Buchberger-ov algoritam i vizuelizacija monomijalnih ideala

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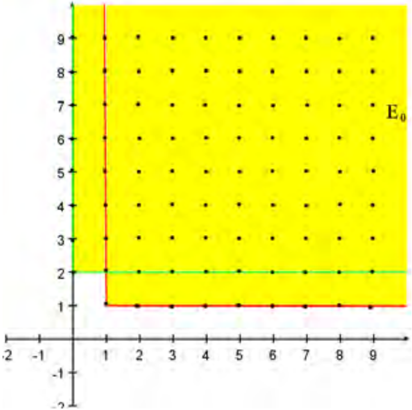
Razvijena je Java aplikacija za računanje Gröbner-ove baze pomoću Buchberger-ovog algoritma, korak po korak, uz vizuelizaciju monomijalnih ideala.

Zadatak

Neka su dati polinomi $f_1 = xy + 1$ i $f_2 = y^2 + 1$ u leksikografskom poretku $\succeq_{LEX}$.

1. *Formirati skup $E_0 \subseteq \mathbb{N}_0^2$ takav da važi $(\alpha, \beta) \in E_0 \Leftrightarrow x^\alpha y^\beta \in \langle LT(f_1), LT(f_2) \rangle$.*
2. *Primenom Buchberger-ovog algoritma odrediti standardnu Gröbner-ovu bazu G ideala $I = \langle f_1, f_2 \rangle$.*
3. *Formirati skup $E_1 \subseteq \mathbb{N}_0^2$ takav da važi $(\alpha, \beta) \in E_1 \Leftrightarrow x^\alpha y^\beta \in \langle LT(I) \rangle$.*

- $x^\alpha y^\beta \in \langle LT(f_1), LT(f_2)\rangle = \langle xy, y^2\rangle \Leftrightarrow$
$(\alpha,\beta) \in \{(1,1)+(p,q) \mid p,q \in \mathbb{N}_0\} \cup \{(0,2)+(p,q) \mid p,q \in \mathbb{N}_0\}$



- $F = \{f_1, f_2\}, \;\; G = \{f_1, f_2\}$

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Step 1: $S(f_1, f_2) = \frac{LCM(xy, y^2)}{xy}(xy+1) - \frac{LCM(xy, y^2)}{y^2}(y^2+1)$
$= \frac{xy^2}{xy}(xy+1) - \frac{xy^2}{y^2}(y^2+1) = xy^2 + y - xy^2 - x = -x + y$

- $F = \{f_1, f_2\}, \;\; G = \{f_1, f_2\}$

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$= \frac{xy^2}{xy}(xy+1) - \frac{xy^2}{y^2}(y^2+1) = xy^2 + y - xy^2 - x = -x + y$

$S(f_1, f_2) : (f_1, f_2) = (-x+y) : (xy+1, y^2+1) = (0,0)$

$$\frac{0}{-x+y}$$

$f_3 = REM(S(f_1, f_2); f_1, f_2) = -x + y, \;\; G = \{f_1, f_2, f_3\}$

- $F=\{f_1,f_2\},\ \ G=\{f_1,f_2\}$

Step 1: $S(f_1,f_2)=\frac{LCM(xy,y^2)}{xy}(xy+1)-\frac{LCM(xy,y^2)}{y^2}(y^2+1)$

$=\frac{xy^2}{xy}(xy+1)-\frac{xy^2}{y^2}(y^2+1)=xy^2+y-xy^2-x=-x+y$

$S(f_1,f_2):(f_1,f_2)=(-x+y):(xy+1,y^2+1)=(0,0)$

$$\begin{array}{l} \underline{0\quad\quad} \\ -x+y \end{array}$$

$f_3=REM(S(f_1,f_2);f_1,f_2)=-x+y,\ \ G=\{f_1,f_2,f_3\}$

Step 2:

- $S(f_1,f_2)=-x+y$

- $F = \{f_1, f_2\}, \;\; G = \{f_1, f_2\}$

Step 1: $S(f_1, f_2) = \frac{LCM(xy, y^2)}{xy}(xy + 1) - \frac{LCM(xy, y^2)}{y^2}(y^2 + 1)$

$= \frac{xy^2}{xy}(xy + 1) - \frac{xy^2}{y^2}(y^2 + 1) = xy^2 + y - xy^2 - x = -x + y$

$S(f_1, f_2) : (f_1, f_2) = (-x + y) : (xy + 1, y^2 + 1) = (0, 0)$

$$\frac{0}{-x + y}$$

$f_3 = REM(S(f_1, f_2); f_1, f_2) = -x + y, \;\; G = \{f_1, f_2, f_3\}$

Step 2:

- $S(f_1, f_2) = -x + y$

$S(f_1, f_2) : (f_1, f_2, f_3) = (-x + y) : (xy + 1, y^2 + 1, -x + y) = (0, 0, 1)$

$$\frac{-x + y}{0}$$

$REM(S(f_1, f_2); f_1, f_2, f_3) = 0$

- $S(f_1, f_3) = \frac{LCM(xy,x)}{xy}(xy+1) - \frac{LCM(xy,x)}{-x}(-x+y)$
  $= \frac{xy}{xy}(xy+1) - \frac{xy}{-x}(-x+y) = xy+1-xy+y^2 = y^2+1$

- $S(f_1,f_3)=\frac{LCM(xy,x)}{xy}(xy+1)-\frac{LCM(xy,x)}{-x}(-x+y)$
  $=\frac{xy}{xy}(xy+1)-\frac{xy}{-x}(-x+y)=xy+1-xy+y^2=y^2+1$

$$S(f_1,f_3):(f_1,f_2,f_3)=(y^2+1):(xy+1,y^2+1,-x+y)=(0,1,0)$$

$$\begin{array}{c}\underline{y^2+1}\\0\end{array}$$

$$REM(S(f_1,f_3);f_1,f_2,f_3)=0$$

- $S(f_1,f_3) = \frac{LCM(xy,x)}{xy}(xy+1) - \frac{LCM(xy,x)}{-x}(-x+y)$
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$$S(f_1,f_3) : (f_1,f_2,f_3) = (y^2+1) : (xy+1, y^2+1, -x+y) = (0,1,0)$$

$$\frac{y^2+1}{0}$$

$$REM(S(f_1,f_3); f_1,f_2,f_3) = 0$$

- $S(f_2,f_3) = \frac{LCM(y^2,x)}{y^2}(y^2+1) - \frac{LCM(y^2,x)}{-x}(-x+y)$
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$$S(f_1, f_3) : (f_1, f_2, f_3) = (y^2+1) : (xy+1, y^2+1, -x+y) = (0, 1, 0)$$

$$\frac{y^2+1}{0}$$

$$REM(S(f_1, f_3); f_1, f_2, f_3) = 0$$

- $S(f_2, f_3) = \frac{LCM(y^2,x)}{y^2}(y^2+1) - \frac{LCM(y^2,x)}{-x}(-x+y)$
  $= \frac{xy^2}{y^2}(y^2+1) - \frac{xy^2}{-x}(-x+y) = xy^2 + x - xy^2 + y^3 = x + y^3$

$$S(f_2, f_3) : (f_1, f_2, f_3) = (x+y^3) : (xy+1, y^2+1, -x+y) = (0, y, 1)$$

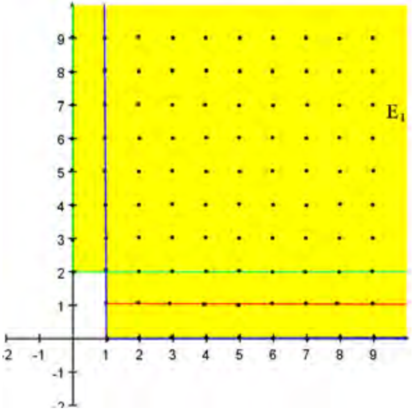
$$\frac{y^3+y}{x-y}$$

$$\frac{-x+y}{0}$$

$$REM(S(f_2, f_3); f_1, f_2, f_3) = 0$$

$G = \{f_1, f_2, f_3\}$ je Gröbner-ova baza ideala I.

- $x^\alpha y^\beta \in \langle LT(I)\rangle = \langle xy, y^2, -x\rangle \Leftrightarrow$
$(\alpha,\beta) \in E_0 \cup \{(1,0)+(p,q) \mid p,q \in \mathbb{N}_0\}$



Zadatak

Neka su dati polinomi $f_1 = x^2z - y^2$, $f_2 = yz^2 + z$ *i* $f_3 = y - z$ *u leksikografskom poretku* $\succeq_{LEX}$.

1. *Formirati skup* $E_0 \subseteq \mathbb{N}_0^3$ *takav da važi* $(\alpha,\beta,\gamma) \in E_0 \Leftrightarrow x^\alpha y^\beta z^\gamma \in \langle LT(f_1), LT(f_2), LT(f_3)\rangle$.
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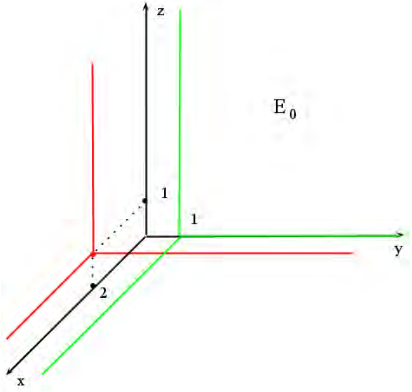
- $x^\alpha y^\beta z^\gamma \in \langle LT(f_1), LT(f_2), LT(f_3)\rangle = \langle x^2z, yz^2, y\rangle = \langle x^2z, y\rangle \Leftrightarrow$
$$(\alpha,\beta,\gamma) \in \{(2,0,1)+(p,q,r) \mid p,q,r \in \mathbb{N}_0\} \cup \{(0,1,0)+(p,q,r) \mid p,q,r \in \mathbb{N}_0\}$$



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$= x^2yz^2 - y^3z - x^2yz^2 - x^2z = -x^2z - y^3z$

- $F = \{f_1, f_2, f_3\}, \ \ G = \{f_1, f_2, f_3\}$

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  $= x^2yz^2 - y^3z - x^2yz^2 - x^2z = -x^2z - y^3z$
  $S(f_1,f_2) : (f_1,f_2,f_3) =$
  $(-x^2z-y^3z) : (x^2z-y^2, yz^2+z, y-z) = (-1, -y, -y^2z-y)$

$$
\begin{array}{l}
\underline{-x^2z+y^2} \\
\quad -y^3z-y^2 \\
\quad \underline{-y^3z+y^2z^2} \\
\qquad -y^2z^2-y^2 \\
\qquad \underline{-y^2z^2-yz} \\
\qquad\quad -y^2+yz \\
\qquad\quad \underline{-y^2+yz} \\
\qquad\qquad 0
\end{array}
$$

$REM(S(f_1,f_2); f_1,f_2,f_3) = 0$

- $S(f_1,f_3) = \frac{LCM(x^2z,y)}{x^2z}(x^2z-y^2) - \frac{LCM(x^2z,y)}{y}(y-z)$
  $= \frac{x^2yz}{x^2z}(x^2z-y^2) - \frac{x^2yz}{y}(y-z)$
  $= x^2yz - y^3 - x^2yz + x^2z^2 = x^2z^2 - y^3$

