescribable.
Preprints


THANK YOU
FOR YOUR
ATTENTION!