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  $= x^2yz - y^3 - x^2yz + x^2z^2 = x^2z^2 - y^3$
  $S(f_1,f_3) : (f_1,f_2,f_3) =$
  $(x^2z^2-y^3) : (x^2z-y^2, yz^2+z, y-z) = (z, 0, -y^2)$

$$
\begin{array}{l}
x^2z^2 - y^2z \\
\hline
\qquad -y^3 + y^2z \\
\qquad -y^3 + y^2z \\
\hline
\qquad\quad 0
\end{array}
$$

  $REM(S(f_1,f_3); f_1,f_2,f_3) = 0$

- $S(f_2, f_3) = \frac{LCM(yz^2, y)}{yz^2}(yz^2 + z) - \frac{LCM(yz^2, y)}{y}(y - z)$
  $= \frac{yz^2}{yz^2}(yz^2 + z) - \frac{yz^2}{y}(y - z)$
  $= yz^2 + z - yz^2 + z^3 = z^3 + z$

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  $= \frac{yz^2}{yz^2}(yz^2 + z) - \frac{yz^2}{y}(y - z)$
  $= yz^2 + z - yz^2 + z^3 = z^3 + z$
  $S(f_2, f_3) : (f_1, f_2, f_3) =$
  $(z^3 + z) : (x^2 z - y^2, yz^2 + z, y - z) = (0, 0, 0)$
  $\frac{0}{z^3 + z}$
  $f_4 = REM(S(f_2, f_3); f_1, f_2, f_3) = z^3 + z, \; G = \{f_1, f_2, f_3, f_4\}$

Step 2:

- $S(f_1, f_2) = -x^2z - y^3z$

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- $S(f_1, f_2) = -x^2z - y^3z$

  $S(f_1, f_2) : (f_1, f_2, f_3, f_4) =$

  $(-x^2z - y^3z) : (x^2z - y^2, yz^2 + z, y - z, z^3 + z) =$

  $$\begin{array}{l}
  \underline{-x^2z + y^2} \qquad\qquad\qquad\qquad (-1, -y, -y^2z - y, 0) \\
  \quad -y^3z - y^2 \\
  \quad \underline{-y^3z + y^2z^2} \\
  \qquad -y^2z^2 - y^2 \\
  \qquad \underline{-y^2z^2 - yz} \\
  \qquad\quad -y^2 + yz \\
  \qquad\quad \underline{-y^2 + yz} \\
  \qquad\qquad 0
  \end{array}$$

  $REM(S(f_1, f_2); f_1, f_2, f_3, f_4) = 0$

- $S(f_1, f_3) = x^2 z^2 - y^3$

- $S(f_1,f_3) = x^2z^2 - y^3$

  $S(f_1,f_3) : (f_1,f_2,f_3,f_4) =$
  $(x^2z^2 - y^3) : (x^2z - y^2, yz^2 + z, y - z, z^3 + z) = (z, 0, -y^2, 0)$

  $$\begin{array}{l} \underline{x^2z^2 - y^2z} \\ -y^3 + y^2z \\ \underline{-y^3 + y^2z} \\ 0 \end{array}$$

  $REM(S(f_1,f_3); f_1,f_2,f_3,f_4) = 0$

- $S(f_1, f_3) = x^2z^2 - y^3$

  $S(f_1, f_3) : (f_1, f_2, f_3, f_4) =$
  $(x^2z^2 - y^3) : (x^2z - y^2, yz^2 + z, y - z, z^3 + z) = (z, 0, -y^2, 0)$

  $$\begin{array}{l} \underline{x^2z^2 - y^2z} \\ \qquad -y^3 + y^2z \\ \qquad \underline{-y^3 + y^2z \qquad} \\ \qquad\quad 0 \end{array}$$

  $REM(S(f_1, f_3); f_1, f_2, f_3, f_4) = 0$
- $S(f_2, f_3) = z^3 + z$

- $S(f_1,f_3) = x^2z^2 - y^3$

  $S(f_1,f_3) : (f_1,f_2,f_3,f_4) =$
  $(x^2z^2 - y^3) : (x^2z - y^2, yz^2 + z, y - z, z^3 + z) = (z, 0, -y^2, 0)$

  $$\begin{array}{l} \underline{x^2z^2 - y^2z} \\ \quad -y^3 + y^2z \\ \quad \underline{-y^3 + y^2z} \\ \qquad 0 \end{array}$$

  $REM(S(f_1,f_3); f_1,f_2,f_3,f_4) = 0$

- $S(f_2,f_3) = z^3 + z$

  $S(f_2,f_3) : (f_1,f_2,f_3,f_4) =$
  $(z^3 + z) : (x^2z - y^2, yz^2 + z, y - z, z^3 + z) = (0, 0, 0, 1)$

  $$\begin{array}{l} \underline{z^3 + z} \\ \quad 0 \end{array}$$

  $REM(S(f_2,f_3); f_1,f_2,f_3) = 0$

- $S(f_1,f_4)=\frac{LCM(x^2z,z^3)}{x^2z}(x^2z-y^2)-\frac{LCM(x^2z,z^3)}{z^3}(z^3+z)$
$=\frac{x^2z^3}{x^2z}(x^2z-y^2)-\frac{x^2z^3}{z^3}(z^3+z)$
$=x^2z^3-y^2z^2-x^2z^3-x^2z=-x^2z-y^2z^2$

- $S(f_1, f_4) = \frac{LCM(x^2z, z^3)}{x^2z}(x^2z - y^2) - \frac{LCM(x^2z, z^3)}{z^3}(z^3 + z)$
  $= \frac{x^2z^3}{x^2z}(x^2z - y^2) - \frac{x^2z^3}{z^3}(z^3 + z)$
  $= x^2z^3 - y^2z^2 - x^2z^3 - x^2z = -x^2z - y^2z^2$

  $S(f_1, f_4) : (f_1, f_2, f_3, f_4) =$
  $(-x^2z - y^2z^2) : (x^2z - y^2, yz^2 + z, y - z, z^3 + z) =$

$$
\begin{array}{lr}
\underline{-x^2z + y^2} & (-1, -y, -y, 0) \\
\quad -y^2z^2 - y^2 & \\
\quad \underline{-y^2z^2 - yz} & \\
\qquad -y^2 + yz & \\
\qquad \underline{-y^2 + yz} & \\
\qquad\quad 0 &
\end{array}
$$

  $REM(S(f_1, f_4); f_1, f_2, f_3, f_4) = 0$

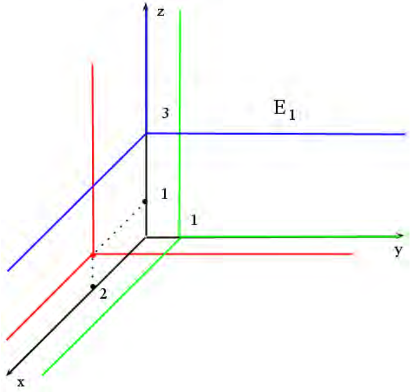
- $S(f_2, f_4) = \frac{LCM(yz^2, z^3)}{yz^2}(yz^2 + z) - \frac{LCM(yz^2, z^3)}{z^3}(z^3 + z)$
  $= \frac{yz^3}{yz^2}(yz^2 + z) - \frac{yz^3}{z^3}(z^3 + z)$
  $= yz^3 + z^2 - yz^3 - yz = -yz + z^2$

- $S(f_2, f_4) = \frac{LCM(yz^2, z^3)}{yz^2}(yz^2 + z) - \frac{LCM(yz^2, z^3)}{z^3}(z^3 + z)$
  $= \frac{yz^3}{yz^2}(yz^2 + z) - \frac{yz^3}{z^3}(z^3 + z)$
  $= yz^3 + z^2 - yz^3 - yz = -yz + z^2$
  $S(f_2, f_4) : (f_1, f_2, f_3, f_4) =$
  $(-yz + z^2) : (x^2z - y^2, yz^2 + z, y - z, z^3 + z) = (0, 0, -z, 0)$
  $$\frac{-yz + z^2}{0}$$
  $REM(S(f_2, f_4); f_1, f_2, f_3, f_4) = 0$

- $S(f_3, f_4) = \frac{LCM(y, z^3)}{y}(y - z) - \frac{LCM(y, z^3)}{z^3}(z^3 + z)$
  $= \frac{yz^3}{y}(y - z) - \frac{yz^3}{z^3}(z^3 + z)$
  $= yz^3 - z^4 - yz^3 - yz = -yz - z^4$

- $S(f_3, f_4) = \frac{LCM(y,z^3)}{y}(y - z) - \frac{LCM(y,z^3)}{z^3}(z^3 + z)$
  $= \frac{yz^3}{y}(y - z) - \frac{yz^3}{z^3}(z^3 + z)$
  $= yz^3 - z^4 - yz^3 - yz = -yz - z^4$
  $S(f_3, f_4) : (f_1, f_2, f_3, f_4) =$
  $(-yz - z^4) : (x^2 z - y^2, yz^2 + z, y - z, z^3 + z) = (0, 0, -z, -z)$
  $$\begin{array}{l} \underline{-yz + z^2} \\ \quad\quad -z^4 - z^2 \\ \quad\quad \underline{-z^4 - z^2 \quad\quad} \\ \quad\quad\quad\quad 0 \end{array}$$
  $REM(S(f_3, f_4); f_1, f_2, f_3, f_4) = 0$
  $G = \{f_1, f_2, f_3, f_4\}$ je Gröbner-ova baza ideala I.

- $x^\alpha y^\beta z^\gamma \in \langle LT(I)\rangle = \langle x^2z, yz^2, y, z^3\rangle = \langle x^2z, y, z^3\rangle \Leftrightarrow$
$$(\alpha,\beta,\gamma) \in \{(2,0,1)+(p,q,r) \mid p,q,r \in \mathbb{N}_0\}$$
$$\cup\{(0,1,0)+(p,q,r) \mid p,q,r \in \mathbb{N}_0\}$$
$$\cup\{(0,0,3)+(p,q,r) \mid p,q,r \in \mathbb{N}_0\}$$

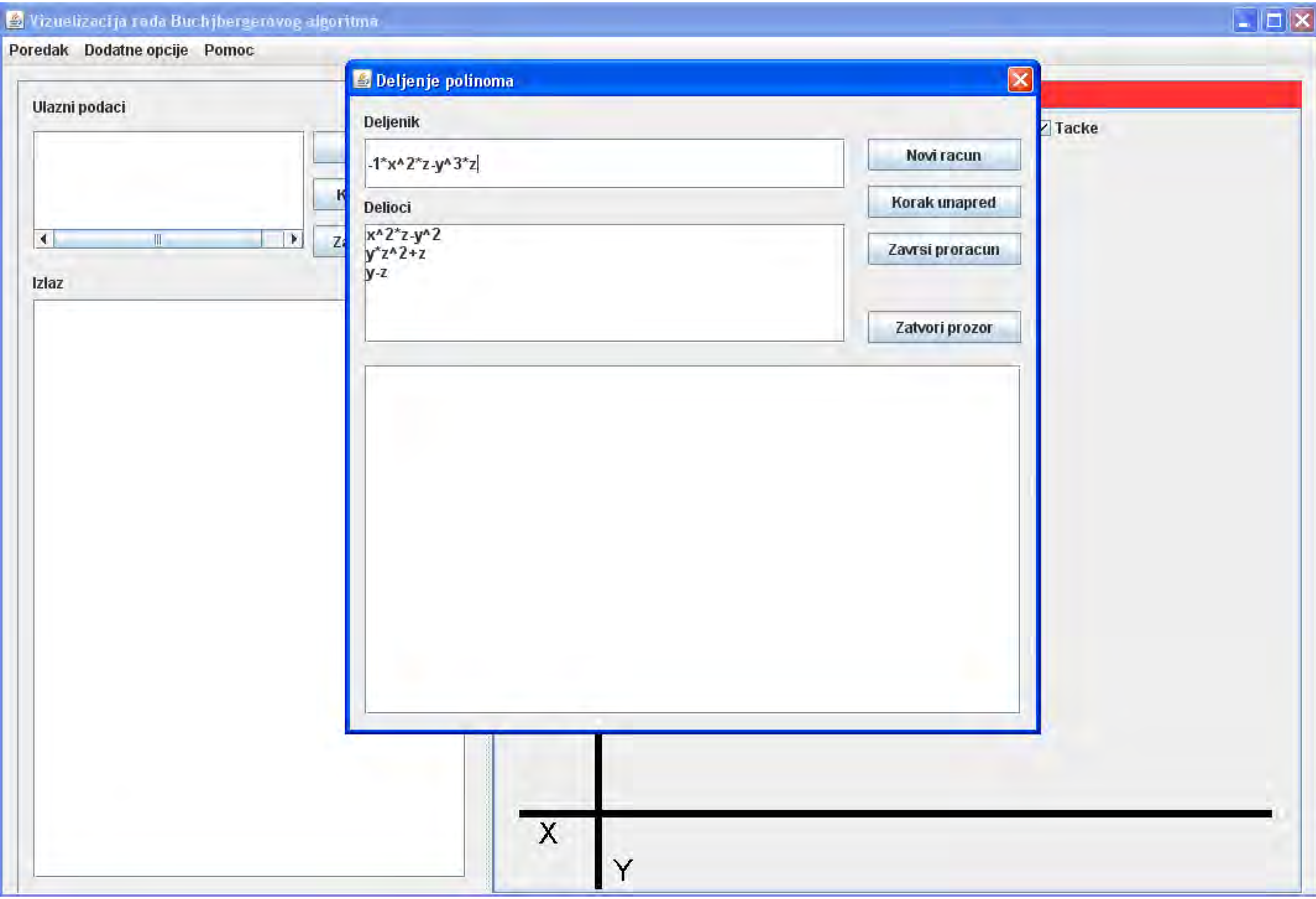


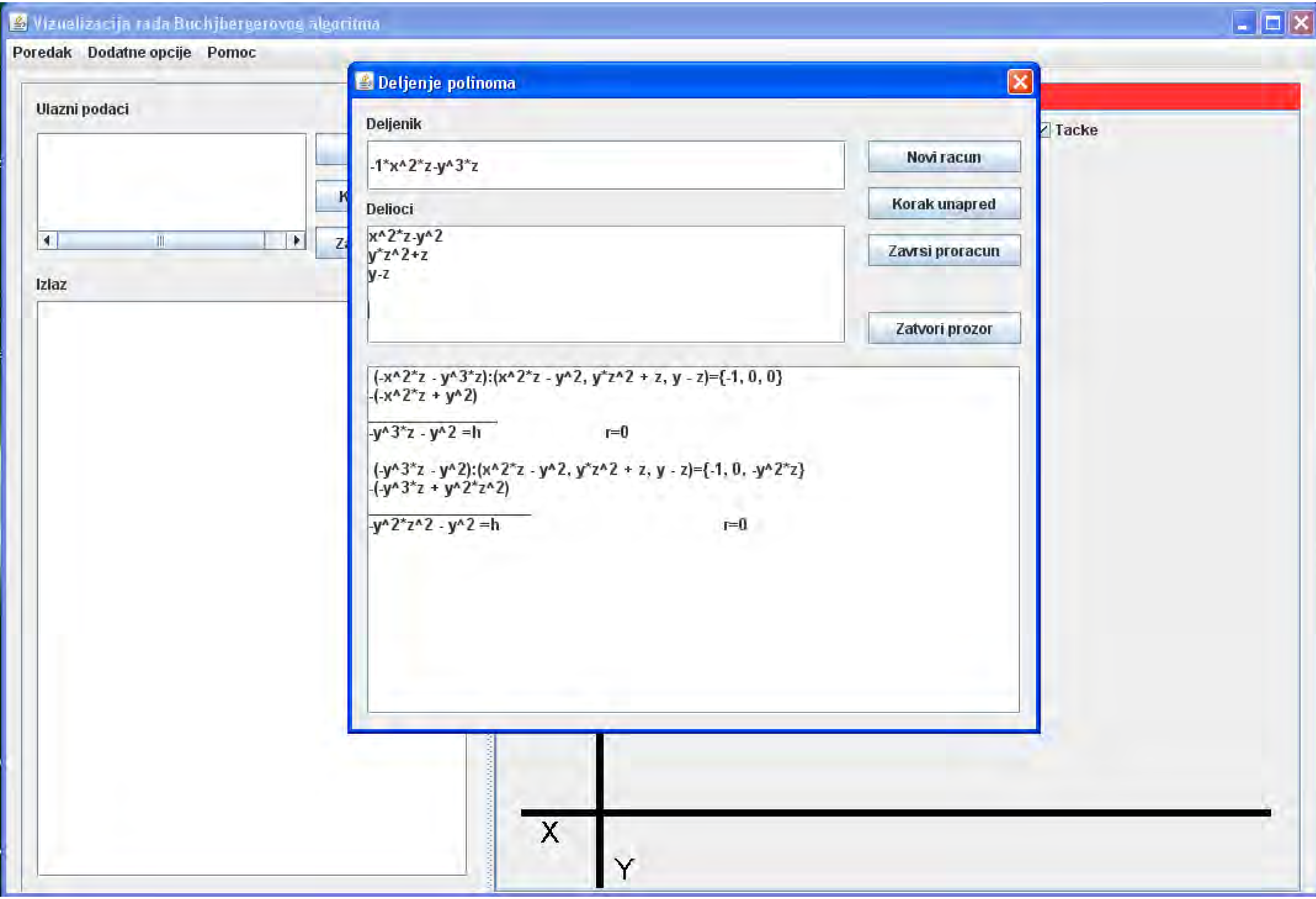
Algoritam deljenja:

Input: $f, f_1, f_2, \ldots, f_k$
Output: $a_1, a_2, \ldots, a_k, r$
$a_1 := 0; a_2 := 0; \ldots; a_k := 0; r := 0$
$h := f;$
while $h \neq 0$ if $(\exists i) LT(f_i) \mid LT(h)$
    then for min index i such that $LT(f_i) \mid LT(h)$
        $a_i := a_i + LT(h)/LT(f_i)$
        $h := h - LT(h)/LT(f_i) \cdot f_i$
    else
        $r := r + LT(h)$
        $h := h - LT(h)$

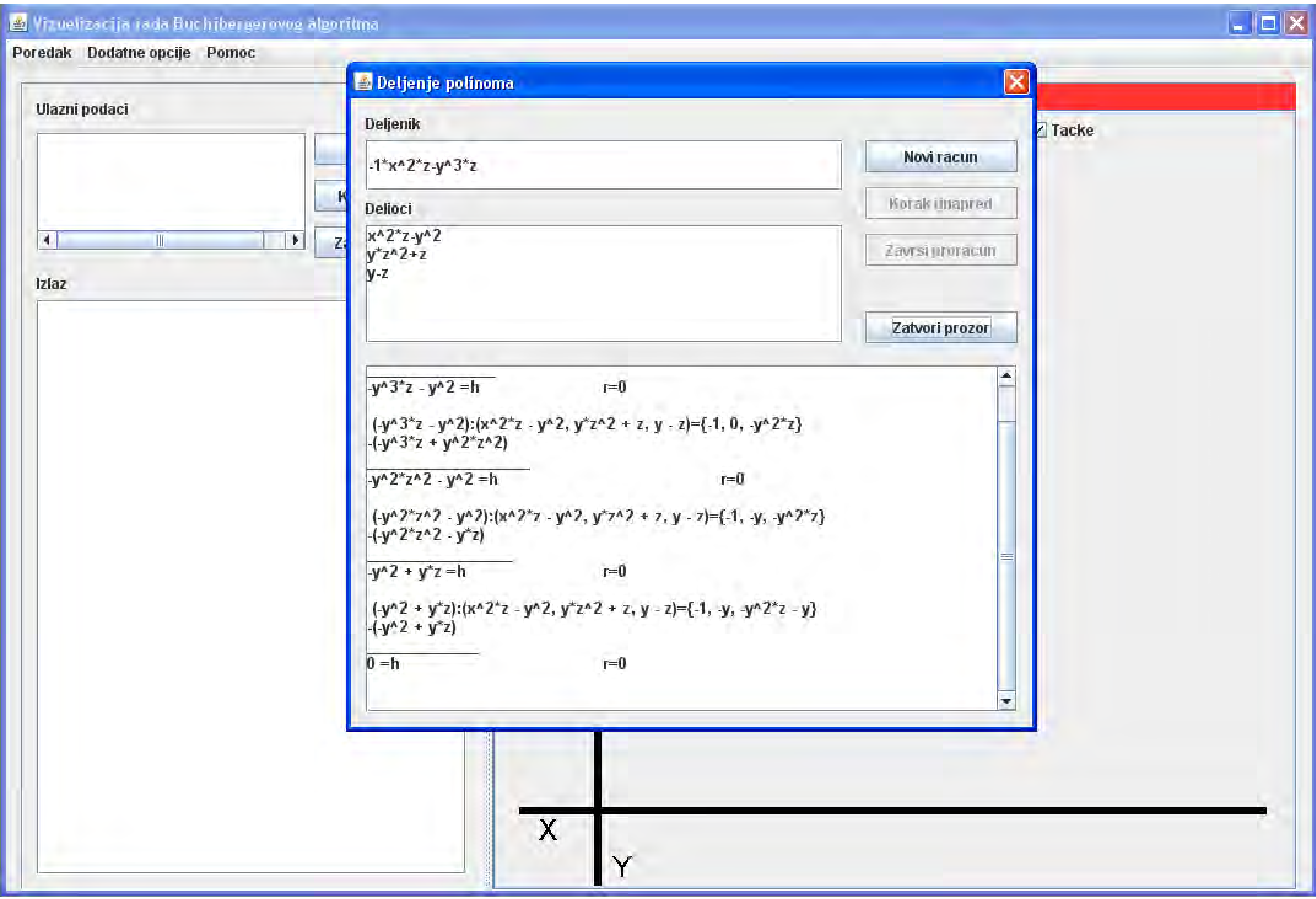
Buchberger-ov algoritam:

Input: $F = \{f_1, f_2, \ldots, f_k\}$ skup generatora ideala I
Output: $G = \{g_1, g_2, \ldots, g_m\}$ Gröbner-ova baza ideala I
$G := F$;
repeat
    $G' := G$
    for $\{p, g\} \subseteq G', p \neq q$ do
        $S := S(p, q)$
        $h := REM(S; G')$
        if $h \neq 0$ then $G := G \cup \{h\}$
until G=G'

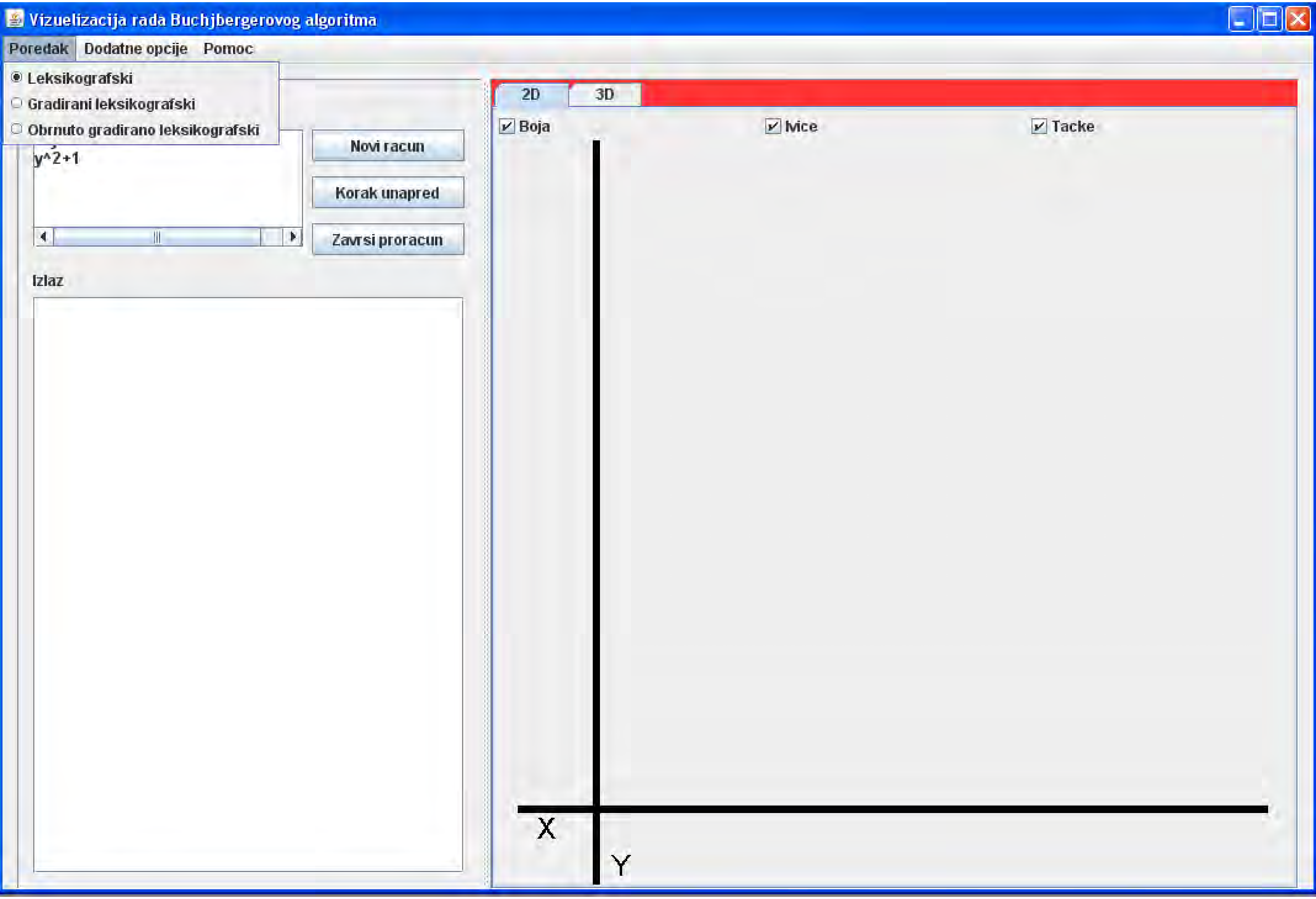

Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
Izlaz
Tacke
Deljenje polinoma
Deljenik
-1*x^2*z-y^3*z
Delioci
x^2*z-y^2
y*z^2+z
y-z
Novi racun
Korak unapred
Zavrsi proracun
Zatvori prozor
X
Y

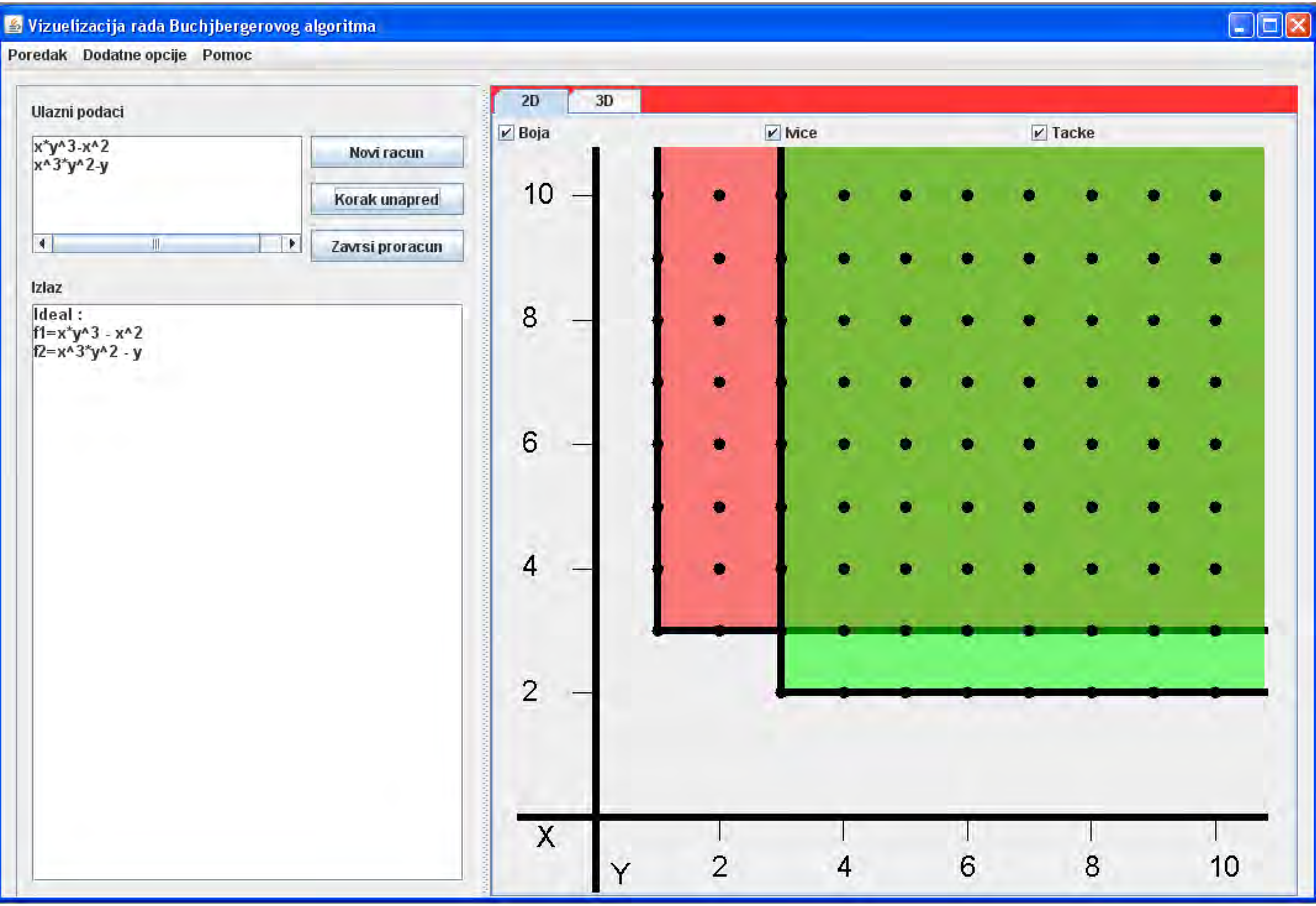


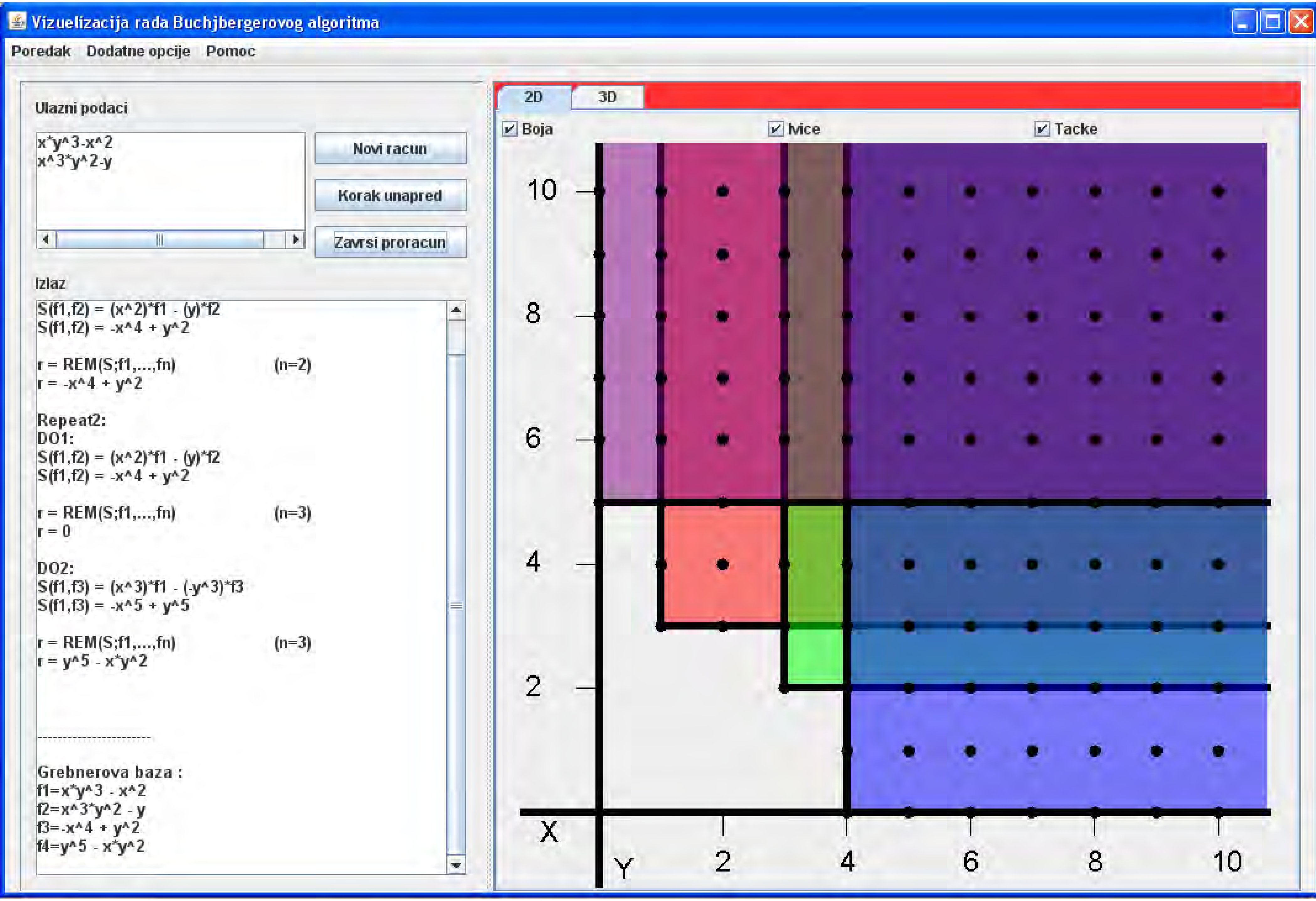
Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
Izlaz
Deljenje polinoma
Deljenik
-1*x^2*z-y^3*z
Delioci
x^2*z-y^2
y*z^2+z
y-z
Novi racun
Korak unapred
Zavrsi proracun
Zatvori prozor
(-x^2*z - y^3*z):(x^2*z - y^2, y*z^2 + z, y - z)={-1, 0, 0}
-(-x^2*z + y^2)
-y^3*z - y^2 =h
r=0
(-y^3*z - y^2):(x^2*z - y^2, y*z^2 + z, y - z)={-1, 0, -y^2*z}
-(-y^3*z + y^2*z^2)
-y^2*z^2 - y^2 =h
r=0
Tacke
X
Y



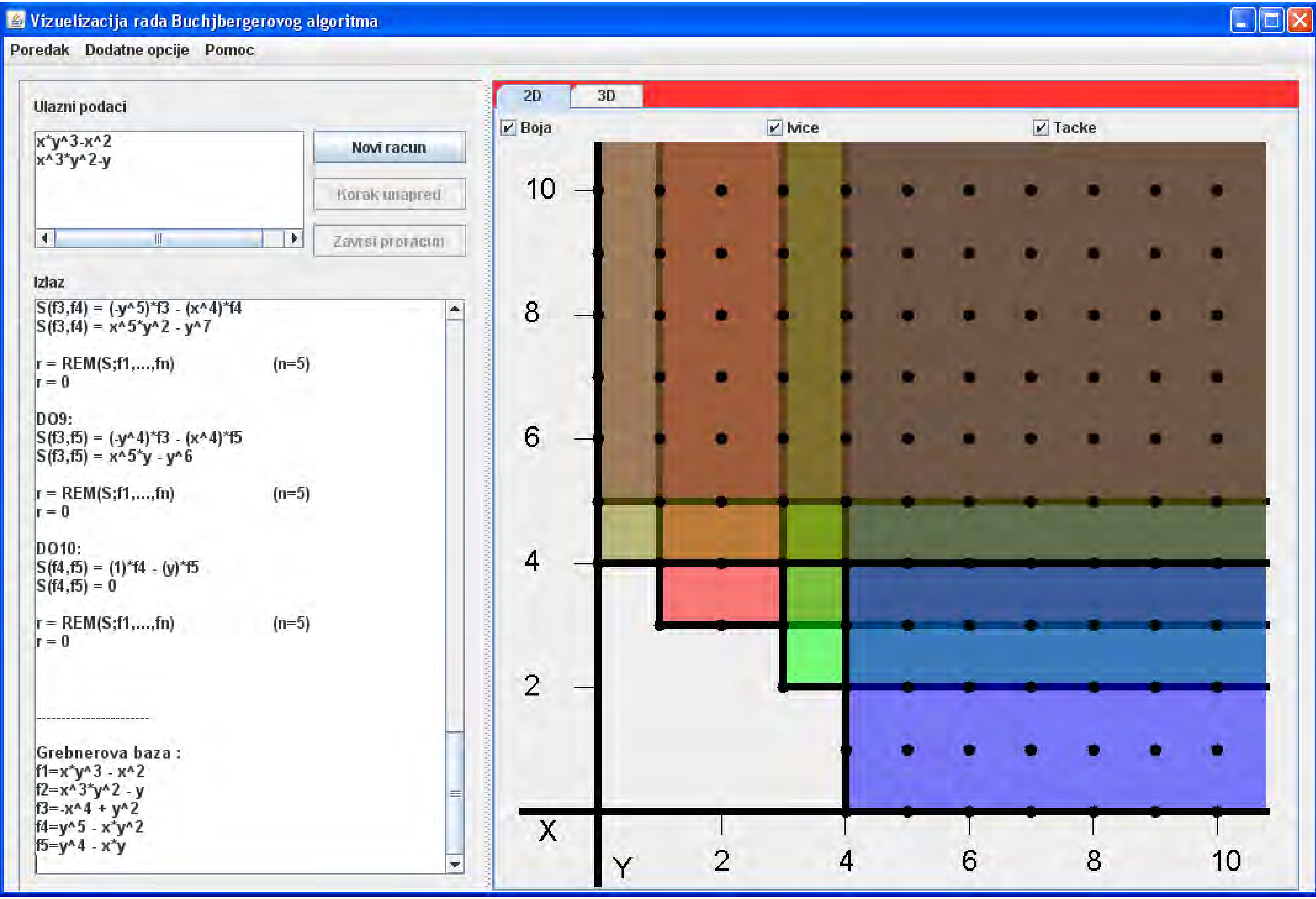
Vizuelizacija rada Buchbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
Izlaz
Deljenje polinoma
Deljenik
-1*x^2*z-y^3*z
Delioci
x^2*z-y^2
y*z^2+z
y-z
Novi racun
Korak unapred
Zavrsi proracun
Zatvori prozor
Tacke
-y^3*z - y^2 =h r=0
(-y^3*z - y^2):(x^2*z - y^2, y*z^2 + z, y - z)={-1, 0, -y^2*z}
-(-y^3*z + y^2*z^2)
-y^2*z^2 - y^2 =h r=0
(-y^2*z^2 - y^2):(x^2*z - y^2, y*z^2 + z, y - z)={-1, -y, -y^2*z}
-(-y^2*z^2 - y*z)
-y^2 + y*z =h r=0
(-y^2 + y*z):(x^2*z - y^2, y*z^2 + z, y - z)={-1, -y, -y^2*z - y}
-(-y^2 + y*z)
0 =h r=0
X
Y

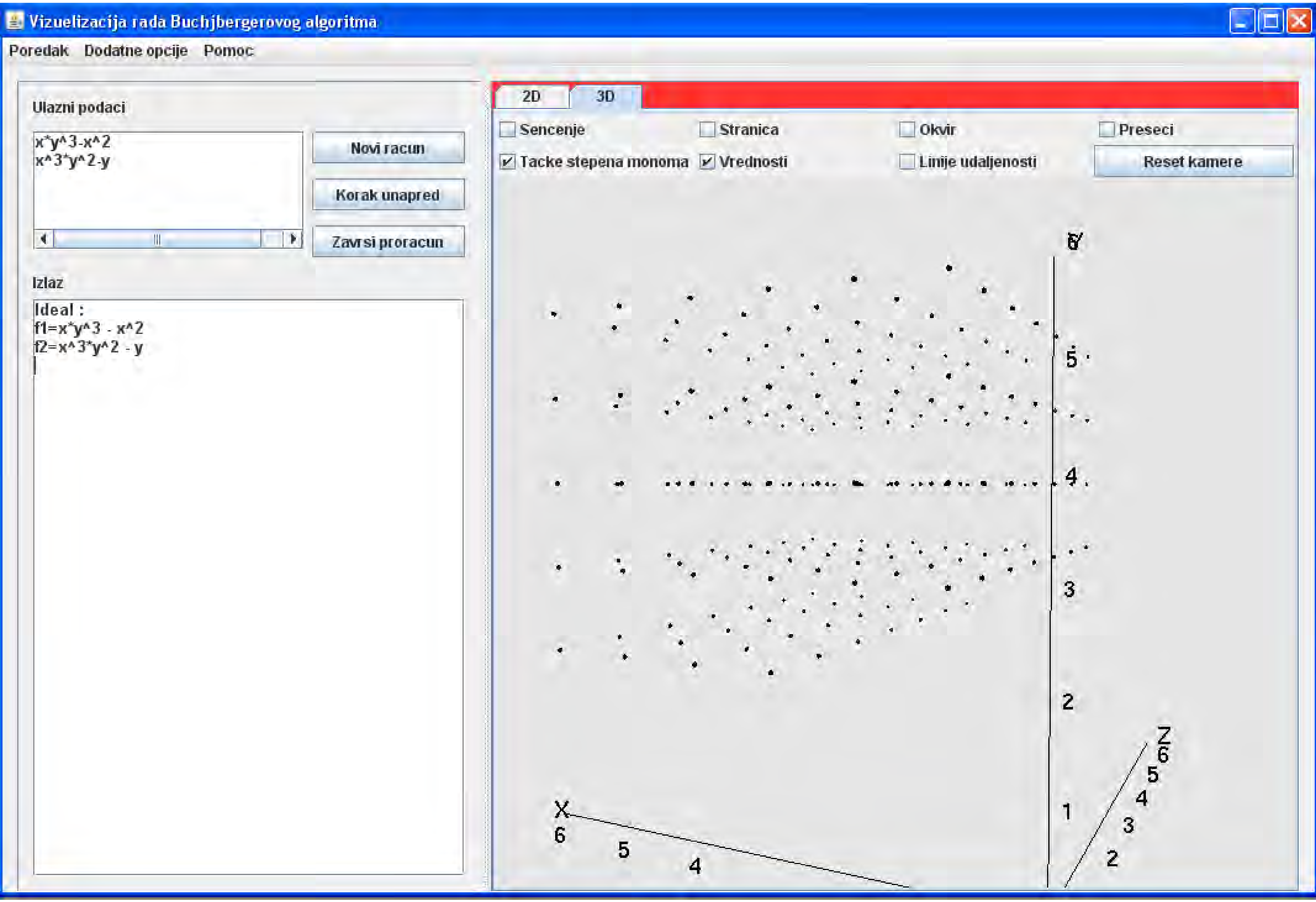

Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Leksikografski
Gradirani leksikografski
Obrnuto gradirano leksikografski
y^2+1
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
2D
3D
Boja
Ivice
Tacke
X
Y

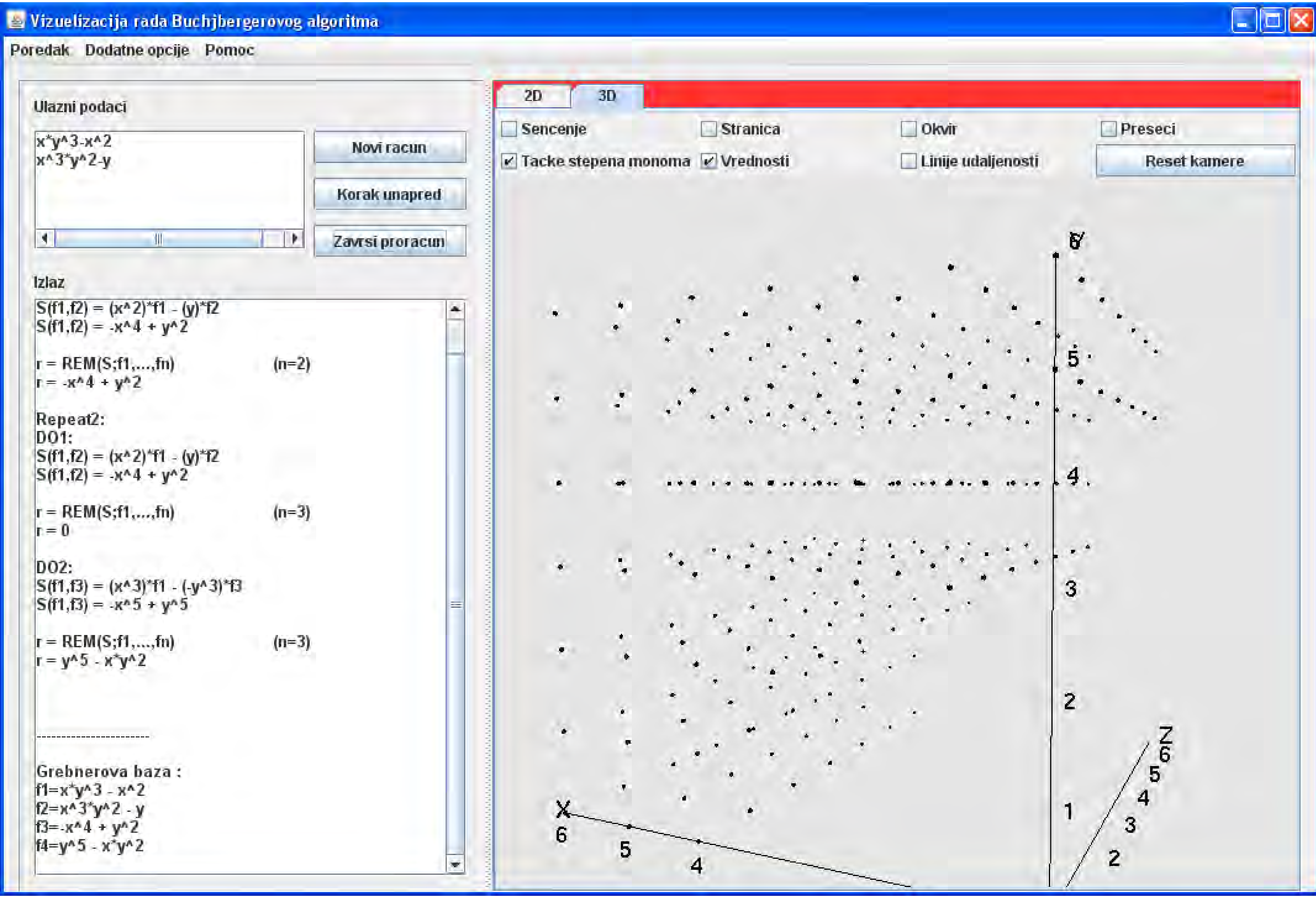

Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x*y^3-x^2
x^3*y^2-y
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
Ideal :
f1=x*y^3 - x^2
f2=x^3*y^2 - y
2D
3D
Boja
Ivice
Tacke
10
8
6
4
2
X
Y
2
4
6
8
10


Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x*y^3-x^2
x^3*y^2-y
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
S(f1,f2) = (x^2)*f1 - (y)*f2
S(f1,f2) = -x^4 + y^2
r = REM(S;f1,...,fn) (n=2)
r = -x^4 + y^2
Repeat2:
DO1:
S(f1,f2) = (x^2)*f1 - (y)*f2
S(f1,f2) = -x^4 + y^2
r = REM(S;f1,...,fn) (n=3)
r = 0
DO2:
S(f1,f3) = (x^3)*f1 - (-y^3)*f3
S(f1,f3) = -x^5 + y^5
r = REM(S;f1,...,fn) (n=3)
r = y^5 - x*y^2

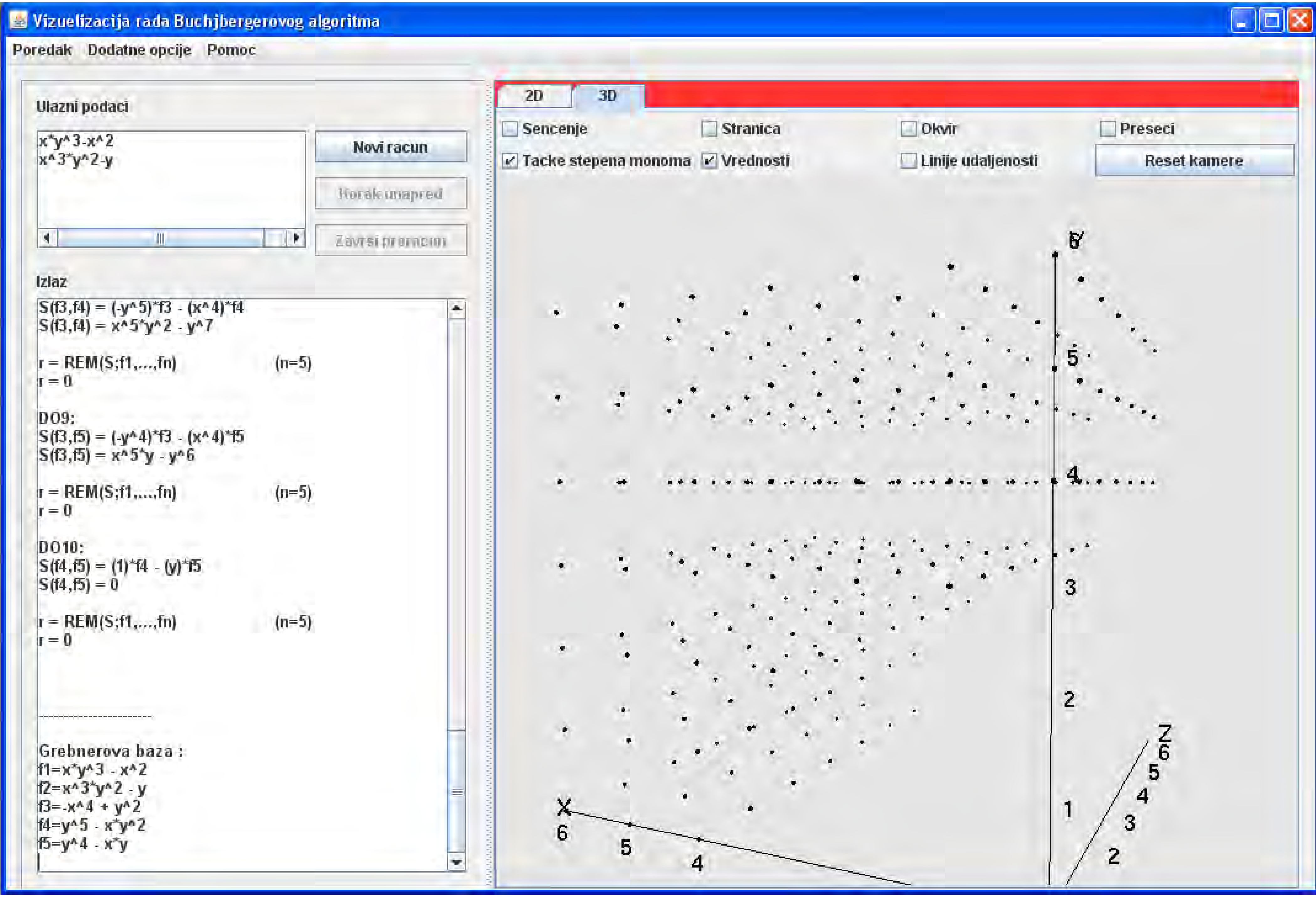
Grebnerova baza :
f1=x*y^3 - x^2
f2=x^3*y^2 - y
f3=-x^4 + y^2
f4=y^5 - x*y^2
2D
3D
Boja
Mice
Tacke
10
8
6
4
2
X
Y
2
4
6
8
10


Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x*y^3-x^2
x^3*y^2-y
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
S(f3,f4) = (-y^5)*f3 - (x^4)*f4
S(f3,f4) = x^5*y^2 - y^7
r = REM(S;f1,...,fn) (n=5)
r = 0
DO9:
S(f3,f5) = (-y^4)*f3 - (x^4)*f5
S(f3,f5) = x^5*y - y^6
r = REM(S;f1,...,fn) (n=5)
r = 0
DO10:
S(f4,f5) = (1)*f4 - (y)*f5
S(f4,f5) = 0
r = REM(S;f1,...,fn) (n=5)
r = 0
Grebnerova baza :
f1=x*y^3 - x^2
f2=x^3*y^2 - y
f3=-x^4 + y^2
f4=y^5 - x*y^2
f5=y^4 - x*y
2D
3D
Boja
Ivice
Tacke
10
8
6
4
2
X
Y
2
4
6
8
10

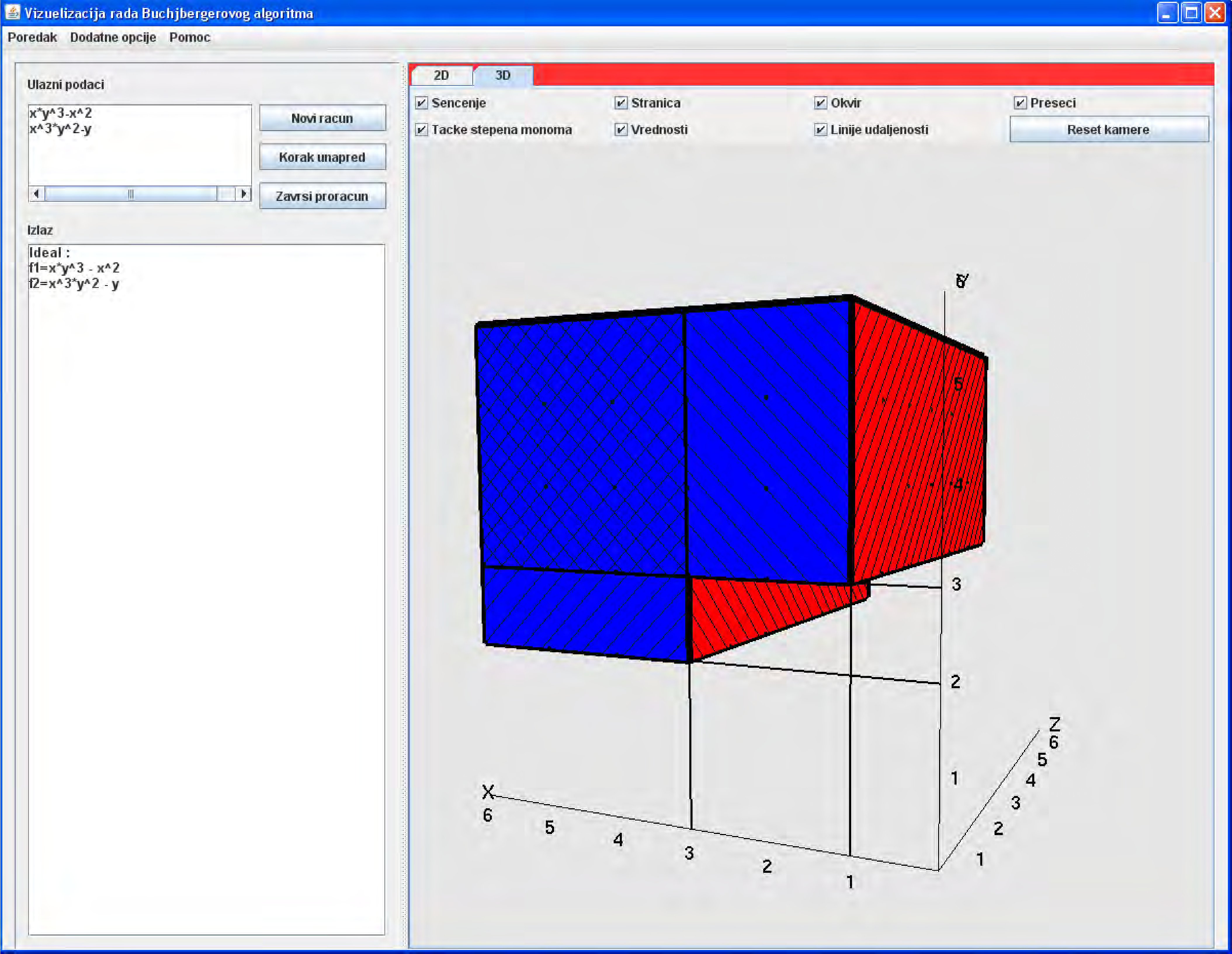

Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x*y^3-x^2
x^3*y^2-y
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
Ideal :
f1=x*y^3 - x^2
f2=x^3*y^2 - y
2D
3D
Sencenje
Stranica
Okvir
Preseci
Tacke stepena monoma
Vrednosti
Linije udaljenosti
Reset kamere
X
Y
Z

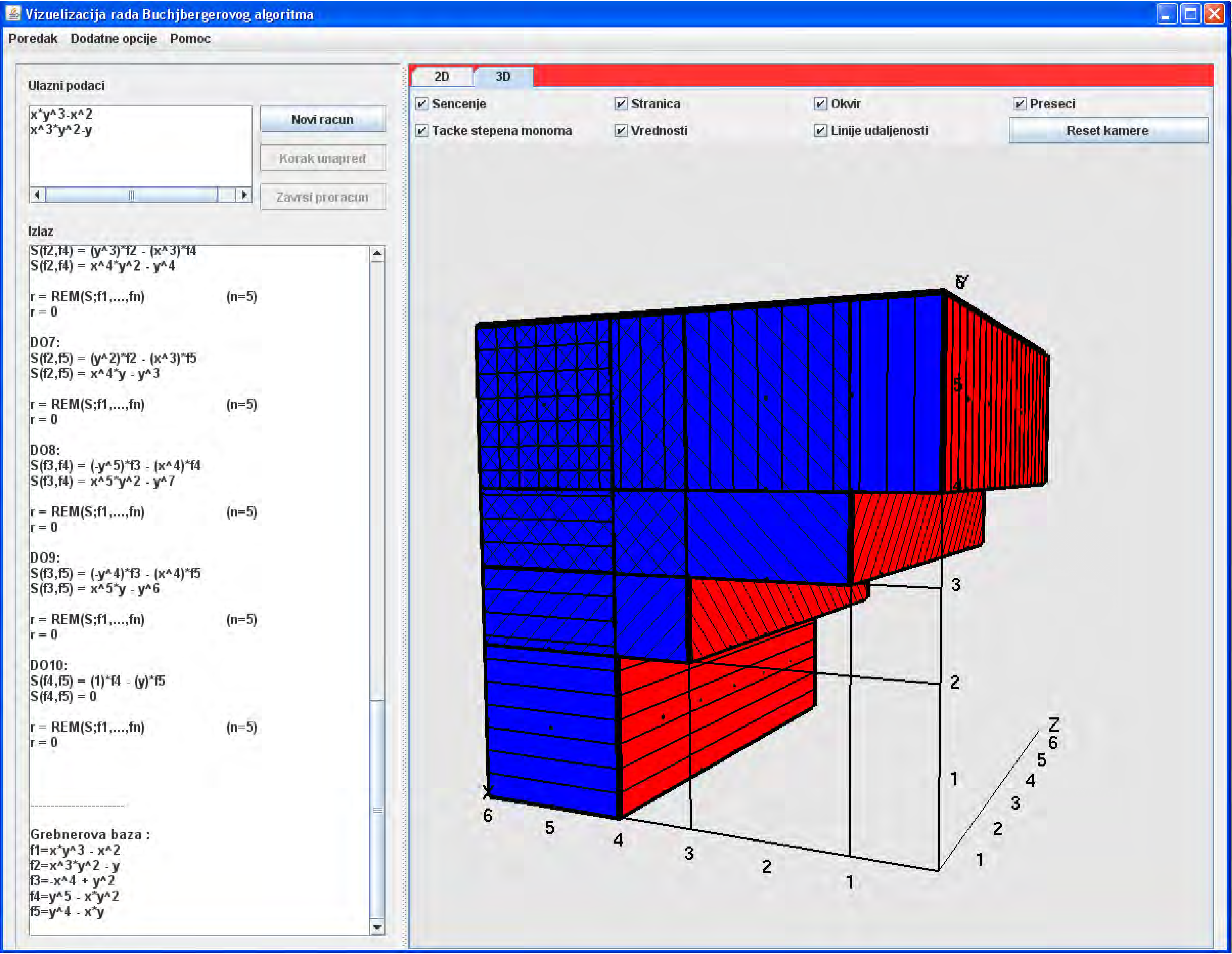

Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x*y^3-x^2
x^3*y^2-y
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
S(f1,f2) = (x^2)*f1 - (y)*f2
S(f1,f2) = -x^4 + y^2
r = REM(S;f1,...,fn) (n=2)
r = -x^4 + y^2
Repeat2:
DO1:
S(f1,f2) = (x^2)*f1 - (y)*f2
S(f1,f2) = -x^4 + y^2
r = REM(S;f1,...,fn) (n=3)
r = 0
DO2:
S(f1,f3) = (x^3)*f1 - (-y^3)*f3
S(f1,f3) = -x^5 + y^5
r = REM(S;f1,...,fn) (n=3)
r = y^5 - x*y^2

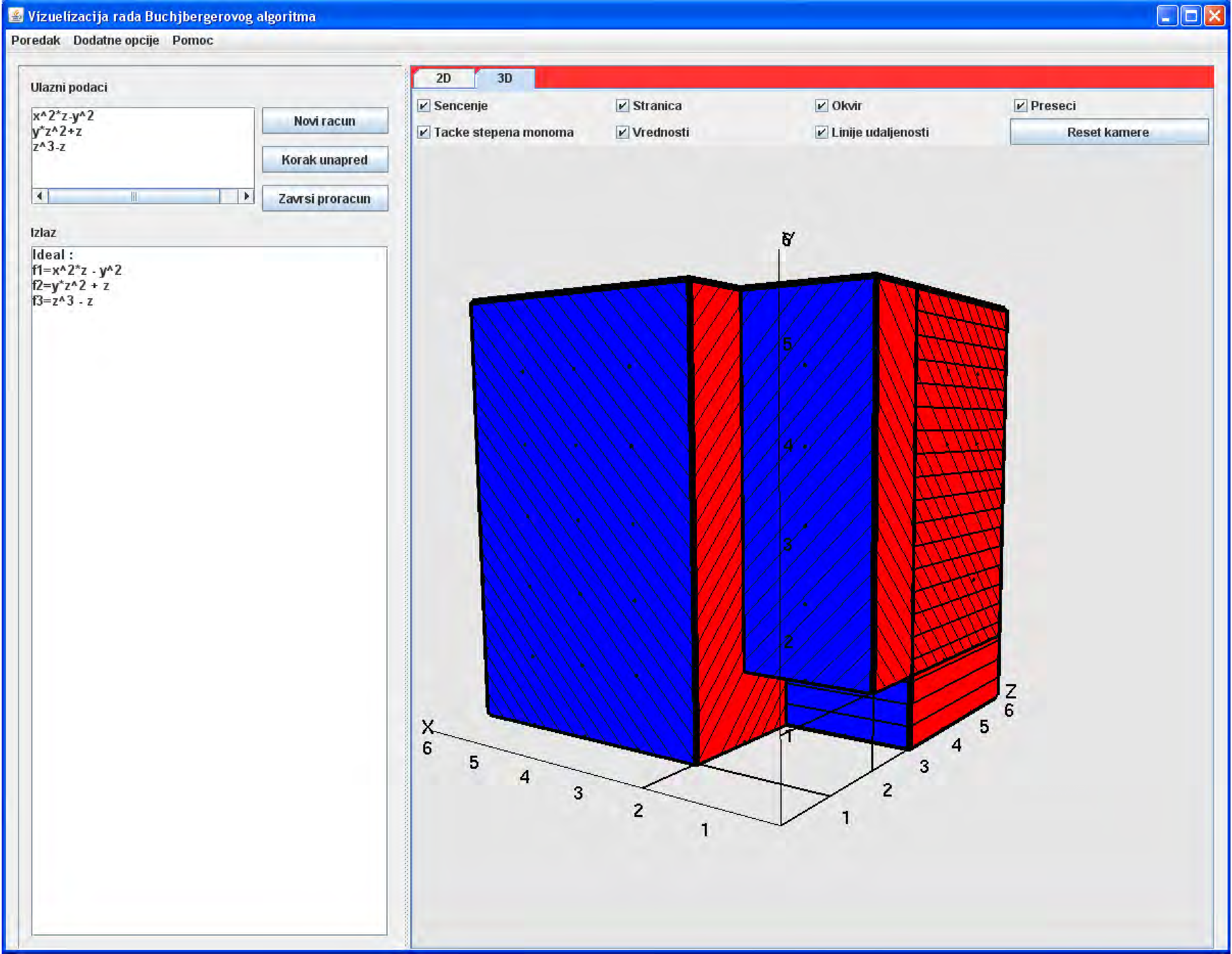
Grebnerova baza :
f1=x*y^3 - x^2
f2=x^3*y^2 - y
f3=-x^4 + y^2
f4=y^5 - x*y^2
2D
3D
Sencenje
Stranica
Okvir
Preseci
Tacke stepena monoma
Vrednosti
Linije udaljenosti
Reset kamere
X
Z

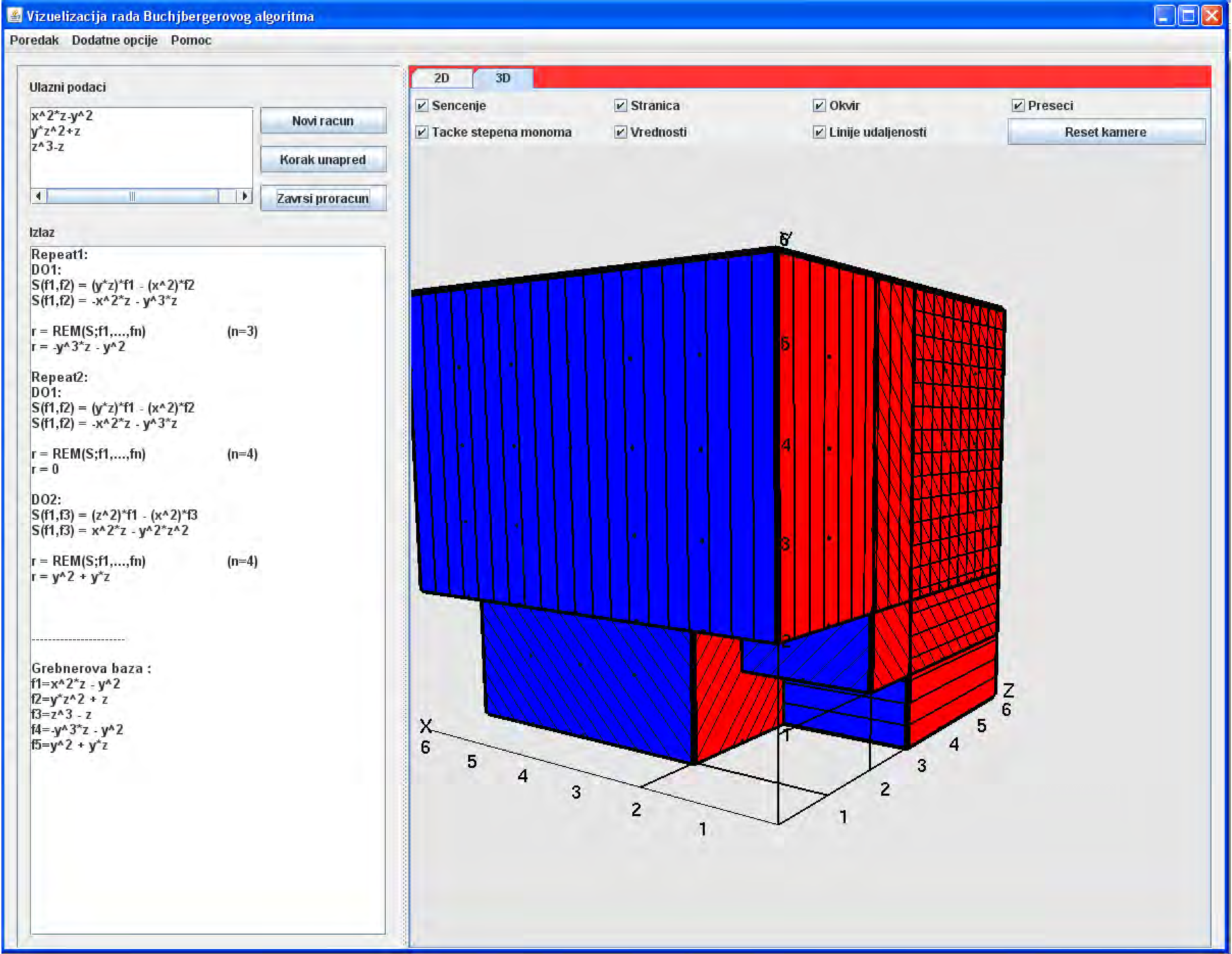

Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x*y^3-x^2
x^3*y^2-y
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
S(f3,f4) = (-y^5)*f3 - (x^4)*f4
S(f3,f4) = x^5*y^2 - y^7
r = REM(S;f1,....,fn) (n=5)
r = 0
DO9:
S(f3,f5) = (-y^4)*f3 - (x^4)*f5
S(f3,f5) = x^5*y - y^6
r = REM(S;f1,....,fn) (n=5)
r = 0
DO10:
S(f4,f5) = (1)*f4 - (y)*f5
S(f4,f5) = 0
r = REM(S;f1,....,fn) (n=5)
r = 0

Grebnerova baza :
f1=x*y^3 - x^2
f2=x^3*y^2 - y
f3=-x^4 + y^2
f4=y^5 - x*y^2
f5=y^4 - x*y
2D
3D
Sencenje
Stranica
Okvir
Preseci
Tacke stepena monoma
Vrednosti
Linije udaljenosti
Reset kamere
Y
6
5
4
3
2
1
X
6
5
4
Z
6
5
4
3
2

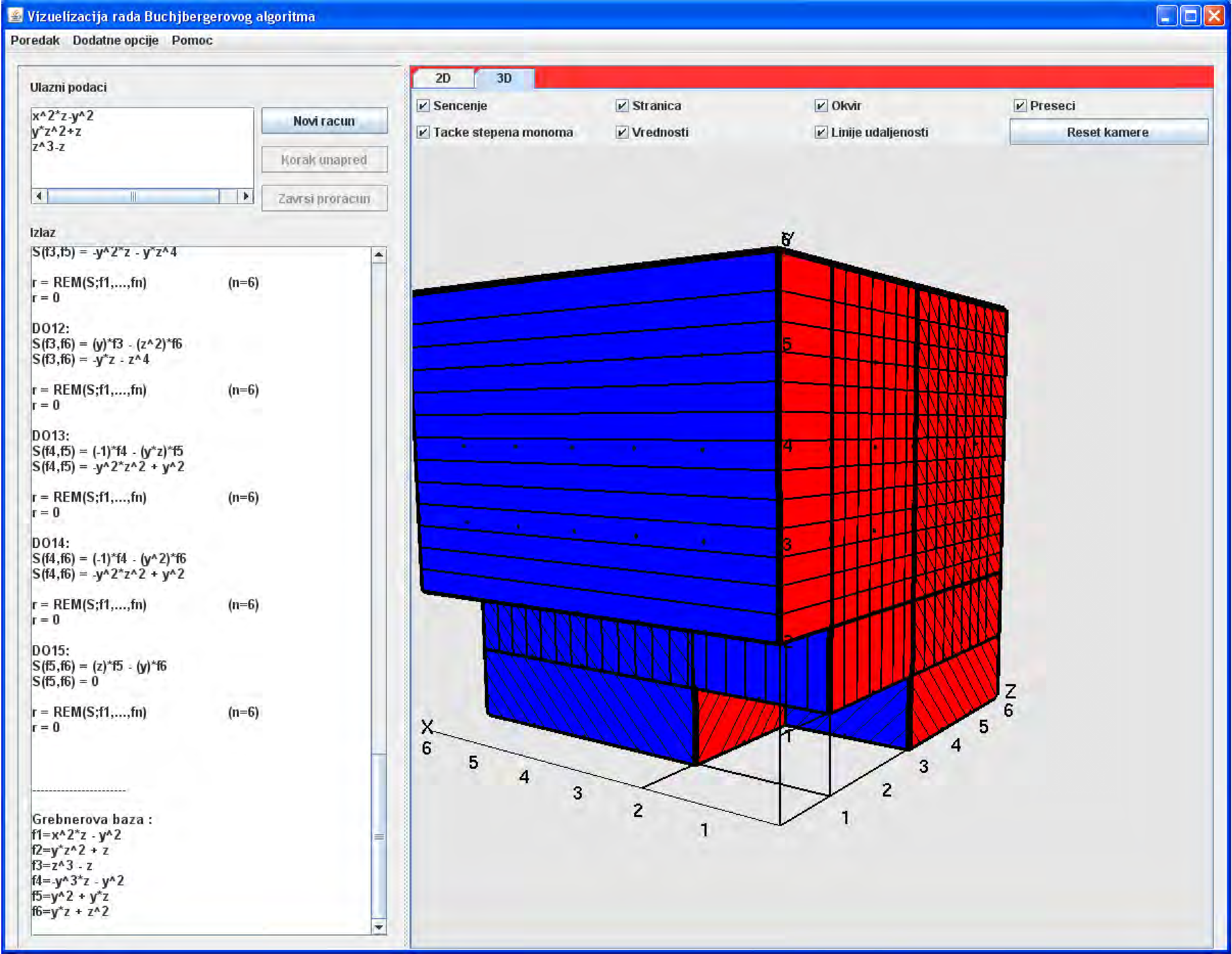

Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x*y^3-x^2
x^3*y^2-y
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
Ideal :
f1=x*y^3 - x^2
f2=x^3*y^2 - y
2D
3D
Sencenje
Stranica
Okvir
Preseci
Tacke stepena monoma
Vrednosti
Linije udaljenosti
Reset kamere
X
Z


Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x*y^3-x^2
x^3*y^2-y
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
S(f2,f4) = (y^3)*f2 - (x^3)*f4
S(f2,f4) = x^4*y^2 - y^4
r = REM(S;f1,...,fn) (n=5)
r = 0
DO7:
S(f2,f5) = (y^2)*f2 - (x^3)*f5
S(f2,f5) = x^4*y - y^3
r = REM(S;f1,...,fn) (n=5)
r = 0
DO8:
S(f3,f4) = (-y^5)*f3 - (x^4)*f4
S(f3,f4) = x^5*y^2 - y^7
r = REM(S;f1,...,fn) (n=5)
r = 0
DO9:
S(f3,f5) = (-y^4)*f3 - (x^4)*f5
S(f3,f5) = x^5*y - y^6
r = REM(S;f1,...,fn) (n=5)
r = 0
DO10:
S(f4,f5) = (1)*f4 - (y)*f5
S(f4,f5) = 0
r = REM(S;f1,...,fn) (n=5)
r = 0
Grebnerova baza :
f1=x*y^3 - x^2
f2=x^3*y^2 - y
f3=-x^4 + y^2
f4=y^5 - x*y^2
f5=y^4 - x*y
2D
3D
Sencenje
Stranica
Okvir
Preseci
Tacke stepena monoma
Vrednosti
Linije udaljenosti
Reset kamere
X
Y
Z
6
5
4
3
2
1


Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x^2*z-y^2
y*z^2+z
z^3-z
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
Ideal :
f1=x^2*z - y^2
f2=y*z^2 + z
f3=z^3 - z
2D
3D
Sencenje
Stranica
Okvir
Preseci
Tacke stepena monoma
Vrednosti
Linije udaljenosti
Reset kamere
X
Y
Z


Vizuelizacija rada Buchjbergerovog algoritma
Poredak
Dodatne opcije
Pomoc
Ulazni podaci
x^2*z-y^2
y*z^2+z
z^3-z
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
Repeat1:
DO1:
S(f1,f2) = (y*z)*f1 - (x^2)*f2
S(f1,f2) = -x^2*z - y^3*z
r = REM(S;f1,...,fn) (n=3)
r = -y^3*z - y^2
Repeat2:
DO1:
S(f1,f2) = (y*z)*f1 - (x^2)*f2
S(f1,f2) = -x^2*z - y^3*z
r = REM(S;f1,...,fn) (n=4)
r = 0
DO2:
S(f1,f3) = (z^2)*f1 - (x^2)*f3
S(f1,f3) = x^2*z - y^2*z^2
r = REM(S;f1,...,fn) (n=4)
r = y^2 + y*z

Grebnerova baza :
f1=x^2*z - y^2
f2=y*z^2 + z
f3=z^3 - z
f4=-y^3*z - y^2
f5=y^2 + y*z
2D
3D
Sencenje
Stranica
Okvir
Preseci
Tacke stepena monoma
Vrednosti
Linije udaljenosti
Reset kamere
X
Y
Z


Vizuelizacija rada Buchjbergerovog algoritma
Poredak Dodatne opcije Pomoc
Ulazni podaci
x^2*z-y^2
y*z^2+z
z^3-z
Novi racun
Korak unapred
Zavrsi proracun
Izlaz
S(f3,f5) = -y^2*z - y*z^4
r = REM(S;f1,...,fn) (n=6)
r = 0
DO12:
S(f3,f6) = (y)*f3 - (z^2)*f6
S(f3,f6) = -y*z - z^4
r = REM(S;f1,...,fn) (n=6)
r = 0
DO13:
S(f4,f5) = (-1)*f4 - (y*z)*f5
S(f4,f5) = -y^2*z^2 + y^2
r = REM(S;f1,...,fn) (n=6)
r = 0
DO14:
S(f4,f6) = (-1)*f4 - (y^2)*f6
S(f4,f6) = -y^2*z^2 + y^2
r = REM(S;f1,...,fn) (n=6)
r = 0
DO15:
S(f5,f6) = (z)*f5 - (y)*f6
S(f5,f6) = 0
r = REM(S;f1,...,fn) (n=6)
r = 0

Grebnerova baza :
f1=x^2*z - y^2
f2=y*z^2 + z
f3=z^3 - z
f4=-y^3*z - y^2
f5=y^2 + y*z
f6=y*z + z^2
2D
3D
Sencenje
Stranica
Okvir
Preseci
Tacke stepena monoma
Vrednosti
Linije udaljenosti
Reset kamere
X
Y
Z
6
5
4
3
2
1